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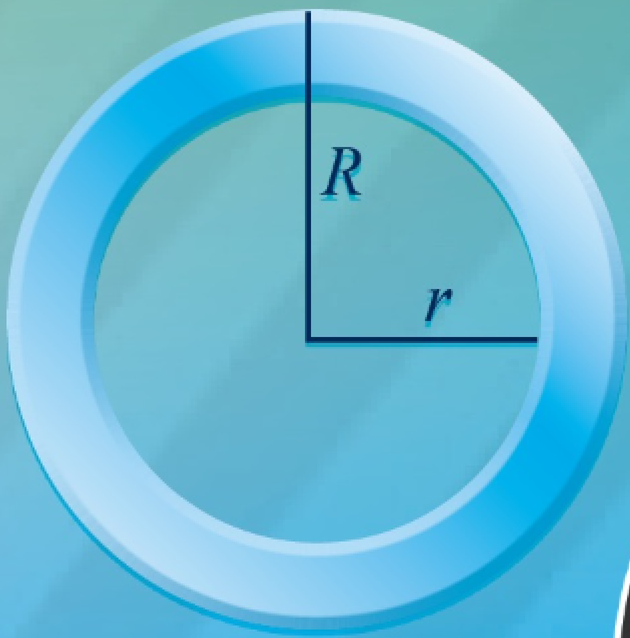

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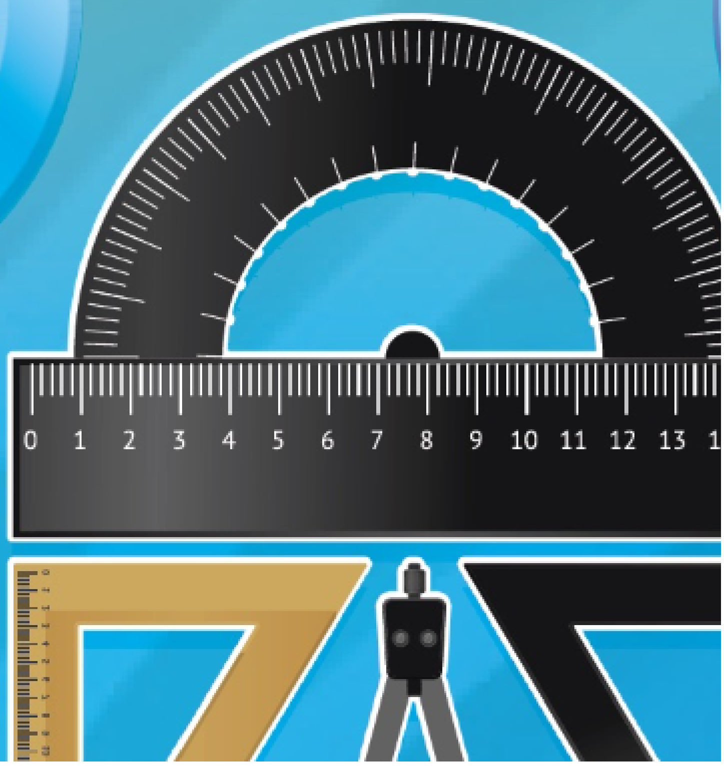

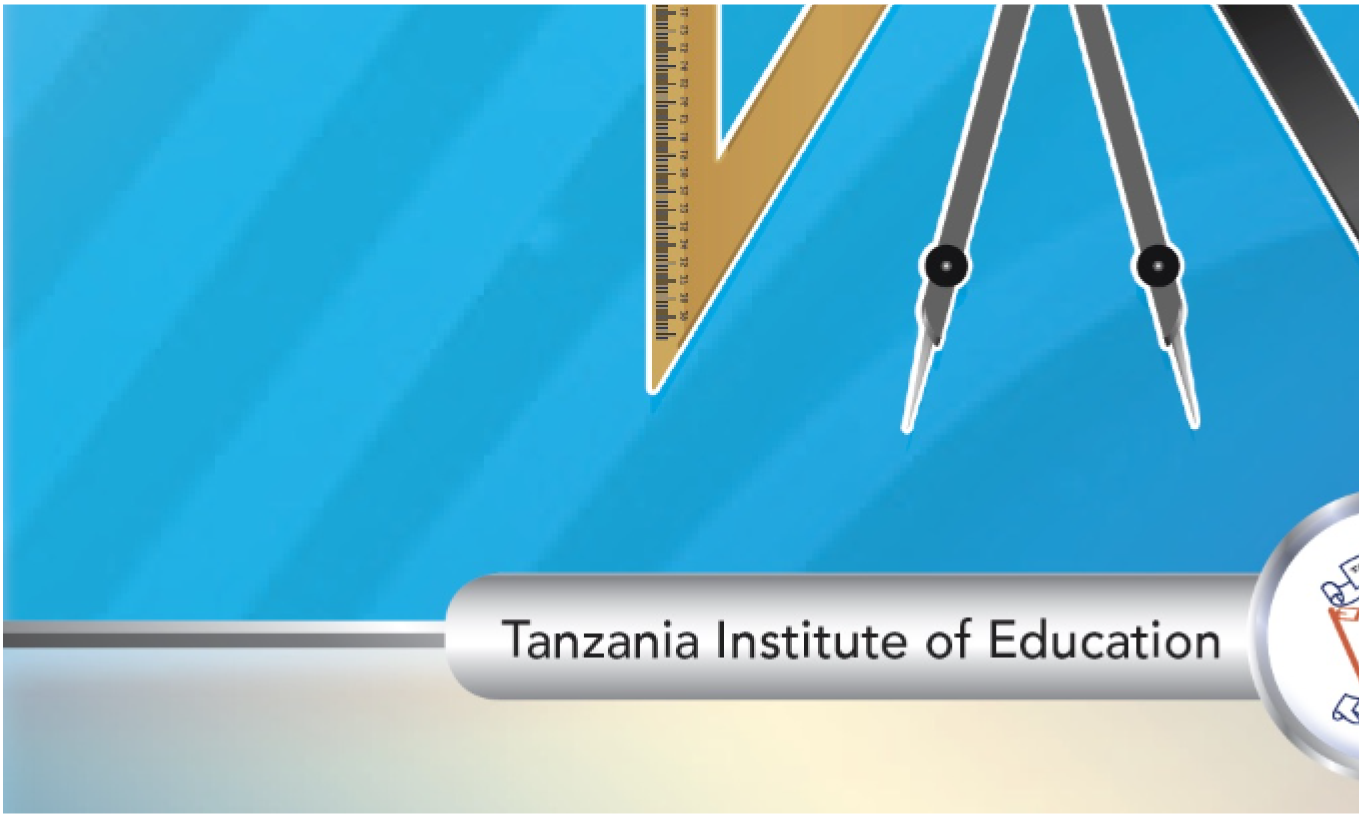
Tanzania Institute of Education

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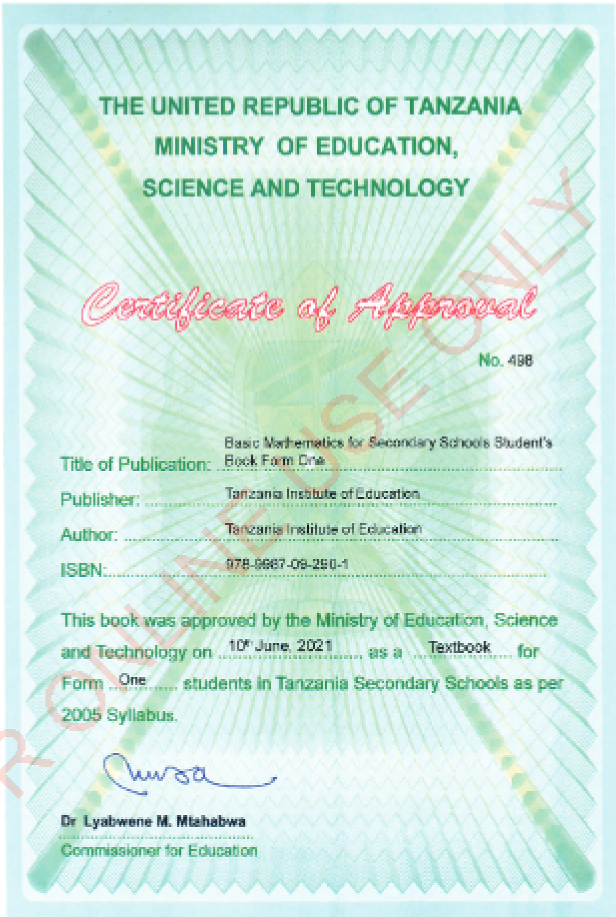

THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION,
SCIENCE AND TECHNOLOGY

Certificate of Approval

No. 498

Title of Publication: Basic Mathematics for Secondary Schools Student's Book Form One

Publisher: Tanzania Institute of Education

Author: Tanzania Institute of Education

ISBN: 978-9987-09-290-1

This book was approved by the Ministry of Education, Science and Technology on 10th June, 2021 as a Textbook for Form One students in Tanzania Secondary Schools as per 2005 Syllabus.

Dr Lyabwene M. Mtahabwa
Commissioner for Education

Tanzania Institute of Education

Published 2021

ISBN: 978-9987-09-290-1

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Table of Contents

Acknowledgements

The Tanzania Institute of Education (TIE) would like to aknowledge the contributions of all the organisations and individuals who participated in designing and developing this textbook. In particular, TIE wishes to thank the University of Dar es Salaam (UDSM), University of Dodoma (UDOM), Ardhi University (ARU), Dar es Salaam University College of Education (DUCE), Catholic University College of Mbeya (CUCoM), Sokoine University of Agriculture (SUA), and The State University of Zanzibar (SUZA), The National Examinations Council of Tanzania (NECTA), school quality assurance offices, teacher colleges, and secondary schools.

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TIE also appreciates secondary school teachers and students who participated in the trial phase of the manuscript. Likewise, the Institute would like to thank the Ministry of Education, Science and Technology for facilitating the writing and printing of this textbook.

Dr Aneth A. Komba
Director General
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Preface

This textbook, *Mathematics for Secondary Schools* is written specifically for Form One students in the United Republic of Tanzania. The book is prepared in accordance with the 2005 Basic Mathematics Syllabus for Ordinary Level Secondary Education Form I-IV, issued by the then, Ministry of Education and Vocational Training (MoEVT).

The book consists of twelve chapters, namely Concept of mathematics, Numbers I, Fractions, Decimals and percentages, Metric units, Approximations, Introduction to geometry, Algebra, Numbers II, Ratios, profit, and loss, Coordinate geometry as well as Perimeters and areas. In addition to the contents, each chapter contains activities, illustrations, and exercises. You are encouraged to do all the activities and attempt all questions in the exercises. This will enhance your understanding and development of the intended competencies for this level.

Tanzania Institute of Education

Concept of mathematics

Introduction

Mathematics is all around us. Almost everything we do involves mathematics. Mathematics is applied in various fields and disciplines such as science, engineering, social studies, economics, and many other applications. Numerous devices such as mobile phones, computers, television sets, and satellites are designed and manufactured on the basis of mathematical knowledge. In this chapter, you will learn about the meaning of mathematics, branches of mathematics, relationships between mathematics and other subjects, and importance of studying mathematics. The competencies developed through learning mathematics will enable you to apply mathematics knowledge and skills to solve daily life problems related to various things including food, soil, sports and games, saving money, and entrepreneurship.

Meaning of mathematics

The word 'mathematics' comes from the Greek word '*mathema*', meaning 'which is learnt' or 'science, knowledge or learning'. Numbers, measurements, shapes of physical objects, and equations form a small part of it. Mathematics can be thought of as the science of the structures, orders, patterns, and relations that has evolved from elementary practices of counting, measuring, and describing the shapes of objects. Mathematics can also be thought of as a language of science because by using mathematical reasoning, one can develop an insight and be able to predict nature. It has the ability to provide precise expression for every concept that can be formulated using mathematical symbols and structures.

Mathematics is termed as the 'Queen of science', 'the science of all sciences', and 'the art of all arts'. The knowledge and skills of mathematics play a crucial role in understanding the contents of other subjects, both sciences and arts.

Branches of mathematics

Mathematics can be categorized into different branches depending on the level of the learners. At a lower secondary level, the following are some of the branches of mathematics:

(a) Arithmetic
(b) Algebra
(c) Geometry

(a) Arithmetic

Arithmetic is a branch of mathematics that deals with properties and manipulations of numbers. Manipulation of numbers is achieved through the use of basic mathematical operations, namely: addition, subtraction, multiplication, and division.

(b) Algebra

Algebra is a branch of mathematics in which arithmetic operations are applied to symbols rather than specific numbers. The symbols or letters in algebra represent quantities with no fixed values, commonly known as variables.

(c) Geometry

The word geometry was derived from the Greek word 'Geo', which means 'earth' and 'metry', which means 'measurement'. Geometry is, therefore, a branch of mathematics which deals with the study of the sizes, shapes, positions, angles, and dimensions of different physical objects. In geometry, properties of points, lines, planes, similarities, congruence, and shapes of different regular objects are studied.

Activity: The concept of mathematics in daily life

Perform the following tasks individually or in groups:

1. Identify ten activities which use the concepts of mathematics in daily life.
2. Identify any ten daily life activities which do not involve mathematics.
3. Write T for a true statement and F for a false statement:
   (a) Some activities like cooking do not involve mathematics concepts.
   (b) When you want to cross a road, you have to do some mathematics.
   (c) Language subjects do not need mathematical knowledge at all.

Relationship between mathematics and other subjects

Mathematics is related to other subjects in a very unique way. The relationship between Mathematics and other subjects is briefly discussed below:

Mathematics and agriculture

Agriculture is closely related to mathematics. For example; when farmers want to buy seeds, they need to understand the ratio of seeds that is sufficient per piece of land. Similarly, the determination of the number of bags of fertilizers needed per acre requires one to do some calculations. In these two examples, mathematics enables farmers to avoid wastage of financial resources by purchasing only the required amount of inputs. Mathematics is also used in agriculture to determine the suitable amount of water to be used in irrigation, and the spacing between seedlings to get optimal yields. Similarly, mathematics is used to determine the investment, expenditure, and savings in cultivating a specific crop, dividing a piece of land, calculating the cost of labour, and so forth.

Mathematics and biology

There is a direct relationship between mathematics and biology. For example; normal animal weights, calorific value, rate of respiration, nutritive values of food, and transpiration are few quantities in biology that can be calculated using mathematical concepts. Mathematics can also be applied to estimate the number of blood cells present in the body, measurement of blood pressure, and counting sex chromosomes, among many others.

Mathematics in business and commerce

Mathematics lies at the heart of business and commerce as all the commercial principles depend on the understanding of the ways numbers work, how they interact with reality, and how certain equations would normally have simple solutions. If a business person wants to succeed in any business, he or she must understand the mathematics behind the investment he or she wants to make. All banks and other financial institutions operate using some well-established models which primarily involve the concepts of mathematics.

Mathematics and chemistry

With the concepts and skills from mathematics, the molecular weights of organic compounds can be calculated. Mathematics is also used to measure the constituents of mixtures and chemical compounds, calculate empirical or molecular formulas, balance chemical equations and electronic configurations of atoms of elements, and many others.

Mathematics and geography

Geography requires mathematical calculations to find the distance from one place to another, to find gradients, and to find altitudes of hills and mountains. Through mathematical calculations, the layout of areas can be predicted. Geographical locations of different places are determined using latitudes and longitudes, which can be measured and manipulated using the concepts of mathematics. There are so many other concepts in geography that need mathematics for easy understanding and computation.

Mathematics and history

In history, mathematics helps in determining and calculating historical dates. For example; the duration of different colonial rules in Tanganyika and Zanzibar, and the time taken for the first and the second World wars. Also, mathematics is used in history to determine or estimate dates and ages of fossils by using some mathematical principles, such as carbon-14.

Mathematics in information and communications technology

The information and communications technology (ICT) is strongly related with mathematics. Computer programs, application, softwares, and different computer languages are impossible without mathematics. All the computer and mobile parts are assembled using advanced mathematics techniques.

Mathematics and literature

Literature might be seen as a far cry from mathematics. But, mastering basic arithmetic can enable a student to better understand fiction. For instance, determining the number of words per sentence, the number of sentences per paragraph, and the number of paragraphs per page require the knowledge of mathematics. The linear and logical thinking used in mathematical problems can also help a student to write more clearly and logically.

Mathematics and music

In music, mathematics can be used to develop, express and communicate ideas. Mathematics can help to explain how strings vibrate at certain frequencies, and how sound waves are produced to give desired beats.

Mathematics and physics

Physics involves the study of laws, principles, and theorems which are expressed by mathematical formulas. In order to understand how to apply the formulas to solve some physical problems, the knowledge of mathematics is needed. All the quantities in physics are expressed in numbers, and units which have to be manipulated using the concepts and skills of mathematics. For example; to determine the velocity of a moving object, one has to measure the units of displacement and time, by using the knowledge of mathematics. Also, the understanding of complex numbers is a pre-requisite to learn many concepts in electronics.

Mathematics in sports and games

Mathematics is important in all forms of competitive sports and games organized to improve physical abilities and skills while providing entertainment to participants and fans. Examples of such sports and games include football, netball, playing cards, athletics, and golf. The winners are determined by counting the scores written in numbers. Also, the positions of players in some games such as football are determined by numbers.

Importance of mathematics

The following are some importance of mathematics:

(i) Mathematics for brain development

Mathematics provides an effective way of building mental discipline and encouraging logical reasoning and mental vigour. It is scientifically proved that, students who solve mathematics problems in their daily life have higher logical thinking skills than those who do not solve mathematics problems.

(ii) Mathematics in finance

Using mathematics, it is easy to make financial budget by calculating how much money you have and how much you can spend. You can also calculate expenses, profit, loss, and loans.

(iii) Better problem-solving skills

Problem-solving is one of the most important skills in life. Mathematics is one of the most effective ways to increase analytical and logical thinking, which helps us to become better problem solvers.

(iv) Mathematics in food and human nutrition

In food and human nutrition, mathematics can be used to estimate the quantity of food varieties to be consumed for a proper balanced diet. With the concept of mathematics, we can determine the amount of food required daily in order to get enough fats, proteins, carbohydrates, and vitamins, among many other nutrients. Mathematics can also be used to determine the amount of each ingredient required in a meal.

(v) Professions and occupations

In most careers, employers want to employ people who can solve complex problems. If someone is good at mathematics and has a keen ability to solve complex problems, he or she is at a better chance of being employed for many good jobs ranging from engineers, medical doctors, bankers, scientists, and economists, to mention a few.

(vi) Time management

Time management is a key to success for everyone. Therefore, we have to be more careful in time management. Mathematics helps us to calculate the time that we spend in every activity so as to make wise decision on how we can manage time effectively.

(vii) Money saving

Mathematics helps us to calculate how much money we can lose by buying some stuff or how much money we can save by avoiding buying unnecessary stuff. To become a successful business person, one has to take and calculate financial risks before investing money in a certain project.

(viii) Entrepreneurship

Mathematics skills encourage problem-solving, self-reliance, and empowering individuals to solve their problems so as to enhance entrepreneurial skills. For example; people who deal with financial transactions and money transfer need mathematical skills. These include people who work in banks, mobile money transactions, kiosks, stores and other businesses.

Chapter Two

Numbers I

Introduction

In the early stages, numbers were introduced to refer to objects, symbols as well as words. In this chapter, you will learn about base-ten numeration, natural, and whole numbers. You will also learn about operations on whole numbers, factors and multiples of whole numbers, and integers. The competencies developed in this chapter can be applied to count and measure in a variety of situations. For example; a carpenter uses numbers in measuring lengths of the pieces of wood while a class teacher uses numbers to process examination results and attendance of students in the class. Furthermore, the competencies developed will enable you to label and order objects.

Base ten numeration

Base ten numeration or decimal system of numeration is the system where a group of ten symbols which are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are used to represent numbers. These symbols are called numerals. A single symbol in numeral form is called a digit. A numeral is formed by one or more digits. For example; the number 46 468 408 has digits 0, 4, 6, and 8. There are only ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 which can be used to represent any number. The numeral for the number ten is 10.

Place value of a digit in a number

Each digit in a number has a different value depending on its position in that number. This position is called the place value of a digit. The place value of a digit in numerals which is used for counting, including zero, starts with a small value in position from the right of a numeral to a large value in position to the left of a numeral.

The values in position start with ones, tens, hundreds, thousands, ten thousands and so on.

Consider the number 154 396 728. We can write this numeral as;

$$154\,396\,728 = 1\times100\,000\,000 + 5\times10\,000\,000 + 4\times1\,000\,000 + 3\times100\,000 + 9\times10\,000 + 6\times1\,000 + 7\times100 + 2\times10 + 8\times1.$$

This is called the expanded form of the number 154 396 728.

The place values of the digits in the number 154 396 728 are as follows:

1 is in hundred millions
5 is in ten millions
4 is in millions
3 is in hundred thousands
9 is in ten thousands
6 is in thousands
7 is in hundreds
2 is in tens
8 is in ones

Example 1

Write 987 543 124 in expanded form.

Solution

$$987\,543\,124 = 9\times100\,000\,000 + 8\times10\,000\,000 + 7\times1\,000\,000 + 5\times100\,000 + 4\times10\,000 + 3\times1\,000 + 1\times100 + 2\times10 + 4\times1.$$

Example 2

Write 700 000 000 + 80 000 000 + 4 000 000 + 900 000 + 50 000 + 2 000 + 600 + 10 + 3 in short form.

Answer

700 000 000 + 80 000 000 + 4 000 000 + 900 000 + 50 000 + 2 000 + 600 + 10 + 3 = 784 952 613.

Exercise 1

1. Write each of the following numbers in expanded form:
   (a) 285 176 932 (b) 862 554 917 (c) 306 940 681

2. For each of the following numbers, write the place value of the digit in the brackets. For example; 762 891 016; (9) Ten thousands.
   (a) 281 724 956; (8) (c) 978 152 347; (9)
   (b) 190 172 865; (7) (d) 462 587 913; (5)

3. Write the following numbers in short form:
   (a) $9 \times 100\,000\,000 + 7 \times 10\,000\,000 + 8 \times 1\,000\,000 + 2 \times 100\,000 + 4 \times 10\,000 + 1 \times 1\,000 + 7 \times 100 + 6 \times 10 + 5 \times 1$
   (b) $500\,000\,000 + 40\,000\,000 + 9\,000\,000 + 800\,000 + 40\,000 + 7\,000 + 500 + 40 + 9$
   (c) $200\,000\,000 + 0 + 5\,000\,000 + 0 + 80\,000 + 0 + 400 + 90 + 5$

4. Write the following numbers in expanded form:
   (a) 254 946 104 (c) 368 100 097
   (b) 954 625 817 (d) 755 556 156

5. Write the following numbers in short form:
   (a) $700\,000\,000 + 40\,000\,000 + 8\,000\,000 + 400\,000 + 70\,000 + 9\,000 + 700 + 60 + 8$
   (b) $900\,000\,000 + 0 + 7\,000\,000 + 0 + 90\,000 + 8\,000 + 0 + 50 + 4$
   (c) $400\,000\,000 + 80\,000\,000 + 0 + 900\,000 + 90\,000 + 9\,000 + 900 + 0 + 9$

Total value

The product of the place value and its digit is called total value. When a number is written in an expanded form, the total value of each digit is given. For example; the expanded form, place value, digit, and total value of a numeral 517 029 864 is as shown in the following table.

Digit	Place value	Expanded form	Total value
5	Hundred millions	5 × 100 000 000	500 000 000
1	Ten millions	1 × 10 000 000	10 000 000
7	Millions	7 × 1 000 000	7 000 000
0	Hundred thousands	0 × 100 000	0
2	Ten thousands	2 × 10 000	20 000
9	Thousands	9 × 1 000	9 000
8	Hundreds	8 × 100	800
6	Tens	6 × 10	60
4	Ones	4 × 1	4

The position of each digit determines the total value of that digit in the number.

Note that: The total value of a digit in a number increases from the right-hand side to the left-hand side of the number. Numerals with more than three digits can be written by grouping in threes from the right-hand side. For example; 982406215 can be written as 982 406 215. A space separates the groups.

Example 1

Write the total value of the underlined digit in each of the following numbers:

(a) 758 629 142 (b) 916 207 158

Answer

(a) The total value of 8 in 758 629 142 is 8 000 000.

(b) The total value of 2 in 916 207 158 is 200 000.

Example 2

Construct a table of place values and total values of 300 469 837.

Solution

Place value	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
Digit	3	0	0	4	6	9	8	3	7
Total value	300 000 000	0	0	400 000	60000	9 000	800	30	7

Writing numbers in words and in numerals

It is simple to write a number in words if you know the place value of each digit of the given number in numerals. A number in numerals shows all the digits forming the number and their place values. For example; number 3 041, can be written in words as three thousand and forty-one.

The following table shows numbers in words and in numerals:

Numbers in words	Numbers in numerals
Forty-six million four hundred sixty-eight thousand four hundred and eight	46 468 408
Eight hundred forty-six million six hundred ninety thousand four hundred and forty-five	846 690 445

Example 1

Write the following numbers in numerals:

(a) Four hundred thirty-two million two hundred thirty-five thousand three hundred and fifty-seven.

(b) Six hundred forty-five million five hundred twenty-four thousand eight hundred and thirty-two.

(c) One billion.

Answer

(a) 432 235 357 (b) 645 524 832 (c) 1 000 000 000

Example 2

Write the following numbers in words:

(a) 281 423 865 (b) 230 403 104

Answer

(a) 281 423 865 in words is two hundred eighty-one million four hundred twenty-three thousand eight hundred and sixty-five.

(b) 230 403 104 in words is two hundred thirty million four hundred three thousand one hundred and four.

Exercise 2

1. Write the following numerals in words:
   (a) 672 190 854 (c) 68 775 146 (e) 415 982 704
   (b) 230 403 204 (d) 80 690 400 (f) 900 000 000

2. Write the total value of the underlined digit in each of the following numbers:
   (a) 465 $\underline{9}$57 164 (c) 104 72$\underline{5}$ 098
   (b) 6$\underline{8}$4 312 506 (d) 117 948 $\underline{2}$45

3. Write down the largest four-digit number.
4. Write down the largest four-digit number when the digits are not repeating.
5. Write down the smallest three-digit number without using a zero.
6. Change the order of the digits in 47 986 to make:
   (a) The largest possible number.
   (b) The smallest possible number.

7. Write the following numbers in words and in numerals:
   (a) Smallest number of six digits
   (b) Largest number of six digits
   (c) Smallest number of nine digits
   (d) Largest number of nine digits

8. Write the place value of the underlined digit in each of the following numbers:
   (a) 264 1$\underline{8}$2 911 (c) 142 914 $\underline{6}$28
   (b) 300 $\underline{6}$24 945 (d) $\underline{8}$17 216 125

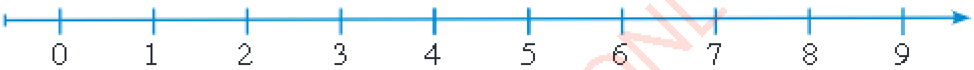
9. Write the following numbers in numerals:
   (a) Three hundred forty-eight million seven hundred forty thousand eight hundred and thirty.
   (b) Nine hundred five million eight hundred ninety-nine thousand five hundred and seventy-two.
   (c) Three hundred forty-six million eight hundred fifty thousand eight hundred and forty-seven.
   (d) Forty-nine million two hundred six thousand and fifty-one.

Natural and whole numbers

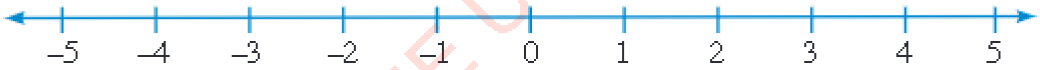
Natural numbers are part of the number system which include all positive numbers from 1 and increase by one to infinity. These numbers are also used for counting. They do not include zero (0). Therefore, 1, 2, 3, 4, 5, 6, 7, 8, 9, . . . are called natural numbers or counting numbers, and they are denoted by the symbol ℕ. When these numbers are shown on a horizontal number line they appear as shown in the following figure.

This line is called a ray. Natural numbers are represented by points 1, 2, 3, . . . on the number line. When zero is included in the set of natural numbers, another set of numbers 0, 1, 2, 3, . . . called whole numbers is formed. Whole numbers are denoted by the symbol 𝕎. Therefore, the set of natural numbers ℕ is 1, 2, 3, . . . and the set of whole numbers 𝕎 is 0, 1, 2, 3, 4, . . .

Whole numbers can be shown on a ray as shown in the following figure.



A number line is a line consisting of negative and positive numbers including zero. Numbers can be represented on a number line as follows.



Note:

(i) A ray is a part of a line which extends without an end in one direction only.

(ii) A number line is a line which extends without an end in both directions.

(iii) Every natural number is a whole number; 0 is a whole number, but it is not a natural number.

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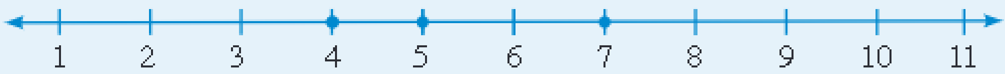
Example

Show the following numbers on the number line: 4, 5, 7.

Solution



Even, odd, and prime numbers

An even number is a number which is divisible by 2. For example; 2, 4, 6, 8, . . . are even numbers. All even numbers end with any of the digits 0, 2, 4, 6, or 8. An odd number is a number which is not divisible by 2. For example; 1, 3, 5, 7, 9, . . . are odd numbers. All odd numbers end with any of the numbers 1, 3, 5, 7, or 9. A prime number is a natural number which is divisible by itself and one only. For example; 2, 3, 5, 7, 11, . . . are prime numbers.

Note: Number one (1) is not a prime number because it is divisible by one only. Number two (2) is both even and prime.

Exercise 3

1. Consider the following list of numbers:
   2, 9, 15, 17, 25, 34, 36, 37, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70.
   From the given list, write the numbers which are:
   (a) Even (b) Prime (c) Odd
2. Write the prime numbers between 70 and 90.
3. Write a number which is both even and prime.
4. Write all odd numbers between 140 and 160.
5. Show even, odd, and prime numbers less than 10 on separate number lines.
6. Among the numbers 3, 5, 7, 9, 11, 13 which number is not prime?

Operations on whole numbers

There are four operations that can be performed on whole numbers. These are: addition (+), subtraction (–), multiplication (×), and division (÷).

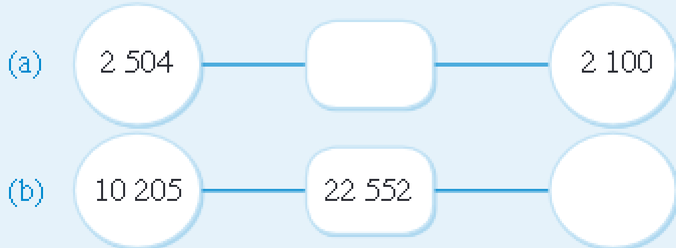
Addition and subtraction of whole numbers

Activity: Recognising addition and subtraction of whole numbers

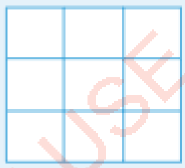
Individually or in groups, perform the following tasks:

In the diagram above, the number in the box equals the sum of the two numbers in the circles on both sides.

1. Complete the following questions in the same way:



2. Arrange the whole numbers from 1 to 9 in the following 3×3 grid so that each row, column, or diagonal adds up to the same total.



Addition of whole numbers

When two or more numbers are added, a sum is obtained. The symbol used for addition is '+', and is read as plus. Addition can be done by arranging the numbers horizontally or vertically. Recall that, in order to add two or more numbers you have to consider the place value of each digit in the given numbers. Also, you can add whole numbers by regrouping or without regrouping. When adding whole numbers, start by adding ones, followed by tens, hundreds up to the highest place value of the given numbers.

Addition of whole numbers arranged horizontally

Example

Find the value of each of the following:

(a) 432 651 987 + 565 346 012 = (b) 627 582 174 + 281 629 728 =

Solution

(a) 432 651 987 + 565 346 012 = 997 997 999

(b) 627 582 174 + 281 629 728 = 909 211 902

Addition of whole numbers arranged vertically

When adding numbers, align the numbers vertically by considering the place values of each digit in each number.

Example

Find the value of each of the following:

(a)
```
   234 165 981
 + 453 813 017
 _____________
```

(b)
```
   482 394 576
 + 495 417 345
 _____________
```

Solution

(a)
```
   234 165 981
 + 453 813 017
 _____________
   687 978 998
```

Solution

(b)
```
   482 394 576
 + 495 417 345
 _____________
   977 811 921
```


Subtraction of whole numbers

When two or more numbers are subtracted, a difference is obtained. The symbol used for subtraction is '–', and is read as minus. When subtracting numbers, remember to consider the place value of each digit in the number. Also, you can subtract numbers when arranged horizontally or vertically by regrouping or without regrouping.

When subtracting whole numbers, remember to start by subtracting ones, followed by tens, hundreds, up to the highest place value of the given numbers.

Subtraction of whole numbers arranged horizontally

Example

Find the value of each of the following:

(a) 876 954 264 – 653 842 152 = (b) 884 549 167 – 759 467 893 =

Solution

(a) 876 954 264 – 653 842 152 = 223 112 112

(b) 884 549 167 – 759 467 893 = 125 081 274

Subtraction of whole numbers arranged vertically

Example

Find the value of each of the following:

(a)
```
  758 936 125
– 416 824 014
-------------

-------------
```

(b)
```
  648 752 087
– 499 827 968
-------------

-------------
```

Solution

(a)
```
  758 936 125
– 416 824 014
-------------
  342 112 111
-------------
```

(b)
```
  648 752 087
– 499 827 968
-------------
  148 924 119
-------------
```


Exercise 4

Find the value of each of the following:

1. 624 123 715 + 234 865 183 =
2. 287 364 116 + 819 756 295 =
3. 862 175 954 – 598 658 476 =
4. 891 654 875 – 789 587 169 =
5. 712 151 423 – 132 248 369 =
6. 625 168 365 + 169 564 192 – 609 514 078 =
7.
```
  426 112 367
+ 263 763 231
-------------

-------------
```
8.
```
  765 462 168
+ 154 679 324
-------------

-------------
```

9. $\begin{array}{r} 967\ 684\ 724 \\ -\ 352\ 572\ 623 \\ \hline \\ \hline \end{array}$

10. $\begin{array}{r} 984\ 364\ 102 \\ -\ 679\ 897\ 651 \\ \hline \\ \hline \end{array}$

Multiplication of whole numbers

The result obtained after multiplying two or more numbers is called a product. The symbol used for multiplication is '×', and is read as times. When multiplying numbers, consider the place value of each digit in the number. Also, align the numbers vertically to correspond to the place value of each digit. When multiplying, you can either multiply the number by starting with the lowest place value of the multiplier towards the left-hand side or you can start with the highest place value of the multiplier towards the right-hand side.

The number used in multiplying is called a multiplier and the number which is multiplied is called a multiplicant.

Example

Find the product of each of the following:

(a) 232 124 × 2 312 =

Solution

$$\begin{array}{r} 232124 \\ \times \quad 2312 \\ \hline 464248 \\ 232124 \\ 696372 \\ +\ 464248 \\ \hline 536670688 \\ \hline \end{array}$$

Therefore,
232 124 × 2 312 = 536 670 688.

(b) $\begin{array}{r} 298\ 764 \\ \times \quad 2\ 976 \\ \hline \\ \hline \end{array}$

Solution

$$\begin{array}{r} 298764 \\ \times \quad 2976 \\ \hline 597528 \\ 2688876 \\ 2091348 \\ +\ 1792584 \\ \hline 889121664 \\ \hline \end{array}$$

Therefore, the product is 889 121 664.

Division of whole numbers

The result obtained after dividing two numbers is called the quotient. The symbol for division is '÷', and is read as divide. Division of large numbers becomes easy when the long division method is used.

When a number cannot be exactly divided by another number, the left over number is called a remainder.

Example

Find the quotient of each of the following:

(a) 311 106 912 ÷ 85 728 =

Solution

```
             3 6 2 9
85728 ) 3 1 1 1 0 6 9 1 2
      − 2 5 7 1 8 4
          5 3 9 2 2 9
        − 5 1 4 3 6 8
            2 4 8 6 1 1
          − 1 7 1 4 5 6
              7 7 1 5 5 2
            − 7 7 1 5 5 2
                –   –   –
```

Therefore,
311 106 912 ÷ 85 728 = 3 629.

(b) 9875)936 811 653

Solution

```
              94867
9875 ) 936811653
     − 88875
        48061
      − 39500
         85616
       − 79000
          66165
        − 59250
           69153
         − 69125
              28
```

Therefore, the answer is 94 867 remainder 28.

Exercise 5

Find the value of each of the following:

1. 323 156 × 2 895
2. 65 841 × 12 145
3. 871 293 176 ÷ 3 698
4. 210 634 × 1 957
5. 660 204 696 ÷ 96 324
6. 938 725 467 ÷ 35 847
7. $\begin{array}{r} 352\ 864 \\ \times\ 2\ 215 \\ \hline \\ \hline \end{array}$
8. $\begin{array}{r} 2\ 352\ 864 \\ \times\quad 415 \\ \hline \\ \hline \end{array}$
9. $89\ 542\overline{)584\ 082\ 466}$
10. $129\overline{)115\ 455\ 543}$

Mixed mathematical operations

In some questions, there is a combination of more than one mathematical operation. If there are no brackets ( ), start by division, followed by multiplication, then addition, and finally subtraction.

Evaluating an expression depends on the operations used in the expression. If an expression has two or more similar operations, the order of that operation is from left hand side to right hand side.

Example 1

Find the value of 69 875 × 12 + 789 852 ÷ 12.

Solution

69 875 × 12 + 789 852 ÷ 12
= 69 875 × 12 + 65 821 (Division operation)
= 838 500 + 65 821 (Multiplication operation)
= 904 321. (Addition operation)

Therefore, 69 875 × 12 + 789 852 ÷ 12 = 904 321.

Example 2

Find the value of 134 048 ÷ 568 – 96 045 + 279 455 × 18.

Solution

134 048 ÷ 568 – 96 045 + 279 455 × 18

= 236 – 96 045 + 279 455 × 18 (Division operation)

= 236 – 96 045 + 5 030 190 (Multiplication operation)

= 236 + 5 030 190 – 96 045 (Re-arrange)

= 5 030 426 – 96 045 (Addition operation)

= 4 934 381. (Subtraction operation)

Therefore, 134 048 ÷ 568 – 96 045 + 279 455 × 18 = 4 934 381.

Exercise 6

State whether each of the following statements is True or False:

1. 432 604 ÷ 74 – 4 989 = 857
2. 695 × 64 – 29 645 + 18 676 = 33 511
3. 895 × 726 + 96 856 ÷ 8 = 746 626 ÷ 8
4. 92 146 148 – 58 085 112 × 9 + 64 957 = – 430 619 860 + 64 957
5. 886 442 ÷ 2 – 368 165 = 443 231 – 368 165

Find the value of each of the following:

6. 178 485 ÷ 489 + 62 958 × 25
7. 865 236 – 58 972 × 6 +89 762
8. 58 620 145 + 42 095 814 – 16 × 80 892 ÷ 963
9. 46 292 ÷ 71 × 14 – 3 975 + 82 142
10. 965 841 × 25 + 4 762 148 – 3 164 987

BODMAS

BODMAS rule explains the order of operations to be observed when evaluating or simplifying a mathematical expression with mixed operations.

An expression is evaluated or simplified by following the order of precedence of operations given in the BODMAS rule.

Remember, the order of operations is in accordance with the priority given in the BODMAS rule.

The word BODMAS stands for:

BO Brackets Of
D Division
M Multiplication
A Addition
S Subtraction

Example 1

Find the value of $155 \times (98\,654\,168 - 92\,652\,149)$.

Solution

$155 \times (98\,654\,168 - 92\,652\,149)$
$= 155 \times 6\,002\,019$ (Opening brackets)
$= 930\,312\,945.$ (Multiplication operation).

Therefore, $155 \times (98\,654\,168 - 92\,652\,149) = 930\,312\,945$.

Example 2

Find the value of $(65\,680 + 12\,879) \times (72\,191 - 72\,099)$.

Solution

$(65\,680 + 12\,879) \times (72\,191 - 72\,099)$
$= 78\,559 \times 92$ (Opening brackets)
$= 7\,227\,428.$ (Multiplication operation)

Therefore, $(65\,680 + 12\,879) \times (72\,191 - 72\,099) = 7\,227\,428$.

Example 3

Find the value of $2\,618\,954 + (45\,260 \div 365) \times 68 - (1\,614\,312 - 1\,594\,069)$.

Solution

$2\ 618\ 954 + (45\ 260 \div 365) \times 68 - (1\ 614\ 312 - 1\ 594\ 069)$

$= 2\ 618\ 954 + 124 \times 68 - 20\ 243$ (Opening brackets)

$= 2\ 618\ 954 + 8\ 432 - 20\ 243$ (Multiplication operation)

$= 2\ 627\ 386 - 20\ 243$ (Addition operation)

$= 2\ 607\ 143$ (Subtraction operation).

Therefore,

$2\ 618\ 954 + (45\ 260 \div 365) \times 68 - (1\ 614\ 312 - 1\ 594\ 069) = 2\ 607\ 143$.

Exercise 7

Find the value of each of the following:

1. $248 \times (29\ 168\ 147 - 29\ 097\ 489) =$
2. $(98\ 655 + 42\ 144) - (196\ 486 \div 3) =$
3. $895 + (5\ 894\ 325 \div 85\ 425) - 794 =$
4. $(42\ 865 - 38\ 169) \times (19\ 179 + 11\ 615) =$
5. $(64\ 824 \div 4) + 32\ 163 \times 15 =$
6. $82\ 645 + 24\ 165 - (16\ 672 - 9\ 658) =$
7. $689 \times 15 + (144\ 642 - 81\ 692) - 49\ 161 =$
8. $(62\ 464 + 14\ 179) + 16 \times (89\ 162 - 81\ 160) =$
9. $82(187\ 584 \div 32) + 292\ 864 \times 32 =$

Word problems involving whole numbers

You can use the four basic arithmetic operations to solve word problems on whole numbers.

Example 1

In a school garden, there are 4 rows of cabbage seedlings with 12 cabbage seedlings in each row, and 6 rows of tomato seedlings with 8 tomato seedlings in each row. How many seedlings of each kind are there?

Solution

Each row has 12 cabbage seedlings.

4 rows have 12 cabbage seedlings × 4 = 48 cabbage seedlings.

Thus, there are 48 cabbage seedlings.

Each row has 8 tomato seedlings.

6 rows have 8 tomato seedlings × 6 = 48 tomato seedlings.

Thus, there are 48 tomato seedlings.

Therefore, there are 48 tomato seedlings and 48 cabbage seedlings.

Example 2

Two thousand four hundred Tanzanian shillings are deposited in a bank account each month. How much money will be deposited after two years?

Solution

The deposit per month = Tsh 2 400.

There are 12 months in a year.

Total deposit after one year will be:

$$\begin{array}{r} 2400 \\ \times \quad 12 \\ \hline 4800 \\ +2400 \\ \hline 28800 \\ \hline \end{array}$$

After one year, the deposit is 28 800 Tanzanian shillings.

After two years, the deposit = 28 800 Tanzanian shillings × 2.

= 57 600 Tanzanian shillings.

Therefore, after two years, the deposit will be 57 600 Tanzanian shillings.

Example 3

Dar es Salaam Rapid Transport bus can carry 150 passengers in one trip. If it makes 16 such trips a day,

(a) how many passengers will it carry for 5 days?

(b) how much money will be collected for 16 days when each passenger pays a fare of 1 050 Tanzanian shillings?

Solution

(a) Number of passengers in one trip = 150.

Number of passengers in 16 trips = 150 × 16 = 2 400 passengers.

Thus, for one day it will carry 2 400 passengers.

For 5 days it will carry 2400 × 5 = 12 000 passengers.

Therefore, the Rapid Transport bus will carry 12 000 passengers for 5 days.

(b) The amount of money to be paid by one passenger = Tsh 1 050.

1 day = 2400 passengers

16 days = 16 × 2400 passengers = 38 400 passengers

The amount of money to be paid by 38 400 passengers = Tsh 38 400 × 1 050
= Tsh 40 320 000.

Therefore, 40 320 000 Tanzanian shillings will be collected for 16 days.

Example 4

During the construction of a community market, a constructor set aside 231 671 350 Tanzanian shillings for paying 258 day workers equally.

(a) How much money did each day worker get?

(b) How much money did the contractor remain with?

Solution

(a) The total amount of money = 231 671 350 Tanzanian shillings.

Number of day workers = 258.

Divide using a long division method as follows:

```
          897950
    258)231671350
      -2064
        2527
       -2322
         2051
        -1806
          2453
         -2322
           1315
          -1290
             250
            -000
             250
```

Therefore,

(a) Every day worker got 897 950 Tanzanian shillings.

(b) The contractor remained with 250 Tanzanian shillings.

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Example 5

The population of a city is 13 984 765 people. There are 5 679 482 men and 4 947 652 women, and the remaining ones are children. How many children are there in the city?

Solution

Adding the number of men and women in the city:

$$\begin{array}{r} 5\ 679\ 482 \\ +\ 4\ 947\ 652 \\ \hline 10\ 627\ 134 \\ \hline \end{array}$$

To get the number of children, subtract the total number of men and women from the population of the city as follows:

$$\begin{array}{r} 13\ 984\ 765 \\ -10\ 627\ 134 \\ \hline 3\ 357\ 631 \\ \hline \end{array}$$

Therefore, there are 3 357 631 children in the city.

Exercise 8

1. Richard sold 495 copies of *The Guardian* newspaper for 1 000 Tanzanian shillings each, 355 copies of *Uhuru* newspaper for 1 000 Tanzanian shillings each, and 214 copies of *Champion* magazine for 800 Tanzanian shillings each. How much money did he collect?
2. Each day, a school shop collects 756 550 Tanzanian shillings from the customers. If the collection was made for 92 days, and 18 650 950 Tanzanian shillings of the collected money was used to build a fence for the school, how much money was left?
3. A school collects 2 685 eggs from its poultry farm each day. The price of one egg is 500 Tanzanian shillings. How much money does the school earn per day by selling the eggs?
4. Jim's school is 13 000 metres from his home. If he goes to school daily, how many kilometres does he travel in 196 days?

5. A bakery sells 98 655 loaves of bread every week. If each loaf of bread is sold at 2 500 Tanzanian shillings, find the total amount of money the bakery earns every week.
6. A school with 2 485 students has decided to make uniforms for all students. If a shirt and a pair of trousers need 2 metres and 1.5 metres, respectively how much of each material of cloth will the school need to make uniforms for all the students?
7. In a school garden, there are 452 rows of carrot seedlings with 65 carrot seedlings in each row. How many carrot seedlings are there?
8. Two hundred and forty-six students were given 21 069 900 Tanzanian shillings to share equally among themselves. How much money did each student get?
9. The cost of one kilogram of sugar is 2 500 Tanzanian shillings. If Deborah buys 655 kilograms of sugar, how much does it cost her?
10. The government provided tree seedlings to 895 villages for reforestation. If each village got 96 857 tree seedlings, how many tree seedlings did the government distribute to all villages?
11. A school decided to buy 724, 985, and 1 389 textbooks for three subjects (i.e. history, biology and basic mathematics) respectively. If each textbook costs 9 000 Tanzanian shillings. How much money did the school spend on the textbooks?
12. A woman deposited 2 654 550 Tanzanian shillings in her bank account on Tuesday, and withdrew 1 115 250 Tanzanian shillings on Thursday. If she deposited again 870 900 Tanzanian shillings on Saturday, how much money was in her account?
13. A total of 366 310 560 iron sheets were bought for roofing 5 565 family houses. If each family received an equal number of iron sheets, how many iron sheets did each family receive?
14. A factory made 58 675 sieves in a year. The sieves were sold for a total of 906 528 750 Tanzanian shillings. What was the price of each sieve?
15. In a certain region, a funding agency donated 705 687 500 Tanzanian shillings for renovating primary schools. The money was equally distributed among 1 250 schools. How much money did each school get?

Factors and multiples of numbers

Remember that, 3 divides 36 leaving a remainder of 0. We say that, 3 divides 36 exactly or that, 36 is divisible by 3. In this case, we say that 3 is a factor of 36 and 36 is a multiple of 3.

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Thus, when a number divides another number exactly then, the divisor is called a factor of the dividend, and a dividend is called a multiple of the divisor.

Example 1

Find all the factors of 36.

Solution

We can write 36 as a product of two factors as follows:

$$
\begin{aligned}
36 &= 1 \times 36 \\
&= 2 \times 18 \\
&= 3 \times 12 \\
&= 4 \times 9 \\
&= 6 \times 6.
\end{aligned}
$$

Therefore, the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

Example 2

Find all the factors of 54.

Solution

We can write 54 as a product of two factors as follows:

$$
\begin{aligned}
54 &= 1 \times 54 \\
&= 2 \times 27 \\
&= 3 \times 18 \\
&= 6 \times 9.
\end{aligned}
$$

Therefore, the factors of 54 are 1, 2, 3, 6, 9, 18, 27, and 54.

Factors of a number are presented by a listing method as shown in the examples above.

Prime factorization

The process of writing a number as the product of prime factors is called prime factorization of the given number.

The following are the steps for prime factorization of a number:

1. Divide the given number by the smallest prime factor.
2. Keep dividing each of the subsequent quotients by the smallest prime factor until the last quotient is 1.
3. Express the given number as the product of all these prime factors.

Example 1

Write 42 as a product of its prime factors.

Solution

We can write 42 as a product of its prime factors as follows:

2	42
3	21
7	7
	1

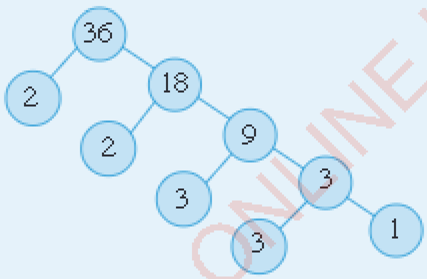
Therefore, $42 = 2 \times 3 \times 7$.

Example 2

Use a factor tree to write 36 as a product of its prime factors.

Solution

36 can be factorized by using a factor tree as follows:



From a factor tree, $36 = 2 \times 2 \times 3 \times 3$.

Therefore, $36 = 2 \times 2 \times 3 \times 3$.

Example 3

Write 420 as the product of its prime factors.

Solution

2	420
2	210
3	105
5	35
7	7
	1

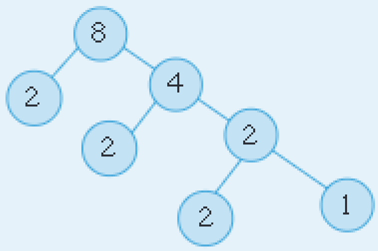
Therefore, $420 = 2 \times 2 \times 3 \times 5 \times 7$.

Example 4

Use a factor tree to write the prime factors of 8.

Solution

A factor tree for 8 is:



From a factor tree, $8 = 2 \times 2 \times 2$.

Therefore, the prime factor of 8 is 2.

Exercise 9

Write each of the following numbers as the product of its prime factors:

1. 256
2. 5 467
3. 343
4. 168
5. 25 088
6. 24
7. 129 600
8. 2 000

9. Use a factor tree to find the prime factors of each of the following numbers:
   (a) 48 (b) 72 (c) 96 (d) 27 (e) 36 (f) 42

10. Is there a number which is a factor of every number? If so, what is the number?

In question 11 to 15, each of the given products represents a number. List all the factors of each number.

11. $2 \times 3 \times 5$
12. $3 \times 3 \times 3$
13. $3 \times 3 \times 5 \times 7$
14. $3 \times 5 \times 5$
15. $2 \times 2 \times 3$

Listing of multiples of numbers

The lowest common multiple (LCM)

LCM is the short form of the lowest common multiple or the least common multiple. The lowest common multiple of two or more numbers is the smallest natural number that is exactly divisible by all given numbers. Since division of natural numbers by zero is undefined, the definition has a meaning only when the given numbers are not equal to zero.

For example; consider the numbers 3 and 7.

Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, . . .

Multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, . . .

Common multiples of 3 and 7 are 21, 42, . . .

We observe that, the least common multiple of 3 and 7 is 21.

Therefore, the LCM of 3 and 7 is 21.

Example 1

Find the first three common multiples of 3 and 5.

Solution

Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, . . .

Multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50 , . . .

Therefore, three common multiples of 3 and 5 are 15, 30, and 45.

There are two ways of finding the least common multiple of the given numbers:

1. By listing the multiples of each of the given numbers, and selecting the smallest common multiple.
2. By prime factorization of each of the numbers.

Example 2

Use the listing method to find the least common multiple of 8 and 12.

Solution

Multiples of 8 are 8, 16, 24, 32, 40, 48 , . . .

Multiples of 12 are 12, 24, 36, 48 , . . .

Therefore, the LCM of 8 and 12 is 24.

Example 3

Find the LCM of 140, 252, and 240 by prime factorization.

Solution

$140 = 2 \times 2 \times 5 \times 7$

$252 = 2 \times 2 \times 3 \times 3 \times 7$

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$= 5\,040.$

Therefore, the LCM of 140, 252, and 240 is 5 040.

Altenatively,

Prime factorization can be used to find the LCM of the numbers 140, 252, and 240 as follows:

1. Arrange the given numbers in columns.
2. Divide the numbers by a prime number which exactly divides at least two of the given numbers, and carry forward the numbers which are not divisible by that prime number. If there is no common prime number to divide the given numbers, work on the given numbers one by one.
3. Repeat the process in step 2 until no two of the numbers are divisible by the same number other than 1.
4. The product of the divisors and the undivided numbers is the required LCM of the given numbers.

2	140	252	240
2	70	126	120
2	35	63	60
2	35	63	30
3	35	63	15
3	35	21	5
5	35	7	5
7	7	7	1
	1	1	1

$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7.$

$= 5\,040.$

Therefore, the LCM of 140, 252 and 240 is 5 040.

Note:
If the number is not divisible by the given prime number, leave it as it is, and go to the next step.

Example 4

Find the smallest number of sweets that can be put into bags which can either contain 9 or 15 or 20 sweets with none left over.

Solution

There are three bags. They can either carry 9 or 15 or 20 sweets, respectively.

$$9 = 3 \times 3$$
$$15 = 3 \times 5$$
$$20 = 2 \times 2 \times 5$$
$$\text{LCM} = 2 \times 2 \times 3 \times 3 \times 5$$
$$= 180.$$

Therefore, the smallest number of sweets which can be put in the three bags with none left over is 180.

Exercise 10

In question 1 to 6, find the LCM of the given numbers by listing:

1. 18, 20, 30
2. 72, 108
3. 15, 18, 24
4. 16, 72
5. 25, 45, 75
6. 48, 64, 120

Find the LCM of the following numbers by prime factorization:

7. 36, 48, 288
8. 124, 240
9. 36, 48, 24
10. 164, 248
11. 45, 90, 125
12. 128, 256, 360

Find the LCM of the following numbers by using either listing or prime factorization method:

13. 2×3, 3×5, $2 \times 2 \times 2 \times 3$
14. $3 \times 3 \times 3 \times 3 \times 7 \times 7$, $2 \times 5 \times 5 \times 7$, $2 \times 5 \times 7 \times 7 \times 11$
15. 3×3, $2 \times 2 \times 3$, 3×7
16. An electronic device beeps after every 25 minutes. Another device beeps after every 45 minutes. If they beep together at 6 am, at what time do they beep together again?
17. Three bells ring together at a certain starting point of time. Then, they ring at the intervals of 20, 25, and 50 minutes, respectively. After what interval of time will they ring together again?

18. Find the smallest number of oranges which can be divided to three people in the amounts of 10 oranges, 15 oranges, and 18 oranges, respectively.

The greatest common factor (GCF)

The greatest common factor (GCF) or the highest common factor (HCF) of two or more natural numbers is the largest number that divides the numbers exactly.

For example; the greatest common factor of 36 and 90 can be found as follows:

Step 1: List all the factors of each number.
All the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.
All the factors of 90 are, 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, and 90.

Step 2: Use an arrow to match the same factors from the top list and the bottom list as shown.

1, 2, 3, 4, 6, 9, 12, 18, 36.

1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90.

The factors common to 36 and 90 are 1, 2, 3, 6, 9, and 18. The largest factor is 18, which is the greatest common factor (GCF) of 36 and 90.

Example 1

Find the GCF of 120 and 192.

Solution

$120 = 2 \times 2 \times 2 \times 3 \times 5$

$192 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$GCF = 2 \times 2 \times 2 \times 3 = 24.$

Therefore,
the GCF of 120 and 192 is 24.

Example 2

Square tiles are used to build a floor measuring 12 metres by 16 metres. If the tiles are of different sizes, and only whole ones are used, what is the greatest size of the tile?

Solution

Different sizes of tiles can be used, but here the biggest size of these is needed.

The product of prime factors of 12 m = $(2 \times 2 \times 3)$ m.

The product of prime factors of 16 m = $(2 \times 2 \times 2 \times 2)$ m.

The product of common prime factors are (2×2) m = 4 m.

Therefore, the greatest length of each tile is 4 metres.

Exercise 11

Use the method of listing to find the GCF of each of the following:

1. 18, 45
2. 425, 200
3. 210, 56
4. 12, 16, 20
5. 66, 108, 136
6. 35, 420, 245

Use the prime factorization method to find the GCF in questions 7 to 12.

7. 16, 56
8. 36, 162
9. 16, 72, 104
10. 24, 156, 180
11. 12, 42, 72
12. 30, 40, 50

13. Find the largest number which divides:
    (a) 63 and 105
    (b) 36 and 72
    (c) 240 and 360

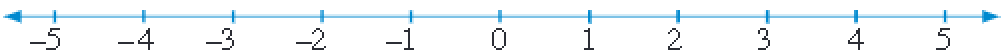
14. A boy walked 120 paces while a girl walked 100 paces from the same home to the market. If they started walking at the same time, and used the same speed, how many times did they step together?

15. Equal squares as large as possible are drawn on a rectangular board measuring 54 centimetres by 78 centimetres. Find the size of each square.

Integers

Integers include whole numbers and opposite of positive whole numbers. The order of the size of the numbers 0, +1, + 2, + 3, + 4, . . . can be represented as points on the number line starting from zero onwards to the right. Other points on the same number line with the same distance from zero, can be represented in the opposite direction as –1, –2, –3, –4, . . .

This is illustrated in the following figure:



The numbers , . . . –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, … are called integers and the set of integers is denoted by $\mathbb{Z}$. The numbers with a minus '–' sign are called negative integers and those with plus '+' sign are called positive integers.

From the number line, we observe the following:

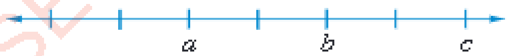
1. 0 lies to the left of every positive integer, thus, 0 is less than every positive integer.
2. 0 lies to the right of every negative integer, thus, 0 is greater than every negative integer.
3. Every negative integer lies to the left of the positive integers, thus, every negative integer is less than every positive integer.

Consider the following number line with fixed points corresponding to some numbers:

Consider two points d and e. Since e is to the right of d, then e is greater than d. On the other hand, if a and b are numbers such that a is less than b, then the point corresponding to a is to the left of the point corresponding to b.

The expression a is greater than b means that the point corresponding to a is to the right of the point corresponding to b.

The sign '≠' is the opposite of '='. Consider three points a, b, and c on the following number line:

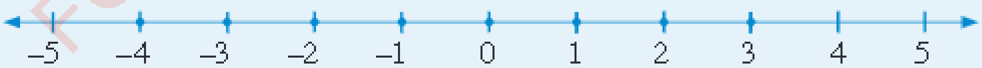


For any three integers a, b, and c, if a is less than b and b is less than c, then a is less than c.

Example 1

Represent –4, –3, –2, –1, 0, 1, 2, 3 on a number line.

Solution



Example 2

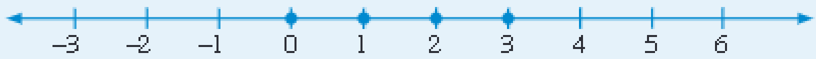
Use a number line to represent each of the following integers:

(a) –1 is less than x and x is less than 4.

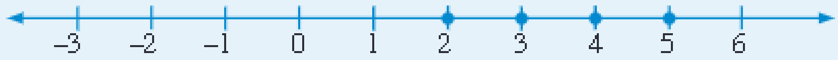
(b) 2 is less or equal to x and x is less than 6.

Solution

(a)



(b)



Exercise 12

1. Use an appropriate sign (>, <, or =) between each of the following pairs of integers to make true statements:
   (a) 25 ☐ 25
   (b) –7 ☐ 6
   (c) 7 ☐ –6
   (d) –7 ☐ –6
   (e) –3 ☐ –4
   (f) 14 ☐ 7
   (g) –14 ☐ 7
   (h) –14 ☐ 2
   (i) –5 ☐ –100
   (j) –1 ☐ –1

2. If Mary is not older than Margaret, and also not younger than Margaret, what conclusion can be made about Mary's and Margaret's ages?

3. If John is taller than Joseph, and Joseph is taller than James, what conclusion can you draw about John and James?

4. In each of the following integers, which is greater, and by how much?
   (a) –5 or –3 (b) +2 or –7

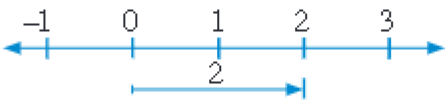
5. Represent the following integers on a number line:
   (a) 3 (d) 0
   (b) –3 (e) –4
   (c) –5 (f) 4

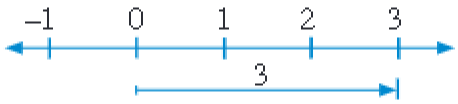
Operations on integers

(a) Addition of integers

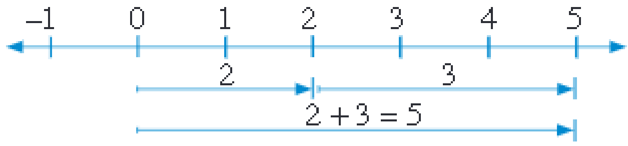
A positive symbol '+' is considered as a command to 'move to the forward direction', while a negative symbol '–' is considered as a command to 'move to the backward direction'.

Consider the addition of integers on the number line. The integers 2 and 3 are indicated on the number line by arrows:



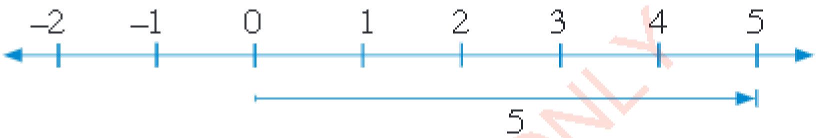


The addition of 2 and 3 can then be performed as shown in the following figure.



The addition of one positive integer and another positive integer is done by moving to the right along the number line.
Arrows for positive numbers move to the right while arrows for negative numbers move to the left.

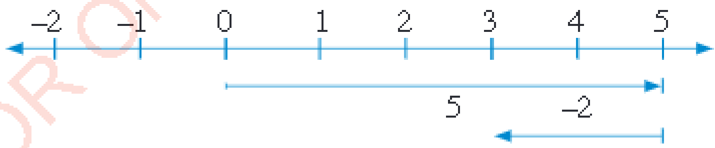
For example; $5 + (-2)$ can be computed as follows:
Using an arrow, indicate 5 units on a number line:



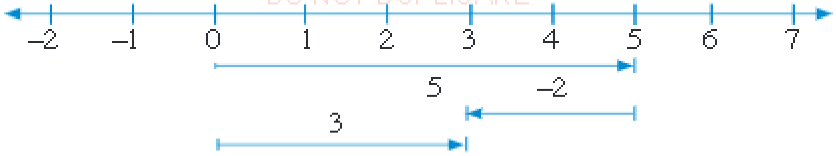
On the same number line, indicate –2 using an arrow.

The arrow that corresponds to –2 has the same length as that for 2, but goes to the opposite direction.

The arrow indicating –2 is drawn starting at the point where 5 ends and moves two units to the left as shown in the following figure:

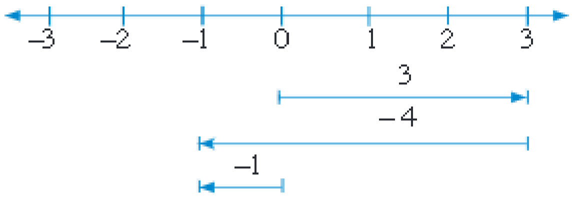


The point corresponding to $5 + (-2)$ is found by starting at 0, moving 5 units to the right, and then 2 units to the left. The result of $5 + (-2) = 3$, as shown in the following figure:



Therefore, $5 + (-2) = +3$.

In the same way, 3 and – 4 can be added as follows:



Therefore, $3 + (-4) = -1$.

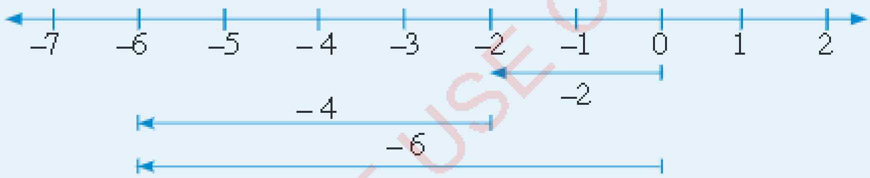
Example 1

Use the number line to find the value of each of the following:

(a) $-2 + (-4)$ (b) $-6 + 2$ (c) $-2 + 6$ (d) $-2 + 2$

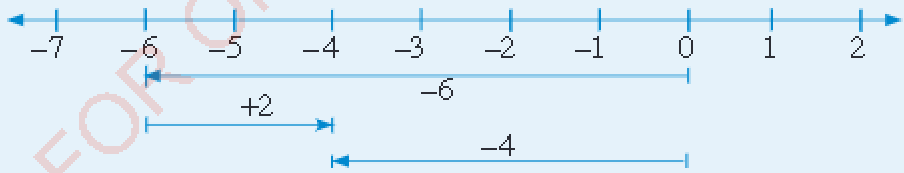
Solution

(a) $-2 + (-4)$



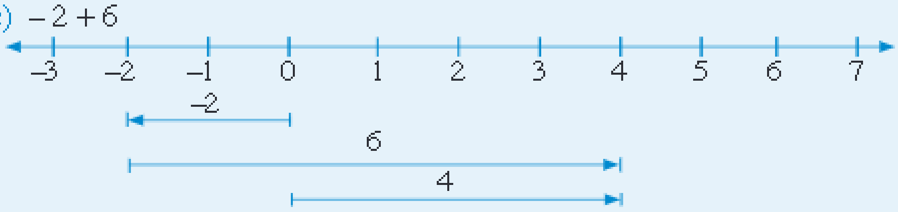
Therefore, $-2 + (-4) = -6$.

(b) $-6 + 2$



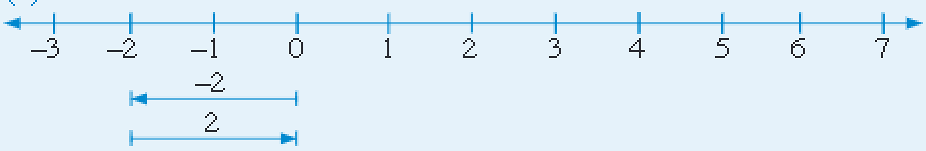
Therefore, $-6 + 2 = -4$.

(c) $-2+6$



Therefore, $-2+6=4$.

(d) $-2+2$



Therefore, $-2+2=0$.

Note:

1. When both numbers are positive, the sum is positive as in $5+3=8$.
2. When both numbers are negative, the sum is negative as in $-5+(-3)=(-8)$.
3. When one number is positive and the other number is negative, it is the number further from 0 that determines whether the sum is positive or negative. If the numbers are at the same distance from 0, the sum is 0.

Example 2

Find the value of each of the following:

(a) $(+64)+(+52)=$

(b) $(-146)+(+98)=$

(c) $(-91)+(-112)=$

(d) $(+194)+(-76)=$

Solution

(a) $(+64)+(+52)=+64+52=+116$

(b) $(-146)+(+98)=-146+98=-48$

(c) $(-91)+(-112)=-91-112=-203$

(d) $(+194)+(-76)=+194-76=+118$

Exercise 13

Find the value of each of the following without using the number line:

1. $+5 + (+6) =$
2. $-57 + (+62) =$
3. $+101 + (+57) =$
4. $+9 + (-9) =$
5. $-12 + (-20) =$
6. $+15 + (-16) =$
7. $-192 + (-56) =$
8. $+4 + (+8) =$
9. $-8 + (+8) =$
10. $+20 + (-10) =$
11. $+107 + (-110) =$
12. $+25 + (+40) =$
13. $+3 + (+6) =$
14. $-13 + (+14) =$
15. $-81 + (+53) =$
16. Use the number line to find the value of each of the following:

    (a) $9 + (-5) =$

    (b) $-7 + 5 =$

    (c) $10 + (-7) =$

    (d) $5 + (-10) =$

    (e) $-5 + (-2) =$

    (f) $3 + (-7) =$

(b) Subtraction of integers

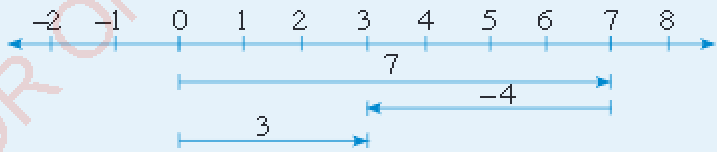
In order to subtract an integer from another integer, we change the sign of the number to be subtracted, and replace the subtraction operation with an addition operation.

Example 1

Use a number line to find the value of $7 - 4$

Solution

$7 - 4 = 7 + (-4)$. A number line can be used as in addition.



Therefore, $7 - 4 = 3$.

When the sum of two numbers is zero, one of the numbers is the opposite of the other.

Example 2

Find the value of each of the following:

(a) (+ 15) – (+ 12) =
(b) (– 18) – (– 16) =
(c) (+ 64) – (– 36) =
(d) (+ 116) – (+ 142) =

Solution

(a) (+ 15) – (+ 12) = +15 – 12 = + 3.
(b) (– 18) – (– 16) = –18 + 16 = – 2.
(c) (+ 64) – (– 36) = +64 + 36 = +100.
(d) (+ 116) – (+ 142) = +116 – 142 = – 26.

Exercise 14

Use the number line to find the value of each of the following:

1. 6 – 4 =
2. 7 – 8 =
3. 4 + (–3) =
4. 6 + (– 4) =
5. 7 – (+4) =
6. 7 + (–8) =

Find the value of each of the following without using the number line:

7. (+117) – (–117) =
8. (+18) – (–34) =
9. (+118) – (+72) =
10. (– 68) – (+76) =
11. –8 – (+10) =
12. (+49) – (+31) =
13. (– 48) – (– 59) =
14. (+ 214) – (+298) =
15. (– 63) + (–81) =

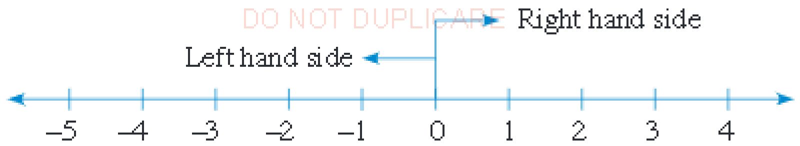
(c) Multiplication of integers

Multiplication of integers can be of + and +, – and –, – and + or + and –. In multiplying integers on a number line, you must consider the signs of the multiplicant as well as that of the multiplier e.g. –4 × +3.

–4 × +3

Sign of the multiplicant   Sign of the multiplier

Step 1:
Check the sign of the multiplicant:
If the sign is positive, face to the right side of 0.
If the sign is negative, face to the left side of 0.



Step 2:

Check the sign of the multiplier:

If the sign is positive, move to the positive direction.

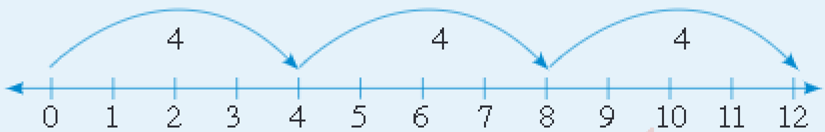
If the sign is negative, move to the negative direction.

Example 1

Use the number line to find the value of 4×3.

Solution

$4 \times 3 = 4 + 4 + 4 = 12$.



4×3 is the same as, starting from 0, then adding 4 three times to the positive direction.

Therefore, $4 \times 3 = 12$.

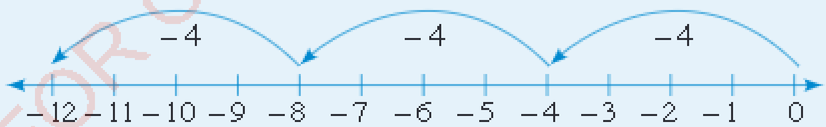
Example 2

Use the number line to find the value of -4×3.

Solution

Starting from 0, add 4 at a time to the negative direction 3 times:

$-4 \times 3 = (-4) + (-4) + (-4) = -12$.



Therefore, $-4 \times 3 = -12$.

If the integers to be multiplied are large, use multiplication rules.

Multiplication rules

1. A product of integers with the same sign is positive, that is,

   (+) × (+) = (+) or (–) × (–) = (+).

2. A product of integers with different signs is negative, that is,

   (–) × (+) = (–) and (+) × (–) = (–).

Example 1

Find the value of each of the following:

(a) (+5) × (+7) =

(b) (– 4) × (– 6) =

Solution

(a) (+5) × (+7) = (+ 35).

(b) (– 4) × (– 6) = (+24).

Example 2

Find the value of each of the following:

(a) (–116) × (+5) =

(b) (+20) × (– 4) =

Solution

(a) (–116) × (+5) = – (116 × 5) = –580.

(b) (+20) × (– 4) = – (20 × 4) = – 80.

(d) Division of integers

Division is the reverse of multiplication. It cannot be illustrated on a number line, but the following rules apply:

1. The quotient of two integers with different signs is negative. That is,

   (+) ÷ (–) = – or (–) ÷ (+) = –.

2. The quotient of two integers with the same sign is positive. That is,

   (–) ÷ (–) = + or (+) ÷ (+) = +.

Example

Find the value of each of the following:

(a) (– 364) ÷ (+ 4) =

(b) (+ 1 728) ÷ (+ 48) =

(c) (– 12 250) ÷ (– 98) =

(d) (+ 4 644) ÷ (– 172) =

Solution

(a) (– 364) ÷ (+ 4) = – 91.

(b) (+ 1 728) ÷ (+ 48) = + 36.

(c) (– 12 250) ÷ (– 98) = + 125.

(d) (+ 4 644) ÷ (– 172) = – 27.

(e) Mixed operations on integers

Sometimes, two, three, or four types of arithmetic operations are used in the same expression with integers. In this case, the BODMAS rule has to be applied.

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Example

Find the value of each of the following:

(a) $464 - 148 + 1\,620 \div 45 =$

(b) $49 \times 18 - 260 \div 4 =$

(c) $195 + 68 \times 72 - 720 \div 36 =$

Solution

(a) $464 - 148 + 1\,620 \div 45$

$= 464 - 148 + 36$

$= 464 + 36 - 148$

$= 500 - 148$

$= 352.$

Therefore,

$464 - 148 + 1\,620 \div 45 = 352.$

(b) $49 \times 18 - 260 \div 4$

$= 49 \times 18 - 65$

$= 882 - 65$

$= 817.$

Therefore,

$49 \times 18 - 260 \div 4 = 817.$

(c) $195 + 68 \times 72 - 720 \div 36$

$= 195 + 68 \times 72 - 20$

$= 195 + 4\,896 - 20$

$= 5\,091 - 20$

$= 5\,071.$

Therefore,

$195 + 68 \times 72 - 720 \div 36 = 5\,071.$

Exercise 15

Use the number line to find the value of each of the following:

1. $3 \times 2 =$
2. $-4 \times 3 =$
3. $-3 \times 2 =$
4. $4 \times (-2) =$
5. $-2 \times -3 =$
6. $6 \times (-3) =$

Find the value of each of the following without using the number line:

7. $-4 \times (-3) =$
8. $-(-4) \times 3 =$
9. $-8 \div (-8) =$
10. $-2 \times (-3) \times (-2) =$
11. $-8 \div 4 =$
12. $-9 \div (-3) =$
13. $324 - 148 + 765 \times 8 =$
14. $61 \times 15 - 1\,550 \div 5 =$
15. $275 + 162 \times 9 - 252 \div 9 =$

Chapter summary

1. A base ten numeration or a decimal numeration system is a method of writing numbers using 10 symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.
2. Digits are the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.
3. A place value means a position of a digit in a numeral.
4. Natural numbers are counting numbers starting with 1, 2, 3, 4, 5, . . .
5. Whole numbers are natural numbers which include digit 0.
6. An even number is a number which is exactly divisible by 2.
7. An odd number is a number which is not exactly divisible by 2.
8. A prime number is a positive number, except one, which is divisible by itself and one only.
9. A sum is obtained by adding together numbers.
10. The difference is an answer obtained after subtracting two numbers.
11. A product is an answer obtained by multiplication of two or more numbers.
12. A quotient is an answer obtained by dividing two numbers.
13. The least common multiple (LCM) of two or more natural numbers is the smallest natural number that is a multiple of each of the given numbers.
14. The greatest common factor (GCF) of two or more natural numbers is the largest number which is a factor of all the given numbers.
15. Integers are whole numbers and their opposites. The set of integers is denoted by $\mathbb{Z}$.
16. A number line is a straight line marked with equal intervals, and labelled with numbers.

Revision exercise

1. Each of the 2 458 students in a certain school drinks a litre of milk every day. If a litre of milk costs 500 Tanzanian shillings, how much money does the school spend per day?
2. How many prime numbers are there between 1 and 20?
3. How many prime factors does the number 225 have?
4. What is the greatest common factor of 672 and 490?
5. If each digit in the number 289 413 756 is used once, rearrange this number by writing in a numeral:
   (a) The largest possible number formed
   (b) The smallest possible number formed
6. Put an appropriate sign (>, <, or =) between each of the following pairs of numbers to make true statements:

(a) If –5 is less than 8, then	–5 × (–3)	☐	8 × (–3).
(b) If 13 is greater than 7, then	13 × (–2)	☐	7 × (–2).
(c) If 8 is greater than –2, then	8 × (–5)	☐	–2 × (–5).
(d) If 6 is greater than –3, then	6 × (– 4)	☐	–3 × (– 4).
(e) If a is greater than b, then	a × (–9)	☐	b × (–9).

7. Calculate the value of each of the following:
   (a) 4 372 × 75 = ______
   (b) 396 125 × 2 114 = ______
   (c) 896 561 × 298 =
   (d) 12 347 062 ÷ 9 854 =
8. Put an appropriate sign (>, <, or =) between each of the following pairs of numbers to make true statements:
   (a) +4 ____ +2 (c) +4 ____ +4 (e) –6 ____ –8
   (b) –3 ____ –2 (d) –11 ____ –1 (f) –9 ____ 0
9. How many packets of cigarettes are left over when 5 292 129 of them are packed into boxes which hold 897 packets each?

10. Find the value of $(-5 + 9) \times (8 - 19)$.

11. Find the value of each of the following:
   (a) $12 \div (2 + 2 \times 1) =$
   (b) $7 \times (3 + 48 \div 8) =$
   (c) $4 \times (2 + (3 \times 5 - 7) \times 2) =$
   (d) $(5 - 3) \times 6 + (20 - 2) \div 9 =$

Find the value of each expression in questions 12 and 13:

12. (a) $12 \times (-14) \times (-8) \div (-3) \times (-4) \times (-7) \times (-2) =$
   (b) $-3 \times 8 \times (-5) \times (-2) \div 4 \times (-6) \times (-10) =$

13. (a) $15 \times (-12) + 5 \times (-12) =$
   (b) $(-7) \times 5 - 13 \times 5 =$
   (c) $12 \times (-4) - 18\ (-4) =$

14. Find the prime factors of the following numbers: 20, 40, and 42.

15. Find the HCF of each of the following numbers:
   (a) 30, 45
   (b) 115, 375, 525

16. Find the LCM of each of the following numbers:
   (a) 24, 60
   (b) 90, 120, 720

17. When two different signs are multiplied, a product is obtained. Multiply the product obtained with a negative sign. Give the last sign you will obtain.

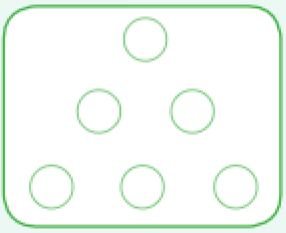
18. Draw a number line, and show the counting numbers 5, 7, 8, and 9.

19. Find the value of each of the following:
   (a) $(+147) + (-121) =$
   (b) $(-261) - (-109) =$
   (c) $(+265) - (-104) =$

20. Use a number line to find the product of -3 and -2.

Project 1

1. Look at the following triangle made of circles. Write number 1 to 6 in the circles so that each side of the triangle adds up to the same number.



(a) In how many different ways can this problem be done? Show the different solutions.

(b) Explain how you could get the sides of the triangle to add up to the smallest number.

(c) Explain how you would get the sides of the triangle to add up to the largest number.

(d) Write down a problem that is similar to this one.

2. Find the prime numbers between 1 and 100 by using the following instructions:

Step 1: Write down the numbers from 1 to 100 in rows of ten.

Step 2: Cross out 1.

Step 3: Circle 2 and cross out the remaining multiples of 2, that is, 4, 6, 8, 10, . . .

Step 4: Circle 3 and cross out the remaining multiples of 3, that is, 6, 9, 12, 15, . . .

Step 5: Circle 5 and cross out the remaining multiples of 5, that is, 10, 15, 20, . . .

Step 6: Circle 7 and cross out the remaining multiples of 7, that is, 14, 21, 28, . . .

Step 7: Now, circle all the remaining numbers, and count all the circled numbers to obtain the total number of the prime numbers between 1 and 100.

Question: How many prime numbers are there between 1 and 100?

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Chapter Three

Fractions

Introduction

The word 'Fraction' comes from the Latin word 'Fractio', which means 'to break'. The first known fractions were used by the Egyptians in the early 1800's BC, though they used pictures to describe fractions. A fraction represents a part of a whole or, more generally, any number of equal parts. When spoken in daily life, a fraction represents the number of parts of a certain size. In this chapter, you will learn about fractions. The competencies gained will help you to allocate resources, divide space and time, as well as develop a foundation for more complex mathematical theories. Hence, you will be able to make parts of objects in real-life activities such as building houses and farming.

Fractions

Activity 1: Recognising fractions

Take an orange and perform the following tasks individually or in groups:

1. Cut the orange into two equal pieces.
2. Draw diagrams of the two pieces.
3. Cut each of the two pieces into two equal pieces to get four equal pieces.
4. Draw diagrams of the four pieces.
5. Cut each of the four pieces into two equal pieces to get eight equal pieces.
6. Draw diagrams of the eight pieces.
7. Write the fraction of:
   (a) One piece obtained in task 2.
   (b) Three pieces obtained in task 4.
   (c) Three pieces obtained in task 6.
8. Share your findings with the rest of the class through presentation.

A fraction is a part or portion of a whole thing. It represents equal parts of the whole. Consider the following figures:

(a)

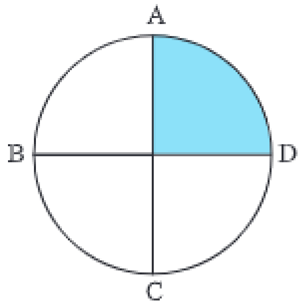
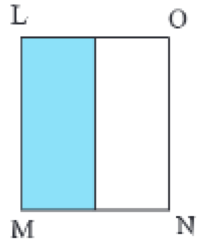


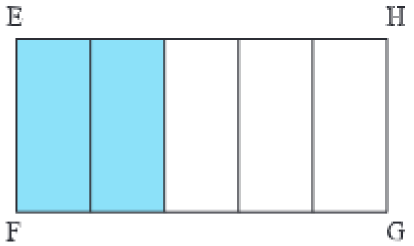
Figure ABCD is divided into 4 equal parts. The shaded part is 1 part out of 4 equal parts, written as $\frac{1}{4}$. It is read as a quarter or one fourth or one over four.

(c)



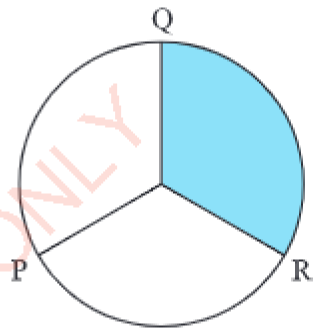
In figure LMNO, the shaded part is 1 out of 2 equal parts, written as $\frac{1}{2}$. It is read as one half or one over two.

(b)



In figure EFGH, the shaded part is 2 out of 5 equal parts, written as $\frac{2}{5}$. It is read as two over five or two fifths.

(d)



In figure PQR, the shaded part is 1 out of 3 equal parts written as $\frac{1}{3}$. It is read as one third or one over three.

A fraction has two parts, namely, a numerator and a denominator. The numerator and the denominator are separated by a horizontal bar. The number at the top is called the numerator and the number at the bottom is called the denominator. For example; in the fraction $\frac{2}{5}$, 2 is a numerator and 5 is a denominator. The horizontal bar separating the numerator and the denominator of a fraction represents the division sign.

For example; in the fraction $\frac{5}{8}$, the horizontal bar means 'divided by', and the fraction is read as 'five divided by eight'.

The number $\frac{a}{b}$ represents a fraction, where a and b are integers, and b is not zero. This is because, if $b = 0$, the fraction $\frac{a}{b}$ will be undefined.

Exercise 1

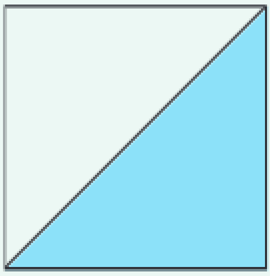
1. Draw a diagram to represent each of the following fractions:

(a) $\frac{1}{6}$ (c) $\frac{1}{5}$ (e) $\frac{3}{5}$ (g) $\frac{3}{8}$

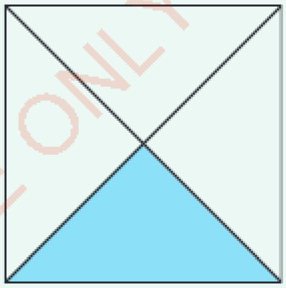
(b) $\frac{1}{2}$ (d) $\frac{1}{4}$ (f) $\frac{3}{4}$

2. In each of the following diagrams, write the fraction of:
   (a) Unshaded regions.
   (b) Shaded regions.

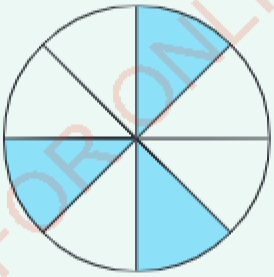
(i)



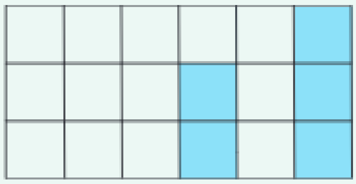
(iii)



(ii)



(iv)



Types of fractions

There are three types of fractions: proper fractions, improper fractions, and mixed fractions.

Proper fractions

A fraction whose denominator is larger than the numerator is called a proper fraction. For example; $\frac{2}{5}$, $\frac{1}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ are proper fractions.

Improper fractions

A fraction whose denominator is smaller than the numerator is called an improper fraction.

For example; $\frac{5}{2}$, $\frac{3}{2}$, $\frac{6}{5}$, $\frac{8}{7}$, and $\frac{16}{15}$ are improper fractions.

Mixed fractions (mixed numbers)

A fraction consisting of a whole part and a proper fraction part is called a mixed fraction (mixed number).

For example; $1\frac{1}{2}$, $-3\frac{5}{6}$, $6\frac{4}{5}$, $4\frac{1}{3}$, and $-2\frac{1}{5}$ are mixed fractions (mixed numbers).

Improper fractions can be converted into mixed fractions and vice versa.

Steps for converting improper fractions into mixed fractions:

1. Divide the numerator by the denominator.
2. Write down the quotient as a whole number.
3. Write down the remainder as the numerator and the divisor as the denominator.

Example 1

Convert the following improper fractions into mixed fractions:

(a) $\frac{5}{2}$ (b) $\frac{69}{5}$

Solution

(a) Divide the numerator by the denominator:

$$\begin{array}{r} 2 \\ 2\overline{)\,5} \\ -4 \\ \hline 1 \end{array}$$

Therefore, $\frac{5}{2} = 2\frac{1}{2}$.

(b) Divide the numerator by the denominator:

$$\begin{array}{r} 13 \\ 5\overline{)\,69} \\ -5 \\ \hline 19 \\ -15 \\ \hline 4 \end{array}$$

Therefore, $\frac{69}{5} = 13\frac{4}{5}$.

Example 2

Convert the following mixed numbers into improper fractions:

(a) $3\frac{3}{4}$ (b) $145\frac{7}{12}$

Solution

Multiply the whole number by the denominator, and add the numerator to the product as follows:

(a) $3\frac{3}{4} = \frac{(3 \times 4) + 3}{4}$

$= \frac{12 + 3}{4}$

$= \frac{15}{4}$.

Therefore, $3\frac{3}{4} = \frac{15}{4}$.

(b) $145\frac{7}{12} = \frac{(145 \times 12) + 7}{12}$

$= \frac{1740 + 7}{12}$

$= \frac{1747}{12}$.

Therefore, $145\frac{7}{12} = \frac{1747}{12}$.

Exercise 2

1. Use the following fractions to answer the questions that follow.

(i) $\frac{70}{99}$ (iii) $\frac{230}{73}$ (v) $\frac{925}{104}$

(ii) $\frac{32}{51}$ (iv) $\frac{115}{59}$

(a) Which fractions are proper?

(b) Which fractions are improper?

2. Convert the following improper fractions into mixed numbers:
   (a) $\frac{8}{5}$ (d) $\frac{104}{9}$ (g) $\frac{123}{10}$
   (b) $\frac{89}{16}$ (e) $\frac{863}{140}$ (h) $\frac{978}{25}$
   (c) $\frac{640}{18}$ (f) $\frac{1465}{562}$

3. Convert the following mixed fractions into improper fractions:
   (a) $3\frac{2}{5}$ (d) $962\frac{3}{16}$ (g) $18\frac{17}{19}$
   (b) $7\frac{5}{6}$ (e) $94\frac{7}{24}$ (h) $364\frac{19}{25}$
   (c) $475\frac{4}{9}$ (f) $152\frac{16}{21}$

4. (a) What type of fraction is $5\frac{1}{9}$? Why?
   (b) Which of the following fractions are improper fractions? Give reasons.
   $9\frac{1}{6}$, $\frac{3}{2}$, $\frac{4}{9}$, $\frac{23}{8}$, $7\frac{2}{7}$, $\frac{15}{6}$, $\frac{7}{9}$, $\frac{16}{7}$.

Equivalent fractions

Equivalent fractions are the fractions with different numerators and denominators that represent the same value of proportion of the whole.

In order to obtain a fraction equivalent to another fraction, multiply or divide the numerator and the denominator of the given fraction by the same non-zero number. For example; $\frac{1}{2}$, $\frac{3}{6}$, $\frac{9}{18}$, and $\frac{27}{54}$ are equivalent fractions. Also, $\frac{3}{2}$, $\frac{9}{4}$, and $\frac{36}{16}$ are equivalent fractions.

Observe that $\frac{3}{6}$ is obtained by multiplying both the numerator and the denominator of $\frac{1}{2}$ by 3. Similarly, $\frac{9}{18}$ is obtained by multiplying both the numerator and the denominator of $\frac{3}{6}$ by 3. All subsequent equivalent fractions can be obtained in the same way.

Example 1

Give four fractions which are equivalent to $\frac{1}{3}$.

Solution

$$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{1 \times 3}{3 \times 3} = \frac{1 \times 4}{3 \times 4} = \frac{1 \times 5}{3 \times 5}$$

$$= \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}.$$

Therefore, the four fractions equivalent to $\frac{1}{3}$ are $\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$, and $\frac{5}{15}$.

Rule: Let $\frac{a}{b}$ and $\frac{c}{d}$ be two fractions.

The two fractions are equivalent if $\frac{a}{b} = \frac{c}{d}$, that is, $ad = bc$. Otherwise, they are not equivalent.

Example 2

Determine whether the fractions $\frac{6}{10}$ and $\frac{9}{15}$ are equivalent.

Solution

Cross multiply the given fractions:

$6 \times 15 = 9 \times 10$.

Thus, $90 = 90$.

Hence, $\frac{6}{10}$ and $\frac{9}{15}$ are equivalent fractions.

Lowest term of a fraction (simplification of a fraction)

A fraction is in its lowest term if the GCF of the numerator and the denominator is 1. Simplification of a fraction into its lowest term is done by dividing both the numerator and the denominator by the same number.

To reduce a fraction to its lowest term, divide both the numerator and the denominator by their GCF.

Example

Simplify each of the following fractions to its lowest term:

(a) $\frac{9}{18}$ (b) $\frac{750}{1250}$ (c) $\frac{648}{1244}$

Solution

Divide the numerator and the denominator by the GCF.

(a) The GCF of 9 and 18 is 9.

$$\frac{9}{18} = \frac{9 \div 9}{18 \div 9}$$

$$= \frac{1}{2}.$$

Therefore, $\frac{9}{18} = \frac{1}{2}.$

(b) The GCF of 750 and 1250 is 250.

$$\frac{750}{1250} = \frac{750 \div 250}{1250 \div 250}$$

$$= \frac{3}{5}.$$

Therefore, $\frac{750}{1250} = \frac{3}{5}.$

(c) The GCF of 648 and 1244 is 4.

$$\frac{648}{1244} = \frac{648 \div 4}{1244 \div 4}$$

$$= \frac{162}{311}.$$

Therefore, $\frac{648}{1244} = \frac{162}{311}.$

Representing fractions on a number line

Fractions can be represented on a number line.

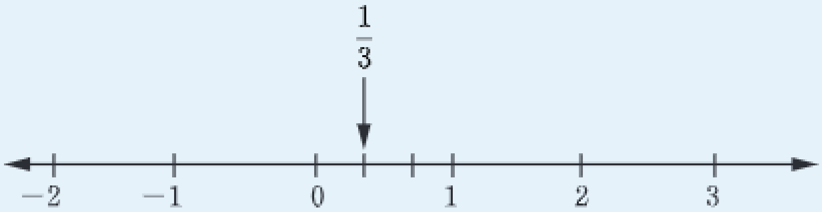
Example 1

Represent $\frac{1}{3}$ on a number line.

Solution

$\frac{1}{3}$ means one divided by three.

This means that, $\frac{1}{3}$ lies between 0 and 1. Divide the length from 0 to 1 into three equal parts. The first part from 0 represents the fraction $\frac{1}{3}$ as shown in the following number line.



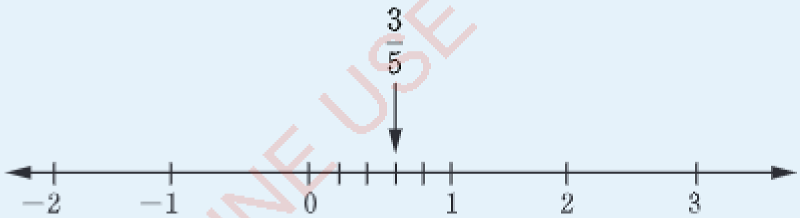
Example 2

Represent $\frac{3}{5}$ on a number line.

Solution

$\frac{3}{5}$ means three divided by five.

This implies that, $\frac{3}{5}$ lies between 0 and 1. Divide the distance from 0 to 1 into five equal parts, then count the first three parts from 0 which represent $\frac{3}{5}$ as shown in the following number line.



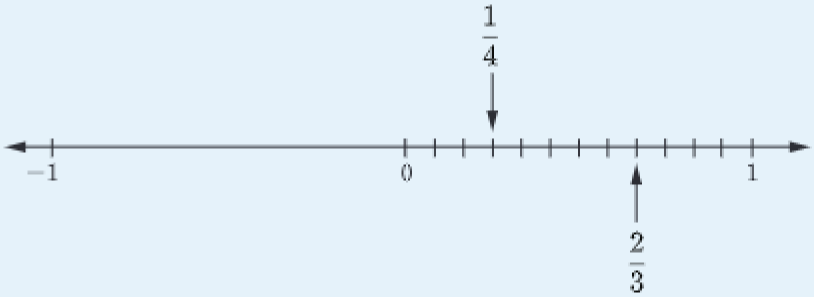
Two or more fractions can also be represented on the same number line.

Example 3

Represent $\frac{1}{4}$ and $\frac{2}{3}$ on a number line.

Solution

Given the fractions $\frac{1}{4}$ and $\frac{2}{3}$ with different denominators. Find the LCM of their denominators. The LCM of 4 and 3 is 12. The LCM shows that you can divide the length from 0 to 1 into 12 equal parts as shown in the following number line.



Each part represents $\frac{1}{12}$. Thus, moving three steps from 0, you arrive at $\frac{1}{4}$, and moving eight steps from 0, you arrive at $\frac{2}{3}$.

Exercise 3

1. Which of the following are proper fractions, improper fractions or mixed fractions? List four equivalent fractions for each of the given fractions.

(a) $\frac{5}{7}$ (b) $\frac{13}{17}$ (c) $\frac{4}{5}$ (d) $\frac{4}{3}$ (e) $\frac{6}{5}$

(f) $\frac{2}{6}$ (g) $\frac{8}{24}$ (h) $\frac{3}{4}$ (i) $\frac{14}{42}$ (j) $\frac{16}{48}$

(k) $\frac{15}{19}$ (l) $\frac{153}{75}$ (m) $\frac{14}{5}$ (n) $1\frac{5}{6}$ (o) $\frac{57}{93}$

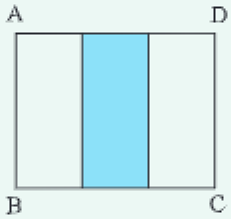
(p) $3\frac{1}{2}$

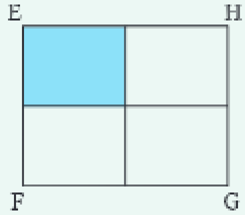
2. Write the following fractions in words:

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{5}{6}$

(e) $\frac{9}{10}$ (f) $\frac{1}{4}$ (g) $\frac{1}{5}$ (h) $\frac{2}{3}$

3. Write the name of the fraction of the shaded part in each of the figures ABCD and EFGH.





4. Simplify each of the following fractions to its lowest term:
   (a) $\frac{456}{88}$ (b) $\frac{4645}{37160}$ (c) $\frac{985}{5910}$

5. Show whether the fractions $\frac{124}{270}$ and $\frac{1488}{3240}$ are equivalent or not.

6. Represent each of the following fractions on the same number line:
   (a) $\frac{1}{2}$ and $\frac{3}{4}$
   (b) $\frac{1}{7}, \frac{3}{7}, \frac{5}{7}$, and $\frac{6}{7}$

7. Represent $\frac{9}{10}$ and $\frac{3}{5}$ on the same number line.

8. What is the condition for a fraction to be called improper?

9. Convert each of the following improper fractions into mixed numbers:
   (a) $\frac{3}{2}$ (c) $\frac{875}{17}$
   (b) $\frac{98}{12}$ (d) $\frac{3674}{2122}$

10. Convert each of the following mixed numbers into improper fractions:
   (a) $3\frac{4}{5}$ (c) $24\frac{15}{23}$
   (b) $172\frac{16}{25}$ (d) $1640\frac{12}{17}$

Comparison of fractions

It is possible to compare two or more fractions, and arrange them in either ascending or descending order. If fractions have the same denominator, then the numerator determines the sizes of the fractions. The larger the numerator, the larger the fraction, and vice versa. For example; in the fractions $\frac{5}{9}, \frac{3}{9}, \frac{8}{9}$, and $\frac{7}{9}$; $\frac{8}{9}$ is the largest and $\frac{3}{9}$ is the smallest.

The following are the steps for determining the sizes of fractions having different denominators:

Step 1: Find the LCM of their denominators.

Step 2: Multiply the LCM by each fraction to form whole numbers.

Step 3: The greater the whole number the greater the fraction.

Example 1

Compare $\frac{1}{3}$ and $\frac{1}{5}$.

Solution

The LCM of 3 and 5 is 15.

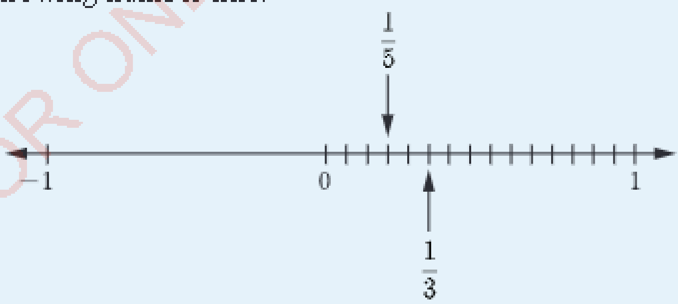
Multiply each fraction by the LCM, that is,

$\frac{1}{3} \times 15 = 5$ and $\frac{1}{5} \times 15 = 3$.

Observe that, despite the fact that 5 is greater than 3, the product of $\frac{1}{3}$ and 15 gives 5, while the product of $\frac{1}{5}$ and 15 gives 3. Therefore, $\frac{1}{3}$ is greater than $\frac{1}{5}$.

Alternatively, the comparison of $\frac{1}{3}$ and $\frac{1}{5}$ can be shown on a number line as follows.

$\frac{1}{3}$ and $\frac{1}{5}$ lie between 0 and 1. Divide the length from 0 to 1 into 15 equal parts (since 15 is the LCM of 3 and 5). Then, count the first three parts from 0 which represent $\frac{1}{5}$ and count the first five parts from 0 which represent $\frac{1}{3}$ as shown in the following number line.



Example 2

Which one is greater between $\frac{15}{24}$ and $\frac{7}{72}$?

Solution

Find the LCM of the denominators of the given fractions.
The LCM of 24 and 72 is 72.
Multiply each fraction by the LCM.

$\frac{15}{24} \times 72 = 45$ and $\frac{7}{72} \times 72 = 7$.

We observe that, 45 is greater than 7.

Therefore, $\frac{15}{24}$ is greater than $\frac{7}{72}$.

Exercise 4

1. Write each of the following fractions in increasing order of their values:
   (a) $\frac{3}{4}, \frac{1}{8}, \frac{3}{8}$
   (b) $\frac{64}{91}, \frac{81}{93}, \frac{16}{41}$
   (c) $\frac{18}{25}, \frac{71}{75}, \frac{84}{93}$
   (d) $\frac{16}{30}, \frac{7}{40}, \frac{11}{55}$

2. Insert the words 'is greater than' or 'is less than' between each of the following pairs of fractions:
   (a) $\frac{1}{9}$ ☐ $\frac{3}{8}$
   (b) $\frac{2}{3}$ ☐ $\frac{1}{3}$
   (c) $\frac{4}{5}$ ☐ $\frac{3}{4}$
   (d) $\frac{5}{6}$ ☐ $\frac{3}{4}$
   (e) $\frac{1}{4}$ ☐ $\frac{3}{4}$
   (f) $\frac{15}{20}$ ☐ $\frac{7}{4}$
   (g) $\frac{8}{9}$ ☐ $\frac{5}{6}$
   (h) $\frac{2}{5}$ ☐ $\frac{1}{6}$

3. In each of the following pairs of fractions, determine which one is greater:
   (a) $\frac{81}{94}, \frac{3}{564}$
   (b) $\frac{48}{155}, \frac{30}{465}$

4. Arrange the following fractions in descending order:
   $\frac{7}{2}, \frac{3}{4}, \frac{1}{5}, 2\frac{1}{6}, \frac{19}{7}, 4\frac{1}{3}$, and $\frac{1}{6}$

5. In each of the following pairs of fractions, which one is smaller than the other?
(a) $3\frac{1}{2}$ or $1\frac{1}{4}$? (b) $\frac{1}{5}$ or $2\frac{1}{7}$?

6. Use '<' or '>' sign to compare each of the following pairs of fractions:
(a) $\frac{2}{9} \square \frac{3}{14}$ (b) $\frac{7}{12} \square \frac{5}{9}$ (c) $\frac{141}{350} \square \frac{61}{555}$

Operations on fractions

Activity 2: Recognising multiplication of fractions

Perform the following tasks individually or in groups:

1. Draw a circle on a manila sheet.
2. Divide the circle into three equal parts.
3. Multiply the fractions obtained in task 2 by $\frac{1}{2}$.
4. What did you observe in task 3?
5. Illustrate the answer obtained in task 3 by drawing.
6. Share your findings with your fellow students in the class through presentation.

Addition of fractions

Fractions can be added to get another fraction.

(a) Addition of fractions with the same denominators

Addition of fractions with the same denominator is done through the following steps:

Step 1: Take the denominator of one fraction as common.

Step 2: Write the sum of the fractions as $\frac{\text{sum of numerators}}{\text{common denominator}}$.

Example

Find the value of each of the following and simplify the results:
(a) $\frac{3}{7}+\frac{2}{7}$ (b) $\frac{19}{48}+\frac{7}{48}+\frac{14}{48}$ (c) $9\frac{5}{8}+3\frac{3}{8}+11\frac{7}{8}$

Solution

(a) $\frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}$

$=\frac{5}{7}.$

Therefore, $\frac{3}{7}+\frac{2}{7}=\frac{5}{7}.$

(b) $\frac{19}{48}+\frac{7}{48}+\frac{14}{48}=\frac{19+7+14}{48}$

$=\frac{40}{48}$

$=\frac{5}{6}.$

Therefore, $\frac{19}{48}+\frac{7}{48}+\frac{14}{48}=\frac{5}{6}.$

(c) $9\frac{5}{8}+3\frac{3}{8}+11\frac{7}{8}=9+3+11+\frac{5}{8}+\frac{3}{8}+\frac{7}{8}$ (adding whole numbers first)

$=23+\frac{5+3+7}{8}$

$=23+\frac{15}{8}$

$=23+1\frac{7}{8}$

$=24\frac{7}{8}.$

Therefore, $9\frac{5}{8}+3\frac{3}{8}+11\frac{7}{8}=24\frac{7}{8}.$

(b) Addition of fractions with different denominators

Addition of fractions with different denominators is done through the following steps:

Step 1: Find the LCM of the denominators.

Step 2: Divide the LCM by the denominator of each fraction.

Step 3: Multiply the numerator of each fraction by the respective value obtained in step 2.

Step 4: Add the products obtained in step 3.

Step 5: The sum of the given fractions is equal to $\frac{\text{the value obtained in step 4}}{\text{LCM of the denominator}}$.

Note: When performing operations on fractions, mixed fractions should first be converted into improper fractions.

Example

Find the value of each of the following and simplify the results:

(a) $\frac{3}{5}+\frac{2}{3}$ (b) $\frac{7}{23}+\frac{11}{46}+\frac{31}{92}$ (c) $5\frac{1}{6}+3\frac{1}{8}+4\frac{5}{12}$

Solution

Since the denominators of the fractions in (a), (b), and (c) are not the same, find the LCM. Divide the LCM by the denominators, and multiply by the numerator of each term.

(a) $\frac{3}{5}+\frac{2}{3}=\frac{(3\times 3)+(5\times 2)}{15}$ (LCM of 3 and 5 is 15)

$=\frac{9+10}{15}$

$=\frac{19}{15}$

$=1\frac{4}{15}$.

Therefore, $\frac{3}{5}+\frac{2}{3}=1\frac{4}{15}$.

(b) $\frac{7}{23}+\frac{11}{46}+\frac{31}{92}=\frac{(7\times 4)+(11\times 2)+(31\times 1)}{92}$

(the LCM of 23, 46, and 92 is 92)

$=\frac{28+22+31}{92}$

$=\frac{81}{92}$.

Therefore, $\frac{7}{23}+\frac{11}{46}+\frac{31}{92}=\frac{81}{92}$.

(c) $5\frac{1}{6}+3\frac{1}{8}+4\frac{5}{12}=\frac{(5\times 6)+1}{6}+\frac{(3\times 8)+1}{8}+\frac{(4\times 12)+5}{12}$

$=\frac{31}{6}+\frac{25}{8}+\frac{53}{12}$

$=\frac{124+75+106}{24}$

$=\frac{305}{24}$

$=12\frac{17}{24}$.

Therefore, $5\frac{1}{6}+3\frac{1}{8}+4\frac{5}{12}=12\frac{17}{24}$.

Alternatively,

$5\frac{1}{6}+3\frac{1}{8}+4\frac{5}{12}=5+3+4+\frac{1}{6}+\frac{1}{8}+\frac{5}{12}$

$=12+\frac{4+3+10}{24}$

$=12+\frac{17}{24}$

$=12\frac{17}{24}$.

Therefore, $5\frac{1}{6}+3\frac{1}{8}+4\frac{5}{12}=12\frac{17}{24}$.

Exercise 5

Find the value of each of the following and simplify the answer where necessary:

1. $\frac{3}{8}+\frac{1}{8}$
2. $\frac{2}{21}+\frac{9}{21}+\frac{11}{42}$
3. $3\frac{2}{7}+2\frac{4}{7}$
4. $2\frac{1}{18}+7\frac{5}{18}$
5. $\frac{8}{22}+\frac{12}{22}$
6. $2\frac{21}{100}+4\frac{19}{100}+3\frac{8}{100}$
7. $\frac{3}{7}+\frac{2}{7}+\frac{5}{7}$
8. $1\frac{1}{2}+5\frac{1}{3}+3\frac{1}{4}$
9. $4\frac{2}{7}+8\frac{2}{5}$
10. $4\frac{1}{6}+3\frac{5}{8}+2\frac{5}{12}$
11. $\frac{1}{6}+\frac{3}{4}+\frac{1}{4}$
12. $\frac{7}{20}+\frac{3}{10}+\frac{1}{5}$
13. $\frac{12}{3}+\frac{18}{6}+\frac{22}{12}$
14. $\frac{2}{5}+\frac{3}{4}+\frac{7}{10}$
15. $\frac{3}{18}+\frac{4}{9}+\frac{1}{3}$
16. $\frac{1}{8}+\frac{1}{6}+\frac{1}{4}$
17. $2\frac{1}{3}+\frac{5}{7}$
18. $1\frac{1}{5}+3\frac{2}{9}$

19. $\frac{4}{5} + 6\frac{2}{7}$

20. $\frac{2}{3} + 12\frac{3}{4} + \frac{1}{5}$

Subtraction of fractions

Activity 3: Recognising subtraction of fractions

Take a watermelon and then, perform the following tasks individually or in groups:

1. Cut it into eight equal pieces.
2. Take two pieces and give them to your first friend. Write down the fraction of the two pieces.
3. Take one more piece and give it to another friend. Write down the fraction of the piece.
4. Write the remaining fractions after giving the three pieces to your friends.
5. Write a formula you can use to get the answer obtained in task 4.
6. Share your findings with your fellow students in the class through presentation.

Fractions can be subtracted to get another fraction. When subtracting fractions, use the same steps as those used for adding fractions.

For example; when subtracting $\frac{3}{5}$ from $\frac{4}{5}$, remember that the denominators are the same.

Thus, $\frac{4}{5} - \frac{3}{5} = \frac{4-3}{5} = \frac{1}{5}$.

Example 1

Find the value of each of the following:

(a) $\frac{6}{7} - \frac{2}{7}$

(b) $\frac{17}{35} - \frac{11}{35} - \frac{1}{35}$

Solution

(a) $\frac{6}{7} - \frac{2}{7} = \frac{6-2}{7}$

$= \frac{4}{7}$.

Therefore, $\frac{6}{7} - \frac{2}{7} = \frac{4}{7}$.

(b) $\frac{17}{35} - \frac{11}{35} - \frac{1}{35} = \frac{17 - 11 - 1}{35}$

$= \frac{5}{35}$

$= \frac{5 \div 5}{35 \div 5}$

$= \frac{1}{7}.$

Therefore, $\frac{17}{35} - \frac{11}{35} - \frac{1}{35} = \frac{1}{7}.$

Example 2

Find the value of each of the following:

(a) $\frac{29}{45} - \frac{7}{15} - \frac{1}{30}$ (b) $4\frac{2}{3} - 1\frac{5}{6}$ (c) $4\frac{1}{3} - 2\frac{1}{2}$

Solution

(a) $\frac{29}{45} - \frac{7}{15} - \frac{1}{30} = \frac{(29 \times 2) - (7 \times 6) - (1 \times 3)}{90}$

(the LCM of 45, 15, and 30 is 90)

$= \frac{58 - 42 - 3}{90}$

$= \frac{13}{90}.$

Therefore, $\frac{29}{45} - \frac{7}{15} - \frac{1}{30} = \frac{13}{90}.$

(b) $4\frac{2}{3} - 1\frac{5}{6} = (4 - 1) + \left(\frac{2}{3} - \frac{5}{6}\right)$

$= 3 + \frac{4 - 5}{6}$ (the LCM of 3 and 6 is 6)

$= 2 + 1 + \frac{4 - 5}{6}$ (since 4 is less than 5, split the whole number 3 as 2 + 1).

But, $1 = \frac{6}{6}$

$4\frac{2}{3} - 1\frac{5}{6} = 2 + \frac{6}{6} + \frac{4}{6} - \frac{5}{6}$ since $\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}$

$= 2 + \frac{10}{6} - \frac{5}{6}$

$= 2 + \frac{5}{6}$

$= 2\frac{5}{6}.$

Therefore, $4\frac{2}{3} - 1\frac{5}{6} = 2\frac{5}{6}.$

Alternatively,

$$4\frac{2}{3}-1\frac{5}{6}=\frac{(4\times 3)+2}{3}-\frac{(1\times 6)+5}{6}$$

$$=\frac{14}{3}-\frac{11}{6}$$

$$=\frac{28-11}{6} \text{ (the LCM of 3 and 6 is 6)}$$

$$=\frac{17}{6}$$

$$=2\frac{5}{6}.$$

Therefore, $4\frac{2}{3}-1\frac{5}{6} = 2\frac{5}{6}$.

(c) $4\frac{1}{3}-2\frac{1}{2}=(4-2)+\left(\frac{1}{3}-\frac{1}{2}\right)$

$$=2+\frac{2-3}{6} \text{ (the LCM of 3 and 2 is 6)}$$

$=1+1+\frac{2-3}{6}$ (since 2 is less than 3, split the whole number 2 into 1 + 1).

But, $1=\frac{6}{6}$

$$4\frac{1}{3}-2\frac{1}{2}=1+\frac{6}{6}+\frac{2-3}{6}$$

$$=1+\left(\frac{6}{6}+\frac{2}{6}\right)-\frac{3}{6} \text{ since } \frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$$

$$=1+\left(\frac{8}{6}-\frac{3}{6}\right)$$

$$=1+\frac{5}{6}$$

$$=1\frac{5}{6}.$$

Therefore, $4\frac{1}{3}-2\frac{1}{2} = 1\frac{5}{6}$.

Alternatively,

$$4\frac{1}{3}-2\frac{1}{2}=\frac{(4\times 3)+1}{3}-\frac{(2\times 2)+1}{2}$$

$$=\frac{13}{3}-\frac{5}{2}$$

$$= \frac{26-15}{6}$$

$$= \frac{11}{6}$$

$$= 1\frac{5}{6}.$$

Therefore, $4\frac{1}{3} - 2\frac{1}{2} = 1\frac{5}{6}.$

Exercise 6

Find the value of each of the following, and simplify the answer where necessary:

1. $\frac{15}{16} - \frac{3}{16}$
2. $\frac{59}{107} - \frac{15}{107}$
3. $\frac{7}{8} - \frac{2}{5}$
4. $\frac{5}{12} - \frac{1}{3} - \frac{1}{36}$
5. $\frac{7}{8} - \frac{2}{3}$
6. $\frac{17}{100} - \frac{1}{10}$
7. $3\frac{5}{7} - \frac{6}{7}$
8. $5\frac{2}{3} - \frac{5}{6} - \frac{7}{18}$
9. $6\frac{1}{2} - 3\frac{1}{4}$
10. $7\frac{1}{5} - 2\frac{2}{3} - 2\frac{2}{5}$
11. $9\frac{3}{4} - 5\frac{7}{8}$
12. $8\frac{5}{7} - 3\frac{11}{14} - 2\frac{1}{28}$
13. $13\frac{1}{5} - 2\frac{1}{3}$
14. $4\frac{1}{6} - 2\frac{3}{4}$
15. $9\frac{2}{9} - 5\frac{2}{3}$

Multiplication of fractions

To multiply fractions, first multiply their numerators and then, multiply their denominators. For mixed numbers, change them into improper fractions and then, multiply accordingly.

Rule: Product of fractions $= \frac{\text{product of numerators}}{\text{product of denominators}}.$

Generally,

1. If $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$ are fractions, where $b \neq 0$, $d \neq 0$, $f \neq 0$ then,
$\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} = \frac{a \times c \times e}{b \times d \times f} = \frac{ace}{bdf}$.
2. If $a\frac{b}{c}$, $d\frac{e}{f}$, and $g\frac{h}{i}$ are mixed numbers, where $c \neq 0$, $f \neq 0$, $i \neq 0$ then,
$$a\frac{b}{c} \times d\frac{e}{f} \times g\frac{h}{i} = \frac{ac+b}{c} \times \frac{df+e}{f} \times \frac{gi+h}{i}$$
$$= \frac{(ac+b) \times (df+e) \times (gi+h)}{cfi}.$$
3. If two fractions are such that their product is 1, then each fraction is called the reciprocal of the other. This means, $\frac{c}{d} \times \frac{d}{c} = \frac{c \times d}{d \times c} = \frac{cd}{dc} = 1$. This shows that, $\frac{c}{d}$ is the reciprocal of $\frac{d}{c}$ and vice versa.

A reciprocal of a fraction is obtained by interchanging the numerator and the denominator.

Example 1

Find the value of each of the following:

(a) $\frac{2}{5} \times \frac{3}{5}$ (b) $\frac{30}{51} \times \frac{2}{45}$ (c) $\frac{4}{9} \times \frac{7}{10} \times \frac{7}{2}$

Solution

(a) $\frac{2}{5} \times \frac{3}{5} = \frac{2 \times 3}{5 \times 5}$

$= \frac{6}{25}$.

Therefore, $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$.

(b) $\frac{30}{51} \times \frac{2}{45} = \frac{30 \times 2}{51 \times 45}$

$= \frac{60}{2295}$

$= \frac{4}{153}$.

Therefore, $\frac{30}{51} \times \frac{2}{45} = \frac{4}{153}$.

(c) $\frac{4}{9} \times \frac{7}{10} \times \frac{7}{2} = \frac{4 \times 7 \times 7}{9 \times 10 \times 2}$

$= \frac{196}{180}$

$= \frac{49}{45}$

$= 1\frac{4}{45}$.

Therefore, $\frac{4}{9} \times \frac{7}{10} \times \frac{7}{2} = 1\frac{4}{45}$.

Example 2

Find the value of each of the following, and simplify the answer where necessary:

(a) $1\frac{3}{4} \times 9$ (b) $3\frac{3}{7} \times 9\frac{4}{5}$ (c) $\frac{1}{8} \times \frac{4}{15} \times 6\frac{2}{5}$

Solution

(a) $1\frac{3}{4} \times 9 = \frac{(4 \times 1) + 3}{4} \times 9$

$= \frac{7}{4} \times 9$

$= \frac{7}{4} \times \frac{9}{1}$

$= \frac{7 \times 9}{4 \times 1}$

$= \frac{63}{4}$

$= 15\frac{3}{4}$.

Therefore, $1\frac{3}{4} \times 9 = 15\frac{3}{4}$.

(b) $3\frac{3}{7} \times 9\frac{4}{5} = \frac{(3 \times 7) + 3}{7} \times \frac{(9 \times 5) + 4}{5}$

$= \frac{24}{7} \times \frac{49}{5}$

$= \frac{24 \times 7}{1 \times 5}$

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$= \frac{168}{5}$

$= 33\frac{3}{5}.$

Therefore, $3\frac{3}{7} \times 9\frac{4}{5} = 33\frac{3}{5}.$

(c) $\frac{1}{8} \times \frac{4}{15} \times 6\frac{2}{5} = \frac{1}{8} \times \frac{4}{15} \times \frac{(6 \times 5) + 2}{5}$

$= \frac{1}{8} \times \frac{4}{15} \times \frac{32}{5}$

$= \frac{1 \times 4 \times 4}{1 \times 15 \times 5}$

$= \frac{16}{75}.$

Therefore, $\frac{1}{8} \times \frac{4}{15} \times 6\frac{2}{5} = \frac{16}{75}.$

Example 3

Find the reciprocal of each of the following fractions:

(a) $\frac{2}{3}$ (b) $\frac{1}{5}$ (c) $\frac{4}{7}$

Solution

(a) Interchanging the numerator and the denominator of $\frac{2}{3}$ gives $\frac{3}{2}$.
Therefore, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

(b) Interchanging the numerator and the denominator of $\frac{1}{5}$ gives $\frac{5}{1} = 5$.
Therefore, the reciprocal of $\frac{1}{5}$ is 5.

(c) Interchanging the numerator and the denominator of $\frac{4}{7}$ gives $\frac{7}{4}$.
Therefore, the reciprocal of $\frac{4}{7}$ is $\frac{7}{4}$.

Exercise 7

Find the value of each of the following:

1. $\frac{2}{5} \times \frac{2}{3}$
2. $\frac{16}{45} \times \frac{55}{72}$
3. $\frac{3}{4} \times \frac{3}{5}$
4. $\frac{11}{15} \times \frac{9}{10}$

5. $\frac{7}{9} \times \frac{3}{28}$
6. $8\frac{1}{21} \times 2\frac{5}{18}$
7. $\frac{5}{9} \times \frac{14}{15} \times \frac{3}{7}$
8. $\frac{7}{20} \times \frac{1}{6} \times \frac{18}{7}$
9. $\frac{9}{5} \times \frac{25}{21} \times \frac{14}{5}$
10. $\frac{7}{18} \times \frac{3}{49} \times \frac{25}{27}$
11. $5\frac{1}{4} \times 2\frac{2}{3}$
12. $7\frac{1}{2} \times 4\frac{4}{7} \times 3\frac{14}{15}$
13. $7\frac{1}{2} \times 1\frac{1}{3} \times \frac{9}{10}$
14. $4\frac{1}{2} \times 1\frac{1}{7} \times 2\frac{1}{3}$
15. $\frac{5}{9} \times 1\frac{1}{10} \times 12$

Multiplication of fractions by using illustrations

Activity 4: Recognising multiplication of fractions by using illustrations

Perform the following tasks individually or in groups:

1. Draw two different rectangles of the same size.
2. Construct five equal rows for the first rectangle, and then shade two rows.
3. Construct nine equal columns for the second rectangle, and then shade seven columns.
4. Combine the shaded rows in task 2 and the shaded columns in task 3 in one rectangle.
5. Using task 4, explain with reasons how you can obtain the solution for $\frac{2}{5} \times \frac{7}{9}$.

Two fractions can be multiplied by using illustrations.

Example

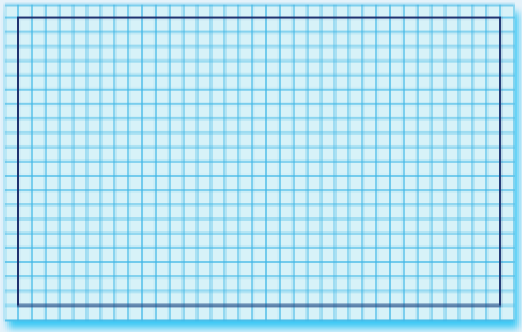
Find the value of $\frac{1}{3} \times \frac{2}{5}$ by using illustrations.

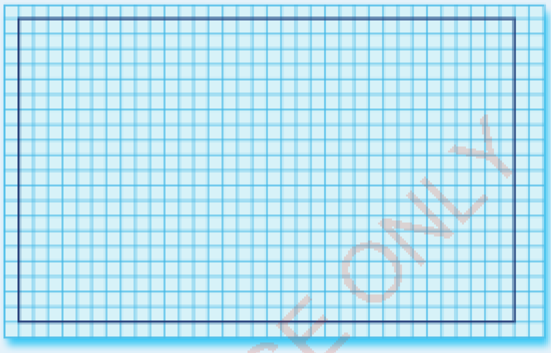
Solution

The solution can be obtained through the following steps:

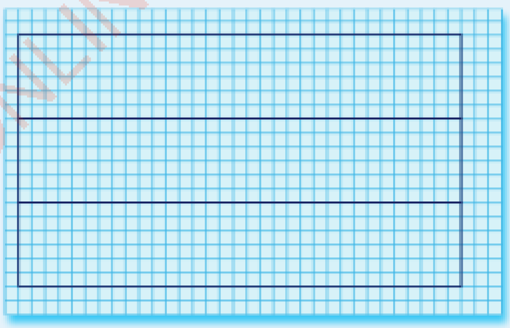
Step 1: Draw two different rectangles of the same size.

$\frac{1}{3} \times \frac{2}{5}$

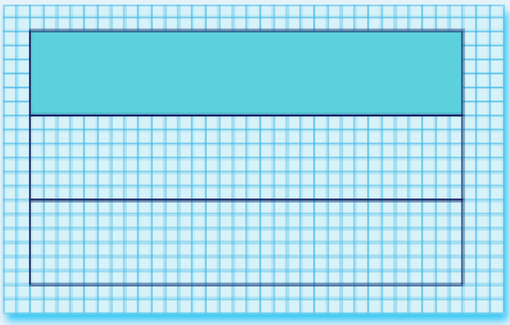




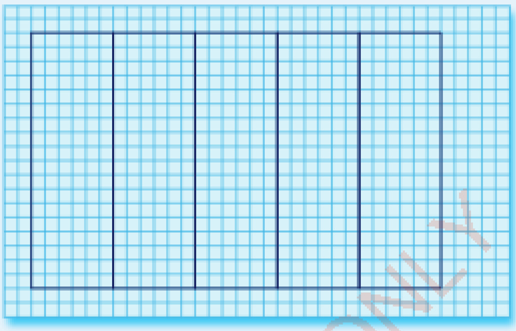
Step 2: Construct 3 equal rows for the first rectangle (3 is the denominator of the first fraction).



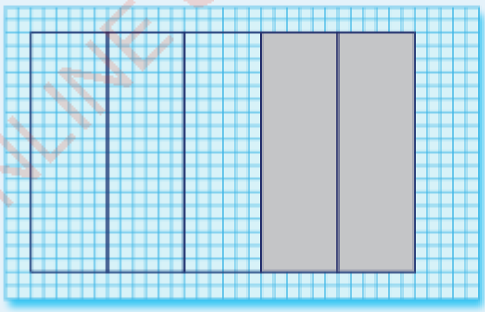
Step 3: Shade 1 row of the first rectangle (1 is the numerator of the first fraction).



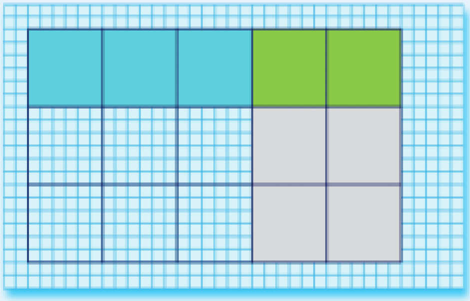
Step 4: Construct 5 equal columns for the second rectangle (5 is the denominator of the second fraction).



Step 5: Shade 2 columns of the second rectangle (2 is the numerator of the second fraction).



Step 6: Combine the two shaded rectangles of step 3 and step 5 in one rectangle.



Step 7: The total number of small rectangles formed in step 6 stands for the denominator. The total number of small rectangles shaded twice stands for the numerator.

Therefore, $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$.

Exercise 8

Find the value of each of the following by using illustrations, and then simplify the answer where necessary:

1. $\frac{1}{3} \times \frac{3}{4}$
2. $\frac{3}{5} \times \frac{5}{7}$
3. $\frac{4}{5} \times \frac{3}{8}$
4. $\frac{2}{3} \times \frac{1}{5}$
5. $\frac{1}{2} \times \frac{7}{7}$
6. $\frac{1}{5} \times \frac{1}{2}$

Division of fractions

In order to divide a fraction by another fraction, we multiply the dividend by the reciprocal of the divisor.

For example: $12 \div \frac{1}{4} = \frac{12}{1} \div \frac{1}{4}$

dividend ↗ ↖ divisor

$\frac{12}{1} \div \frac{1}{4} = \frac{12}{1} \times \frac{4}{1}$

$$= \frac{12 \times 4}{1 \times 1}$$

$$= \frac{48}{1}$$

$$= 48.$$

Therefore, $12 \div \frac{1}{4} = 48.$

Dividing 12 by $\frac{1}{4}$ is the same as finding the total number of quarters in 12. Thus, there are 48 quarters in 12.

Division of fractions is done through the following steps:

Step 1: Find the reciprocal of the divisor.

Step 2: Find the product of the dividend and the reciprocal of the divisor obtained in step 1.

Step 3: Simplify the answer to its lowest term where necessary.

Example 1

Find the value of each of the following, and simplify the answer where necessary:

(a) $6 \div \frac{3}{5}$ (b) $\frac{7}{48} \div \frac{14}{25}$ (c) $\frac{7}{5} \div 21$

Solution

(a) $6 \div \frac{3}{5} = \frac{6}{1} \div \frac{3}{5}$

$$= \frac{6}{1} \times \frac{5}{3}$$

$$= \frac{2 \times 5}{1 \times 1}$$

$$= 10.$$

Therefore, $6 \div \frac{3}{5} = 10.$

(b) $\frac{7}{48} \div \frac{14}{25} = \frac{7}{48} \times \frac{25}{14}$

$$= \frac{7 \times 25}{48 \times 14}$$

$$= \frac{25}{96}.$$

Therefore, $\frac{7}{48} \div \frac{14}{25} = \frac{25}{96}.$

(c) $\frac{7}{5} \div 21 = \frac{7}{5} \div \frac{21}{1}$

$= \frac{7}{5} \times \frac{1}{21}$

$= \frac{7 \times 1}{5 \times 21}$

$= \frac{1}{15}.$

Therefore, $\frac{7}{5} \div 21 = \frac{1}{15}.$

When dividing fractions involving mixed numbers, first change the mixed numbers into improper fractions.

Example 2

Find the value of each of the following, and simplify the answer where necessary:

(a) $4\frac{1}{5} \div 7\frac{2}{3}$ (b) $5\frac{4}{9} \div \frac{14}{27}$ (c) $9\frac{1}{7}$ divide by $1\frac{11}{21}$

Solution

(a) $4\frac{1}{5} \div 7\frac{2}{3} = \frac{(4 \times 5) + 1}{5} \div \frac{(7 \times 3) + 2}{3}$

$= \frac{21}{5} \div \frac{23}{3}$

$= \frac{21 \times 3}{5 \times 23}$

$= \frac{63}{115}.$

Therefore, $4\frac{1}{5} \div 7\frac{2}{3} = \frac{63}{115}.$

(b) $5\frac{4}{9} \div \frac{14}{27} = \frac{(5 \times 9) + 4}{9} \div \frac{14}{27}$

$= \frac{49}{9} \div \frac{14}{27}$

$= \frac{49 \times 27}{9 \times 14}$

$= \frac{7 \times 3}{1 \times 2}$

$= \frac{21}{2}$

$= 10\frac{1}{2}$.

Therefore, $5\frac{4}{9} \div \frac{14}{27} = 10\frac{1}{2}$.

(c) $9\frac{1}{7} \div 1\frac{11}{21} = \frac{(9 \times 7) + 1}{7} \div \frac{(1 \times 21) + 11}{21}$

$= \frac{64}{7} \div \frac{32}{21}$

$= \frac{64 \times 21}{7 \times 32}$

$= 2 \times 3$

$= 6$.

Therefore, $9\frac{1}{7} \div 1\frac{11}{21} = 6$.

Exercise 9

Find the value of each of the following, and simplify the answer where necessary:

1. $18 \div \frac{6}{7}$
2. $40 \div \frac{8}{9}$
3. $14 \div \frac{7}{8}$
4. $72 \div \frac{9}{10}$
5. $35 \div \frac{5}{7}$
6. $\frac{23}{32} \div \frac{3}{4}$
7. $\frac{27}{25} \div \frac{3}{5}$
8. $\frac{8}{25} \div \frac{4}{5}$
9. $\frac{9}{26} \div \frac{12}{13}$
10. $\frac{49}{50} \div \frac{7}{10}$
11. $3\frac{3}{8} \div \frac{15}{4}$
12. $2\frac{2}{7} \div \frac{9}{14}$
13. $7\frac{1}{5} \div 1\frac{7}{20}$
14. $5\frac{5}{8} \div 6\frac{1}{4}$
15. $9\frac{3}{4} \div 1\frac{5}{8}$
16. Divide $10\frac{5}{6}$ by $3\frac{1}{4}$
17. Divide $15\frac{4}{15}$ by $3\frac{1}{5}$
18. Divide $8\frac{2}{3}$ by $5\frac{7}{9}$
19. Divide $4\frac{2}{3}$ by $1\frac{1}{6}$
20. $10\frac{2}{3} \div 1\frac{7}{9}$
21. $8\frac{4}{5} \div 3\frac{3}{10}$

Mixed operations on fractions

Example 1

Find the value of $\frac{3}{8} \times 1\frac{1}{2} \div \frac{12}{16}$.

Solution

Change the mixed number into improper fraction, that is, $1\frac{1}{2}$ to $\frac{3}{2}$.

Change $\frac{12}{16}$ to $\frac{16}{12}$ and change $\div$ into $\times$, that is,

$$\frac{3}{8} \times 1\frac{1}{2} \div \frac{12}{16} = \frac{3}{8} \times \frac{3}{2} \div \frac{12}{16}$$

$$= \frac{3}{8} \times \frac{3}{2} \times \frac{16}{12}$$

$$= \frac{3}{4}.$$

Therefore, $\frac{3}{8} \times 1\frac{1}{2} \div \frac{12}{16} = \frac{3}{4}$.

Example 2

Find the value of each of the following:

(a) $4\frac{1}{2} \times 2\frac{1}{3} \div 1\frac{1}{4}$ (b) $\frac{7}{8} \div \left(1\frac{1}{2} \times 1\frac{5}{9}\right)$ (c) $\frac{2}{3} + \left(\frac{2}{5} - \frac{1}{9}\right) \div \frac{1}{3}$

Solution

(a) $4\frac{1}{2} \times 2\frac{1}{3} \div 1\frac{1}{4} = \frac{9}{2} \times \frac{7}{3} \div \frac{5}{4}$

$$= \frac{9 \times 7 \times 4}{2 \times 3 \times 5}$$

$$= \frac{3 \times 7 \times 2}{1 \times 1 \times 5}$$

$$= \frac{42}{5}$$

$$= 8\frac{2}{5}.$$

Therefore, $4\frac{1}{2} \times 2\frac{1}{3} \div 1\frac{1}{4} = 8\frac{2}{5}$.

(b) $\frac{7}{8} \div \left(1\frac{1}{2} \times 1\frac{5}{9}\right) = \frac{7}{8} \div \left(\frac{3}{2} \times \frac{14}{9}\right)$

$= \frac{7}{8} \div \frac{42}{18}$

$= \frac{7}{8} \times \frac{18}{42}$

$= \frac{1 \times 9}{4 \times 6}$

$= \frac{1 \times 3}{4 \times 2}$

$= \frac{3}{8}.$

Therefore, $\frac{7}{8} \div \left(1\frac{1}{2} \times 1\frac{5}{9}\right) = \frac{3}{8}.$

(c) $\frac{2}{3} + \left(\frac{2}{5} - \frac{1}{9}\right) \div \frac{1}{3} = \frac{2}{3} + \left(\frac{18-5}{45}\right) \div \frac{1}{3}$

$= \frac{2}{3} + \frac{13}{45} \div \frac{1}{3}$

$= \frac{2}{3} + \frac{13}{45} \times \frac{3}{1}$

$= \frac{2}{3} + \frac{13 \times 3}{45 \times 1}$

$= \frac{2}{3} + \frac{39}{45}$

$= \frac{2 \times 15 + 39 \times 1}{45}$

$= \frac{30 + 39}{45}$

$= \frac{69}{45}$

$= \frac{23}{15}$

$= 1\frac{8}{15}.$

Therefore, $\frac{2}{3} + \left(\frac{2}{5} - \frac{1}{9}\right) \div \frac{1}{3} = 1\frac{8}{15}.$

Note: When working out with mixed operations, remember the rules of BODMAS.

Exercise 10

Find the value of each of the following, and simplify the answers where necessary:

1. $\frac{1}{3} - \frac{1}{4} \times \frac{1}{6}$

2. $\dfrac{1\frac{5}{8} - \frac{3}{4} + 6\frac{1}{2}}{8\frac{7}{8} \div 1\frac{1}{2} \times \frac{1}{6} + \frac{1}{4}}$

3. $\frac{5}{6} \times \frac{3}{10} - \frac{3}{16}$

4. $\left(\frac{5}{8} - \frac{1}{3}\right) \times \frac{4}{7}$

5. $\frac{2}{9} + \left(\frac{6}{9} \div \frac{3}{4}\right) \times 3$

6. $\left(6 - 1\frac{1}{2}\right) \div \left(3 \times \frac{3}{5} + 1\frac{3}{5}\right) + 4$

7. $\frac{9}{10} \div \left(\frac{1}{6} + \frac{2}{3}\right)$

8. $2\frac{2}{5} - \frac{7}{10} \times \left(\frac{4}{7} - \frac{1}{3}\right)$

9. $\left(\frac{4}{7} + \frac{1}{3}\right) \times 3\frac{4}{5}$

10. $\frac{7}{10} \div \left(\frac{2}{5} + \frac{4}{15}\right) \times \frac{3}{5}$

Word problems involving fractions

Example 1

A piece of cloth $37\frac{1}{2}$ centimetres long was cut into equal pieces; each was $1\frac{1}{2}$ centimetres long. How many pieces were obtained?

Solution

Length of the cloth is $37\frac{1}{2}$ cm

Length of each piece is $1\frac{1}{2}$ cm

Thus, the number of small pieces = length of the cloth ÷ length of one piece

$$= 37\frac{1}{2}\text{ cm} \div 1\frac{1}{2}\text{ cm}$$

$$= \frac{(37 \times 2) + 1}{2}\text{ cm} \div \frac{(1 \times 2) + 1}{2}\text{ cm}$$

$$= \frac{75}{2}\text{ cm} \div \frac{3}{2}\text{ cm}$$

$$= \frac{75}{2} \times \frac{2}{3}$$

$$= 25.$$

Therefore, 25 pieces were obtained.

Example 2

A bag of rice weighs $2\frac{1}{2}$ kilograms. What is the mass of 16 such bags?

Solution

1 bag weighs $2\frac{1}{2}$ kilograms

16 bags weigh?

$$\frac{2\frac{1}{2}\text{ kilograms} \times 16\text{ bags}}{1\text{ bag}} = \frac{5}{2}\text{ kilograms} \times 16$$

$$= \frac{5 \times 16}{2}\text{ kilograms}$$

$$= \frac{80}{2}\text{ kilograms}$$

$$= 40\text{ kilograms.}$$

Therefore, 16 bags weigh 40 kilograms.

Example 3

A man gave $\frac{7}{25}$ of his money to his son, $\frac{2}{9}$ of the remaining amount to his daughter, and the remaining amount to his wife. If his wife got 11 990 300 Tanzanian shillings;

(a) how much money did the man have in total?

(b) how much money did his daughter get?

Solution

(a) Fraction given to the son = $\frac{7}{25}$.

Fraction that remained $= 1 - \frac{7}{25}$

$= \frac{25}{25} - \frac{7}{25}$

$= \frac{18}{25}$.

Fraction given to the daughter $= \frac{18}{25} \times \frac{2}{9}$

$= \frac{4}{25}$.

Fraction given to the son and the daughter $= \frac{7}{25} + \frac{4}{25}$

$= \frac{11}{25}$.

Fraction left $= 1 - \frac{11}{25}$

$= \frac{25 - 11}{25}$

$= \frac{14}{25}$.

Thus, the fraction given to his wife $= \frac{14}{25}$.

$\frac{14}{25} \times$ total amount of money

= Tsh 11 990 300.

By cross multiplication, we have;

Total amount of money

$= \text{Tsh } \frac{11\ 990\ 300 \times 25}{14}$

= Tsh 21 411 250.

Therefore, the man had 21 411 250 Tanzanian shillings in total.

(b) The amount of money given to the daughter

$= \frac{4}{25} \times$ Tsh 21 411 250

$= 4 \times$ Tsh 856 450

= Tsh 3 425 800.

Therefore, the daughter got 3 425 800 Tanzanian shillings.

Example 4

Linda spent $\frac{7}{9}$ of her income, and remained with 88 100 Tanzanian shillings. What was her income?

Solution

If she spent $\frac{7}{9}$ of her income, then she remained with $1 - \frac{7}{9} = \frac{2}{9}$ of her income.

$\frac{2}{9}$ of Linda's income is Tsh 88 100.

$\frac{7}{9}$ of Linda's income is?

$$= \left(\frac{7}{9} \times \text{Tsh } 88\ 100\right) \div \frac{2}{9}$$

$$= \frac{7}{9} \times \text{Tsh } 88\ 100 \times \frac{9}{2}$$

$$= \frac{7}{2} \times \text{Tsh } 88\ 100$$

$$= 7 \times \text{Tsh } 44\ 050$$

$$= \text{Tsh } 308\ 350.$$

Thus, she spent 308 350 Tanzanian shillings. Her income = income spent + the amount that remained.

$$= \text{Tsh } 88\ 100 + \text{Tsh } 308\ 350$$

$$= \text{Tsh } 396\ 450.$$

Therefore, Linda's income was 396 450 Tanzanian shillings.

Altenatively,

Let Linda's income be Tsh a.

She spent $\frac{7}{9}$ of Tsh a, which equals Tsh $\frac{7}{9}a$.

Remaining amount $= a - \frac{7}{9}a$

$$= \left(1 - \frac{7}{9}\right)a$$

$$= \frac{2}{9}a.$$

Thus, $\frac{2}{9}a = \text{Tsh } 88\ 100$

$$2a = \text{Tsh } 88\ 100 \times 9$$

$$a = \frac{\text{Tsh } 792\ 900}{2}$$

$$a = \text{Tsh } 396\ 450.$$

Therefore, Linda's income was 396 450 Tanzanian shillings.

Exercise 11

1. If a piece of wire is $43\frac{1}{2}$ centimetres long, find the total length of 24 such pieces of wire.
2. Doto had 45 kilograms of sugar. If $15\frac{1}{2}$ kilograms were sold, find the mass of sugar that remained.
3. Mosi bought $8\frac{1}{2}$ kilograms of beans and $5\frac{1}{4}$ kilograms of peas. Find the total mass of the beans and the peas.
4. In a mixed school, $\frac{5}{7}$ are boys. If there are 1 842 girls in the school, find the total number of students in the school.
5. How many packets of sweets, each weighing $\frac{3}{4}$ kilograms, can be made from a packet weighing 138 kg?
6. Beatrice had some money. She spent $\frac{1}{8}$ of it on transport, and $\frac{3}{7}$ of the remainder on clothes. She spent the rest of the money on food. If the amount spent on food was 2 085 750 shillings, how much money did she have in total?
7. An empty bottle weighs 112 grams. If 56 tablets each weighing $\frac{1}{4}$ grams are put in the bottle, what is its total mass?

8. Mponi's bag contains 5 books weighing $\frac{3}{7}$ kilograms each. If Mponi removes 3 books from the bag, what is the mass of the remaining books?

9. The area of a rectangle is 420 square centimetres. If its width is $10\frac{1}{2}$ centimetres, find its length.

10. A car travels $483\frac{1}{2}$ kilometres in $8\frac{2}{3}$ hours. How far does it go in two hours?

Chapter summary

1. A fraction describes parts of a given quantity, usually written as $\frac{a}{b}$, where b is not equal to 0.
2. A numerator is the top number of a fraction.
3. A denominator is the bottom number of a fraction.
4. A proper fraction has the denominator greater than the numerator.
5. An improper fraction has the numerator greater than the denominator.
6. Mixed numbers consist of whole numbers, and proper fractions.
7. Equivalent fractions have the same value.
8. A reciprocal of a fraction is obtained by interchanging the numerator and the denominator.
9. To divide the fractions, multiply the dividend by the reciprocal of the divisor.
10. In order to multiply, or divide mixed fractions, transform them into improper fractions.

Revision exercise

In question 1 to 14, find the value of each expression and simplify the answer where necessary:

1. $\frac{9}{10} + \frac{4}{5}$
2. $\frac{7}{11} - \frac{1}{2}$
3. $4\frac{1}{6} - 2\frac{2}{3}$
4. $8\frac{6}{7} + 6\frac{1}{3} - 2\frac{1}{4}$
5. $\frac{3}{5} \times \frac{5}{3} + \frac{14}{15}$
6. $6\frac{1}{4} \times 3\frac{2}{3} \times 2\frac{1}{8}$
7. $9\frac{2}{5} \div 3\frac{2}{3}$
8. Divide $1\frac{5}{6}$ by $2\frac{5}{6}$
9. $\left(\frac{4}{9} + \frac{1}{4}\right) \times 2\frac{4}{5}$
10. Compare $\frac{9}{25}$ and $\frac{4}{15}$
11. $2\frac{2}{3} - \frac{7}{10} \times \left(\frac{4}{7} - \frac{1}{3}\right)$
12. $\left(\frac{5}{11} - \frac{1}{3}\right) \times \frac{3}{8}$
13. $4\frac{1}{3} \div 2\frac{1}{6} + \frac{3}{4}$
14. $7\frac{1}{3} - 4\frac{1}{8} \div 2\frac{3}{4}$
15. Four boxes weigh $21\frac{2}{3}$ kilograms, $25\frac{1}{2}$ kilograms, $18\frac{3}{4}$ kilograms, and $17\frac{1}{2}$ kilograms respectively. If a porter can carry all the four boxes, what is the total mass carried by the porter?
16. Three sisters shared some money. If the eldest got $\frac{4}{15}$ of the money while the middle sister got $\frac{7}{30}$ of the remainder, what fraction of the money did the youngest sister get?
17. A painter can paint a door for $1\frac{1}{4}$ hours. How long will the painter take to paint 18 doors of the same size?

18. A tank is full of water. When 78 litres are drawn off, it is $\frac{2}{3}$ full. What amount of water will the tank hold when it is full?

19. In a certain class, 8 students take chemistry, 15 take history, 12 take french, and 19 take biology. If 5 students from each subject are selected to represent their fellows in a tour, find the fraction of the students represented by their fellows in that tour from each subject.

20. An 80 metre long rope was cut into four pieces. The four pieces were distributed among Form One, Form Two, Form Three, and Form Four students. In this distribution, Form One got 15 metres, Form Two got 20 metres, Form Three got 40 metres, and the rest of the piece was given to Form Four.
    (a) Write the fraction of the piece of rope distributed to each class.
    (b) Which fraction is the largest of all the fractions?
    (c) Find the difference between the largest fraction and the smallest fraction.

Chapter Four

Decimals and percentages

Introduction

Many numeral systems of ancient civilizations used ten and its power to represent numbers. It was difficult to represent very large numbers in these old numeral systems, and only best mathematicians were able to multiply or divide large numbers. These difficulties were completely solved with the introduction of the Hindu-Arabic numeral system for representing integers. This system has been extended to represent some non-integer numbers called decimals for forming the decimal numeral system. In ancient Rome, long before the existence of the decimal system, computations were often made in fractions in the multiple of $\frac{1}{100}$*. Computation with these fractions was equivalent to computing percentages. In this chapter, you will learn about converting fractions into different types of decimals, expressing quantities as percentages, and converting fractions and decimals into percentages, and vice versa. The competencies developed will help you to measure accurately weight, length, and other quantities. Also, you will be able to calculate an increase or decrease of price or various quantities in percentage over time. This will enable you to report the changes in quantities in percentage over time.*

Decimals

Activity 1: Recognising decimal numbers

Perform the following tasks individually or in groups:

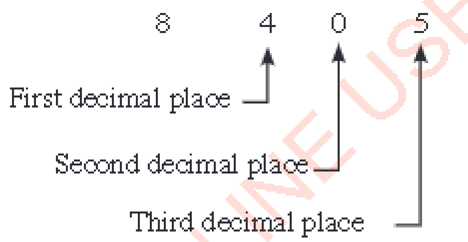
1. Write the masses of any two students in your group.
2. Add the masses of the students in task 1.

3. Write the mass of each student in task 1 as a fraction of their total masses.
4. Write the fractions in task 3 in decimals.
5. Which of the decimals in task 4 is greater than the other?

A decimal is a fraction whose denominator is a multiple of 10, 100, 1 000, 10 000, and so on. When a number cannot be divided exactly by another number, what is left over is called a remainder. The remainder determines the presence of a decimal. The decimals are also considered as fractions which are not written in the form of a numerator and a denominator.

Decimals have two parts separated by the decimal point (•). These two parts are known as whole part and fraction part. The fraction part is also known as the decimal part. The two parts are shown in the decimal number 8.405.

The fraction part is a place where you can get the number of decimal places. The first decimal place is the tenths, the second decimal place is the hundredths, the third decimal place is the thousandths, and so on.

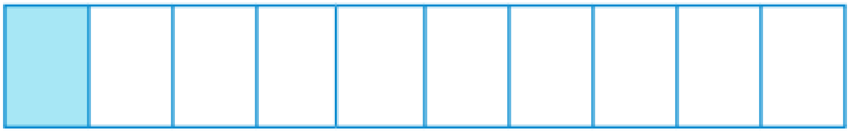


The digits representing the fractional part of the decimal are always read one by one. For example; 8.405 is read as 'eight point four zero five' and not as 'eight point four hundred and five.'

The positions occupied by digits in a decimal number after the decimal point are called decimal places. For example; 8.405 has 3 decimal places.

Any whole number can be written as a decimal. For example; 6 = 6.0, 3 = 3.0, and so on.

Decimals can be shown by illustrations as shown in the following figure.



The shaded section is one out of 10 sections. This is written as $\frac{1}{10}$ in fraction form. It is also written as 0.1 in decimal form, which is read as zero point one. The shaded rectangle can again be divided up into 10 other equal parts.

Every digit of a number, written in a decimal form has its own place value. For example; in 672.543, the place values can be determined as shown in the following table.

Hundreds	Tens	Ones	Point	Tenths	Hundredths	Thousandths
6	7	2	.	5	4	3

5 is in tenths, which means $\frac{5}{10}$.

4 is in hundredths, which means $\frac{4}{100}$.

3 is in thousandths, which means $\frac{3}{1000}$.

Decimals can be expressed in expanded form depending on the place value of each digit.

For example; $0.543 = (0 \times 1) + \left(5 \times \frac{1}{10}\right) + \left(4 \times \frac{1}{100}\right) + \left(3 \times \frac{1}{1000}\right)$.

Alternatively, the expanded form of 0.543 is given by

$$\begin{aligned} 0.543 &= 0 \times 1 + 5 \times 0.1 + 4 \times 0.01 + 3 \times 0.001 \\ &= 0 + 0.5 + 0.04 + 0.003. \end{aligned}$$

Note: Fractions can be converted into decimals and vice versa.

Exercise 1

1. Write each of the following decimals in words:
   (a) 4.128 (b) 367.01 (c) 43.4568

2. Write the place value of 5 in each of the following decimals:
   (a) 8.725 (b) 23.518 (c) 0.0508 (d) 50.461
3. Write each of the following decimals in expanded form:
   (a) 23.105 (b) 4.5201 (c) 0.72 (d) 7.4
4. Write the place value of each digit in the following decimals:
   (a) 0.04 (b) 2.761 (c) 51.4

Conversion of fractions into decimals

To convert a fraction into a decimal, we divide its numerator by its denominator.

Example 1

Convert the following fractions into decimals:

(a) $\frac{2}{5}$ (b) $\frac{5}{8}$

Solution

(a) $\frac{2}{5} = 2 \div 5$

$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ -0 \\ \hline 20 \\ -20 \\ \hline -- \end{array}$$

Therefore, $\frac{2}{5} = 0.4$.

(b) $\frac{5}{8} = 5 \div 8$

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -0 \\ \hline 50 \\ -48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline -- \end{array}$$

Therefore, $\frac{5}{8} = 0.625$.

Example 2

Convert the following mixed fractions into decimals:

(a) $2\frac{2}{3}$ (b) $3\frac{2}{5}$

Solution

(a) Write the mixed fraction as an improper fraction. That is,

$$2\frac{2}{3} = \frac{8}{3}$$

Divide the numerator by the denominator as follows:

$$\begin{array}{r} 2.666\ldots \\ 3\,\overline{)\,8.000} \\ -6 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

Therefore, $2\frac{2}{3} = 2.666\ldots$

(b) $3\frac{2}{5} = \frac{17}{5}$

Divide the numerator by the denominator as follows:

$$\begin{array}{r} 3.4 \\ 5\,\overline{)17.0} \\ -15 \\ \hline 20 \\ -20 \\ \hline -- \end{array}$$

Therefore, $3\frac{2}{5} = 3.4$.

Exercise 2

1. Convert the following fractions into decimals:
   (a) $\frac{1}{8}$ (c) $\frac{4}{17}$
   (b) $\frac{5}{6}$ (d) $\frac{1}{4}$

2. Convert the following fractions into decimals:
   (a) $1\frac{3}{8}$ (c) $7\frac{1}{3}$
   (b) $4\frac{2}{5}$ (d) $11\frac{3}{7}$

3. Convert the following fractions into decimals:
   (a) $\frac{1}{9}$ (c) $\frac{1}{7}$
   (b) $3\frac{2}{7}$ (d) $5\frac{3}{10}$

4. Convert the following fractions into decimals:
   (a) $\frac{17}{19}$ (b) $2\frac{15}{17}$ (c) $\frac{32}{15}$ (d) $\frac{22}{7}$

Types of decimals

There are different types of decimals.

(a) Terminating decimals

A terminating decimal is a number with a finite number of digits after a decimal point.

For example; the decimals 0.25, 0.625, and 0.4 are terminating decimals. They are obtained by converting the fractions $\frac{1}{4}$, $\frac{5}{8}$, and $\frac{2}{5}$ into decimals, respectively.

(b) Recurring or repeating decimals

A repeating decimal or recurring decimal is a decimal representation of a number whose digits are periodic (repeating its values at a regular interval) and the infinitely repeated portion is non-zero.

For example; $1\frac{2}{3} = 1.666\ldots$, $\frac{1}{3} = 0.333\ldots$, and $\frac{71}{111} = 0.639639639\ldots$, are the recurring or repeating decimals.

When writing a repeating decimal, dots or bars are placed over the repeating part.

For example; the decimal 1.6666666. . . can be written as $1.\dot{6}$ or $1.\overline{6}$. This means that, 6 is infinitely repeating.

The decimal 0.474747474. . . is written as $0.\dot{4}\dot{7}$ or $0.\overline{47}$. This means that, 47 is infinitely repeating. The decimal 2.135353535. . . is written as $2.1\dot{3}\dot{5}$ or $2.1\overline{35}$. This means that, 35 is infinitely repeating.

The decimal 0.639639639. . . is written as $0.\dot{6}3\dot{9}$ or $0.\overline{639}$. This means that, 639 is infinitely repeating.

Note: If a group of digits is repeating, a dot should be put over the first and the last repeating digits.

(c) Non-terminating and non-recurring decimals

A non-terminating and non-recurring decimal is a decimal number in which the digits after the decimal point continue endlessly without a defined pattern of numbers. These decimals cannot be represented as fractions, but they can be approximated to fractions. For example; 0.235294. . . , 2.3127613. . ., and 0.368421 . . . are non-terminating and non-recurring decimals.

Conversion of decimals into fractions

The process of converting a decimal into a fraction depends on the type of the decimal.

(a) Terminating decimals

To convert terminating decimals into fractions, use the following steps:

Step 1: Write down the decimal divided by 1. That is, $\frac{\text{decimal}}{1}$

Step 2: Multiply both top and bottom by the multiple of 10 for every digit after the decimal point. That is, if there is one digit after the decimal point, then multiply by 10; if there are two digits multiply by 100; if there are three digits, multiply by 1 000, and so on.

Step 3: Simplify the resulting fraction if necessary.

Example 1

Convert the following decimals into fractions:

(a) 0.25 (b) 1.06

Solution

(a) Write down the decimal divided by 1, that is,

$0.25 = \frac{0.25}{1}$.

Since the decimal has two decimal places, multiply both top and bottom by 100, that is,

$$\frac{0.25}{1} = \frac{0.25 \times 100}{1 \times 100}$$
$$= \frac{25}{100}$$
$$= \frac{1}{4}.$$

Therefore, $0.25 = \frac{1}{4}$.

(b) Write down the decimal divided by 1, that is,

$1.06 = \frac{1.06}{1}$.

Since decimal has two decimal places, multiply both top and bottom by 100, that is,

$$\frac{1.06}{1} = \frac{1.06 \times 100}{1 \times 100}$$
$$= \frac{106}{100}$$
$$= 1\frac{3}{50}.$$

Therefore, $1.06 = 1\frac{3}{50}$.

Example 2

Convert the following decimals into fractions:

(a) 0.48

(b) 0.255

(c) 1.046

Solution

(a) Multiply both top and bottom by 100.

$$\frac{0.48}{1} = \frac{0.48 \times 100}{1 \times 100}$$
$$0.48 = \frac{48}{100}$$
$$= \frac{12}{25}.$$

Therefore, $0.48 = \frac{12}{25}$.

(b) Multiply both top and bottom by 1000.

$$\frac{0.255}{1} = \frac{0.255 \times 1000}{1 \times 1000}$$
$$= \frac{255}{1000}$$
$$= \frac{51}{200}.$$

Therefore, $0.255 = \frac{51}{200}$.

(c) Multiply both top and bottom by 1000.

$$\frac{1.046}{1} = \frac{1.046 \times 1000}{1 \times 1000}$$

$$= \frac{1046}{1000}$$

$$= 1\frac{46}{1000}$$

$$= 1\frac{23}{500}$$

Therefore, $1.046 = 1\frac{23}{500}$.

(b) Recurring or repeating decimals

Conversion of the fraction such as $\frac{2}{3}$ into a decimal gives 0.666666… which is a repeating or recurring decimal. The decimal 0.666666… can also be converted into a fraction. A repeating decimal can be converted into a rational number (fraction) using the following steps:

1. Choose any variable to represent the repeating decimal.
2. Identify the repeating digits next to the decimal point by multiplying both sides of the equation by multiple of 10.
3. Subtract the equation in step 1 from the equation in step 2.
4. From the equation obtained in step 3, solve for a chosen variable in step 1.
5. Simplify, if possible.

Example

Convert the following decimals into fractions:

(a) $0.\dot{3}$ (c) $0.\dot{8}3\dot{5}$

(b) $0.\dot{8}\dot{3}$ (d) $0.8\dot{3}$

Solution

(a) $0.\dot{3}$

Let $x = 0.\dot{3}$ (1)

Since the repeating digit appears at the tenths place, multiply both sides of equation (1) by 10 to obtain

$10x = 3.\dot{3}$ (2)

Subtract equation (1) from equation (2). That is, $10x - x = 3.\dot{3} - 0.\dot{3}$

$$9x = 3.0$$
$$x = \frac{3}{9}$$
$$= \frac{1}{3}.$$

But, $x = 0.\dot{3}$

Therefore, $0.\dot{3} = \frac{1}{3}$.

(b) $0.\dot{8}\dot{3}$

Let $x = 0.\dot{8}\dot{3}$ (1)

Since the repeating digits appear at the tenths and hundredths, multiply both sides of equation (1) by 100 to obtain $100x = 83.\dot{8}\dot{3}$ (2)

Subtract equation (1) from equation (2).

That is,

$$100x - x = 83.\dot{8}\dot{3} - 0.\dot{8}\dot{3}$$
$$99x = 83.0$$
$$x = \frac{83}{99}.$$

But, $x = 0.\dot{8}\dot{3}$

Therefore, $0.\dot{8}\dot{3} = \frac{83}{99}$.

(c) $0.\dot{8}3\dot{5}$

Let $x = 0.\dot{8}3\dot{5}$ (1)

Since the repeating digits appear at the tenths, hundredths, and thousandths, multiply both sides of equation (1) by 1000 to obtain the following equation:

$1000x = 835.\dot{8}3\dot{5}$ (2)

Subtract equation (1) from equation (2)

$$1000x - x = 835.\dot{8}3\dot{5} - 0.\dot{8}3\dot{5}$$
$$999x = 835$$
$$x = \frac{835}{999}.$$

But, $x = 0.\dot{8}3\dot{5}$

Therefore, $0.\dot{8}3\dot{5} = \frac{835}{999}$.

(d) $0.8\dot{3}$

Let $x = 0.8\dot{3}$ (1)

Since the repeating digit occurs only at the hundredth place, multiply both sides of equation (1) by 10 to obtain the following equation:

$10x = 8.3\dot{3}$ (2)

Subtract equation (1) from equation (2)

$$10x - x = 8.3\dot{3} - 0.8\dot{3}$$
$$9x = 7.5$$
$$x = \frac{7.5}{9}$$
$$= \frac{7.5 \times 10}{9 \times 10}$$
$$= \frac{75}{90}$$
$$x = \frac{5}{6}.$$

But, $x = 0.8\dot{3}$.

Therefore, $0.8\dot{3} = \frac{5}{6}$.

Alternative to part (d)

Let $x = 0.8\dot{3}$ (1)

Since the repeating digit occurs only at the hundredth place, multiply both sides of equation (1) by 10 and by 100 to obtain the following equations:

$10x = 8.\dot{3}$ (2)

$100x = 83.\dot{3}$ (3)

Subtract equation (2) from equation (3). That is,

$$100x - 10x = 83.\dot{3} - 8.\dot{3}$$

$$90x = 75$$

$$x = \frac{75}{90}$$

$$x = \frac{5}{6}.$$

But, $x = 0.8\dot{3}$.

Therefore, $0.8\dot{3} = \frac{5}{6}$.

(c) Non-terminating and non-recurring decimals

Non-terminating and non-recurring decimals cannot be converted into fractions, unless the decimals are approximated to a certain number of decimal places to make them terminate.

Exercise 3

In question 1 to 6, convert the given decimals into fractions:

1. (a) 0.475 (c) 11.01
   (b) 1.007 (d) 0.35

2. (a) $1.0\dot{2}$ (c) $80.\dot{2}1\dot{7}$
   (b) $3.1\dot{1}\dot{2}$ (d) $13.0\dot{1}\dot{5}$

3. (a) $0.\dot{8}$ (c) $0.\dot{7}2\dot{3}$
   (b) $0.3\dot{4}$ (d) $2.\dot{4}$

4. (a) $0.7\dot{5}$ (c) $10.\dot{3}\dot{1}$ (e) $10.\dot{1}$
   (b) $5.\dot{1}\dot{3}$ (d) $10.00\dot{1}$

5. (a) 0.45 (c) 0.93 (e) 0.97
   (b) 0.34 (d) 0.215 (f) 0.1034

6. (a) $0.\dot{2}\dot{1}$ (c) $0.\dot{5}6\dot{7}$ (e) $0.\dot{2}1\dot{9}$
   (b) $0.9\dot{3}$ (d) $0.\dot{1}35\dot{2}$ (f) $0.\dot{1}8\dot{6}$

7. Convert the following fractions into decimals. If the decimal repeats, write it by using dots.
   (a) $\frac{1}{3}$ (c) $\frac{4}{12}$ (e) $\frac{7}{13}$
   (b) $\frac{5}{6}$ (d) $\frac{1}{9}$

Operations on decimals

Decimals can be added, subtracted, multiplied, or divided. Since decimals have a decimal point, we use it as a guide when performing these mathematical operations on decimals.

Addition and subtraction of decimals

The following steps are used when adding or subtracting decimals:

Step 1: Arrange the digits of the decimals vertically according to their corresponding place values.

Step 2: Add or subtract by starting from the right to the left.

Example 1

Find the value of 3.5 + 0.08 + 10.75.

Solution

Arrange the digits of the decimals vertically, then find their sum. That is,

$$\begin{array}{r} 3.50 \\ 0.08 \\ +10.75 \\ \hline 14.33 \\ \hline \end{array}$$

Therefore, 3.5 + 0.08 + 10.75 = 14.33.

Example 2

Find the value of 3.84 – 2.45.

Solution

Arrange the digits of the decimals vertically, then find their difference. That is,

$$\begin{array}{r} 3.84 \\ -2.45 \\ \hline 1.39 \\ \hline \end{array}$$

Therefore, 3.84 – 2.45 = 1.39.

Exercise 4

Find the value of each of the following:

1. 4.48 + 3.09
2. 6.8 + 2.003
3. 45.05 + 1.978
4. 98.1 + 89.35
5. 75.8 – 66.67
6. 6.43 – 3.78
7. 8.07 – 4.06
8. 49.001 + 4.06
9. 0.504 + 0.8
10. Find the difference between 0.72 and 0.24.

Multiplication and division of decimals

The following steps are used when multiplying decimals:

Step 1: Ignore the decimal point.

Step 2: Multiply the digits as it is usually done when multiplying whole numbers.

Step 3: Add the resulting products.

Step 4: Count the total number of decimal places of the multiplied decimals.

Step 5: Starting from the right end of the product obtained in step 3, move to the left the same number of decimal places obtained in step 4, and insert the decimal point.

Example 1

Find the value of 0.43×5.208.

Solution

Ignore the decimal point and multiply the numbers as follows:

$$\begin{array}{r} 5208 \\ \times\ 43 \\ \hline 15624 \\ +20832 \\ \hline 223944 \\ \hline \end{array}$$

The total number of decimal places is five (two from 0.43 and three from 5.208). Hence, count 5 decimal places from the right of 223 994 to the left, and then, insert a decimal point after the fifth decimal place to obtain 2.23944.

Therefore, $0.43 \times 5.208 = 2.23944$.

Division of decimals can be done between decimals by decimals, decimals by whole numbers, and whole numbers by decimals.

When dividing decimals by other numbers, or decimals by decimals, the following steps are used:

Step 1: Compare the number of decimal places of the numerator with that of the denominator.

Step 2: Take the highest number of decimal places (that is, if it is one, then multiply by 10 on both the numerator and the denominator. If the number of decimal places is two, then multiply by 100 on both the numerator and the denominator, and so on).

Step 3: Use an appropriate division method to divide the resulting fraction.

Example 2

Find the value of 0.8 ÷ 0.25.

Solution

$0.8 \div 0.25 = \frac{0.8}{0.25}$

The numerator 0.8 has 1 decimal place, and the denominator 0.25 has 2 decimal places.

Thus, multiply by 100 both the numerator and the denominator, because the highest number of decimal places is 2. That is,

$$\frac{0.8 \times 100}{0.25 \times 100} = \frac{80}{25}$$

$$= 3.2.$$

Therefore, 0.8 ÷ 0.25 = 3.2.

Example 3

Find the value of 1.8 ÷ 6.

Solution

$$1.8 \div 6 = \frac{1.8}{6}$$
$$= \frac{1.8 \times 10}{6 \times 10}$$
$$= \frac{18}{60}$$
$$= 0.3.$$

Therefore, $1.8 \div 6 = 0.3$.

Exercise 5

Find the value of each of the following:

1. $38.05 \div 0.005$
2. $0.6 \div 0.02$
3. $0.15 \div 0.3$
4. 49.03×0.54
5. 0.21×0.5
6. $8.55 \div 0.2$
7. 0.34×6.28
8. $(2.4 \times 3.2) \div 2.4$
9. $32.4 \times 4.21 \div 2.5$
10. $(0.62 \div 0.2) \times (2.04 \div 0.2)$
11. $\frac{0.1}{0.01} + \frac{0.04}{0.02} + \frac{2}{0.2}$
12. $\frac{23.4 - 2 \times (5.3 \times 5.2)}{1.2 \times 3.2}$.

Representation of terminating decimals on a number line

A terminating decimal with few decimal places can be represented on a number line.

Activity 2: Representing terminating decimals on a number line

Perform the following tasks individually, and then, share your results with other students in groups.

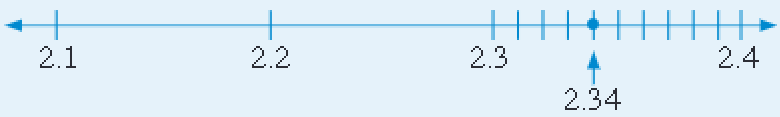
1. Choose a decimal number with 2 decimal places, and a positive whole part not exceeding 5.
2. Draw a number line using appropriate intervals.
3. Represent the decimal number chosen in task 1 on the number line drawn in step 2.
4. Discuss your results with other members of your class under your teacher's guidance.

Example

Represent 2.34 on a number line.

Solution

The number 2.34 lies between 2.3 and 2.4. It can be represented on a number line as follows.



Exercise 6

1. From (a) to (d), find the value of each expression:
   (a) $0.4 + 1.2$
   (b) 4×1.23
   (c) $3.478 - 1.243$
   (d) $1.26 \div 0.2$
   (e) Represent the answers from (a) to (d) on a number line.

2. Represent 0.7 and 2.5 on the same number line.

Word problems involving decimals

Word problems involving decimals are very important because in our daily life experiences, we encounter some tasks which involve decimals. For instance, a rate at which a chemical substance dissolves in water, scores in examinations, and so on.

Example 1

A car uses 7.25 litres of petrol to travel 98.6 kilometres. How many kilometres does it travel using 1 litre of petrol?

Solution

Let x be the number of kilometres travelled using 1 litre.

A car uses 7.25 litres for 98.6 km

A car uses 1 litre for x km

$$x = \frac{1 \text{ litre} \times 98.6 \text{ km}}{7.25 \text{ litres}}$$

$$= \frac{98.6 \text{ km} \times 100}{7.25 \times 100}$$

$$= \frac{9860 \text{ km}}{725}$$

$x = 13.6$ km.

Therefore, the car travels 13.6 kilometres using 1 litre of petrol.

Example 2

If $\frac{2}{5}$ of the population in a village are children, write this fraction as a decimal.

Solution

Divide the numerator by the denominator. That is,

$\frac{2}{5} = 2 \div 5.$

Using a long division method, we have

$$\frac{2}{5} = 5\overline{)\,2\,}\quad \begin{array}{r} 0.4 \\ 2 \\ -0 \\ \hline 20 \\ -20 \\ \hline \hline \end{array}$$

Therefore, $\frac{2}{5} = 0.4$.

Exercise 7

1. A patient lost $\frac{1}{8}$ of his average mass. How much mass did the patient lose in a decimal form?

2. Out of 24 hours, a student spends $\frac{5}{12}$ of the hours for sleeping, $\frac{1}{3}$ of the hours for studying, $\frac{1}{6}$ of the hours for playing, and the remaining fraction of the hours for watching cartoons.
   (a) Find the fraction of hours that the student spends for watching cartoons. Give your answer in a decimal.
   (b) Find the total fraction of the hours that the student spends for sleeping and studying. Write your answer in a decimal.
3. Suppose that, $\frac{3}{8}$ of a basket was full of tomatoes, and $\frac{5}{8}$ of the basket was full of oranges;
   (a) Write the fraction of tomatoes in a decimal.
   (b) Write the fraction of oranges in a decimal.
   (c) Find the sum of the decimals obtained in (a) and (b).
4. If $\frac{7}{100}$ of 2 500 people in a certain village did not attend secondary education, how many people attended secondary education in that village?
5. Out of 2 400 people who tested for COVID-19 in a certain country, $\frac{1}{8}$ were found COVID-19 positive. How many people tested positive?
6. Suppose that maize was planted on $\frac{2}{5}$ of a farm, cassava on $\frac{1}{8}$ of the farm, and beans on $\frac{1}{4}$ of the farm;
   (a) Write each fraction in a decimal.
   (b) Which crop occupied the largest piece of land?
   (c) Add all the decimals obtained in (a) above.

Percentages

A percentage is a number or ratio expressed as a fraction of 100. It is denoted by a symbol %.

For example; $\frac{20}{100}$ is 20 percent, and it is written as 20%.

$\frac{34}{100}$ is 34 percent, and it is written as 34%.

The word percent means per hundred.

Conversion of fractions into percentages

Fractions can be converted into percentages by multiplying the given fraction by 100%, or by dividing the numerator by the denominator, and then multiplying the resulting decimal by 100.

Example

Convert $\frac{1}{8}$ into a percentage.

Solution

$\frac{1}{8} \times 100\% = \frac{100}{8}\%$

$= 12.5\%.$

or

$$\begin{array}{r} 12.5 \\ 8\overline{)\,100} \\ -8 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline -- \end{array}$$

Therefore, $\frac{1}{8} = 12.5\%$.

Conversion of decimals into percentages

Decimals can be converted into percentages by multiplying the given decimal by 100%.

Example

Convert 0.065 into a percentage.

Solution

Multiply 0.065 by 100%. That is,

$0.065 \times 100\% = 6.5\%$.

Therefore, $0.065 = 6.5\%$.

Conversion of percentages into fractions

A percentage can be converted into a fraction by expressing it as a fraction with a denominator of 100. The resulting fraction is simplified to its lowest term (if possible).

Example

Convert 0.026% into a fraction.

Solution

Write the percentage as a fraction as follows:

$$0.026\% = \frac{0.026}{100}$$

$$= \frac{0.013}{50}$$

Since the numerator has 3 decimal places, multiply both the numerator and the denominator by 1 000. That is,

$$0.026\% = \frac{0.013}{50}$$

$$= \frac{0.013}{50} \times \frac{1\,000}{1\,000}$$

$$= \frac{13}{50000}.$$

Therefore, $0.026\% = \frac{13}{50000}$.

Conversion of percentages into decimals

Percentages can be converted into decimals by expressing them as fractions of hundreds, and then converting them into decimals.

Example

Convert 26.4% into a decimal.

Solution

Write 26.4% as a fraction of hundredths, that is,

$$26.4\% = \frac{26.4}{100}$$

Since the numerator has 1 decimal place, multiply both the numerator and the denominator by 10.

That is,

$$26.4\% = \frac{26.4}{100} \times \frac{10}{10}$$

$= \frac{264}{1000}$

$= \frac{33}{125}$

Convert the resulting fraction into a decimal. That is,

$$\begin{array}{r} 0.264 \\ 125\overline{)\,33} \\ -\ 0 \\ \hline 330 \\ -250 \\ \hline 800 \\ -750 \\ \hline 500 \\ -500 \\ \hline -- \end{array}$$

Therefore, 26.4% = 0.264.

Exercise 8

1. Convert the following fractions into percentages:
   (a) $\frac{3}{5}$ (b) $\frac{15}{30}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$
   (e) $\frac{24}{25}$ (f) $\frac{63}{60}$ (g) $3\frac{1}{4}$ (h) $5\frac{2}{3}$
   (i) $8\frac{3}{4}$ (j) $\frac{4}{9}$

2. Convert the following decimals into percentages:
   (a) 0.48 (b) 0.95 (c) 3.42
   (d) 6.05 (e) 0.87 (f) 0.76

3. Convert the following percentages into (i) fractions (ii) decimals:
   (a) 35% (b) 60% (c) 25%
   (d) 16% (e) 15% (f) 3%
   (g) 2% (h) $3\frac{1}{3}$% (i) 98%

4. Write 0.26 as a percentage.
5. If $\frac{3}{4}$ of water in a tank is split, what is this amount of water in percentage?
6. If 85% of Juma's oranges have been sold, what is the number of the sold oranges in a decimal.
7. Convert the following numbers into percentages:
   (a) 78.2 (c) 4
   (b) $3\frac{2}{5}$ (d) 0.75
8. A certain secondary school has a total of 440 Form One students. If 150 students are day scholars;
   (a) What fraction of the students are hostelers?
   (b) Express the number of the day scholars and hostelers in percentage.

Applications of percentages

Percentages are used in many different activities in our everyday life such as in schools, hospitals, businesses, and agriculture. For example; the prices of most quantities increase or decrease by a percentage over time. Profit, loss, discount, rates, commission, and rates of interest can all be expressed as percentages.

Example 1

The cost of a book has increased from 9 000 Tanzanian shillings to 12 000 Tanzanian shillings. By what percentage did the cost increase?

Solution

The increase in cost price = Tsh (12 000 – 9 000)

= Tsh 3 000.

Thus,

$$\text{Percentage increase in cost price} = \frac{3000}{9000} \times 100\%$$

$$= \frac{1}{3} \times 100\%$$

$$\approx 33.3\%.$$

Therefore, the cost increase in percentage is $\approx 33.3\%$.

Example 2

If 81% of passengers in a bus were men, what percentage of the passengers were women?

Solution

Number of all passengers in percentage is 100

Percentage of men = 81

Percentage of women = 100 − 81 = 19

Therefore, 19% of the passengers were women.

Example 3

Dotto scored 55 out of 80 marks in a basic mathematics test. What is his score in percentage?

Solution

Dotto's score = $\frac{55}{80}$

Multiply the fraction by 100%. That is,

$$\text{Dotto's score} = \frac{55}{80} \times 100\%$$

$$= \frac{11}{16} \times 100\%$$

$$= \frac{275\%}{4}$$

$$= 68.75\%.$$

Therefore, the percentage of Dotto's score is 68.75%.

Note:

To find the amount of a given quantity represented by a percentage, the following steps should be followed:

(i) Convert the percentage into a fraction.

(ii) Multiply the resulting fraction by the given quantity.

Alternatively, we can multiply the quantity by the given percentage.

Exercise 9

1. Fill in the blanks in the following table.

Fractions	$\frac{1}{2}$					
Percentages		$2\frac{1}{2}\%$		5%	$66\frac{2}{3}\%$	
Decimals			0.1			0.75

2. The population of Jitegemee village is 3 600 people. If 65% of the population are men, what is the number of women in the village?
3. A boy spent 35% of his money to buy books. If he was left with 26 000 Tanzanian shillings, how much money did he have?
4. There are 35 teachers in a certain secondary school. If 15 teachers are females, what percentage of the teachers are:
   (a) females? (b) males?

Chapter summary

1. A decimal is a fraction whose denominator is a multiple of 10, 100, 1 000, and so on.
2. A terminating decimal has a finite number of digits after the decimal point.
3. A recurring or repeating decimal has an infinite number of digits after the decimal point.
4. The digits after a decimal point in a recurring or repeating decimal are periodic, and have a non-zero infinitely repeated portion.
5. A non-terminating and non-recurring decimal has endless digits after the decimal point with no group of digits repeating.

Revision exercise

1. Write the following fractions in words:
   (a) $\frac{2}{3}$ (b) $\frac{3}{5}$ (c) $\frac{1}{10}$

2. Insert the words "is equal to", "is less than", or "is greater than" between each pair of the following decimals:
   (a) 0.66 ___ 0.7 (b) 0.2 ___ 0.133 (c) 0.15 ___ 0.5

3. Divide 0.0432 by 0.9.

4. Find the value of the following expressions:
   (a) $0.24 + 0.015 + 0.361$ (b) $0.824 + 0.57$

5. Convert the following fractions into decimals:
   (a) $\frac{27}{108}$ (b) $\frac{11}{6}$ (c) $\frac{2}{9}$

6. Convert the following decimals into fractions:
   (a) 0.58 (b) 0.999 (c) 11.75

7. Convert the following decimals into fractions:
   (a) $2.\dot{5}$ (b) $0.\dot{4}\dot{9}$ (c) $0.\dot{1}2\dot{3}$

8. Mariam was given 20 000 Tanzanian shillings by her parents. If she spent 48% of the money to buy shoes, how much money did she remain with?

9. If three fifths of the pupils at Mjimwema Primary School come from the town centre, what percentage of the pupils do not come from the town centre?

10. Find the value of $0.732 + 3.02 + 0.009 + 10.981 - 4.06$.

11. Find the value of $\frac{2.5 \times 0.7}{3.5 \times 4}$.

12. Find the value of $0.00224 \div 0.325 \times 1\,000$.

13. A study conducted among 300 college students showed that 21 students cannot drive. What percentage of the students can drive?

14. In a survey of 1 000 people, it was observed that 60 people did not know how to ride a bike. What is this number in percentage?

Project

Penina was playing with her name and numbers. She let all the consonants equal 0.5 and all the vowels equal 0.3. So, the value of Penina's name was found to be $0.5 + 0.3 + 0.5 + 0.3 + 0.5 + 0.3 = 2.4$. Using Penina's idea:

(a) Find the values of your first name and surname.

(b) Find at least five names in your class that have a value greater than 2.5.

(c) If multiplication is used in place of addition, what are the values of your first name and surname?

(d) What is the value of a name with four consonants and five vowels?

(e) Convert the value of each letter in your first name into a percentage.

Chapter Five

Metric units

Introduction

The standard metric system of measurement was first officially adopted by France in 1799. This system of measurement was called the metric system. In everyday life, measuring things is inevitable. For instance, we can say that, the temperature today is 28°C, the mass of a certain Form One student is 53 kilograms, a distance from Dar es Salaam to Arusha is about 479 kilometres, and so on. These statements involve units of measurements. In this chapter, you will learn about conversions, operations, and solving word problems of metric units of length, mass, time, and capacity. The competencies developed will help you to perform various measurement activities in daily life situations such as cooking, tailoring, carpentry, architecture, engineering, and business. They will also improve your punctuality in different activities such as following school timetable, transport schedules, and completing many other daily life routines by observing time.

Metric units of length

Activity 1: Recognising the metric unit of length

Perform the following tasks individually or in groups:

1. Measure the following:
   (a) The length of your mathematics textbook.
   (b) The width of a mathematical set.
   (c) The thickness of your classroom window.
   (d) The length of a pen.

2. The length from points A to B is 1.5 cm, from points B to C is 4 cm and from points C to D is 6 cm. Find the length from points A to D. How do you obtain your answer? Present your work on a manila sheet. Post your activities on the classroom wall.
3. Share your findings with other groups through class discussion.

Length is a distance between two points. For example; a length of a blackboard, a distance from home to school, or a length of a table.

The basic unit of length is metre, denoted by m. Other units of length include; kilometre (km), hectometre (hm), decametre (dam), decimetre (dm), centimetre (cm), and millimetre (mm).
The following table shows the comparison of these units.

km	hm	dam	m	dm	cm	mm
1	0	0	0	0	0	0
	1	0	0	0	0	0
		1	0	0	0	0
			1	0	0	0
				1	0	0
					1	0
						1

From the table, we observe that,
1km = 1 000 000 mm
1 hm = 100 000 mm
1 dam = 10 000 mm
1 m = 1 000 mm
1 dm = 100 mm
1 cm = 10 mm

The conversion of the metric units of length can be done as shown in the following table.

Unit	Abbreviation	Conversion	Conversion into metres
Millimetre	mm	10 mm = 1 cm	1 m = 1 000 mm
Centimetre	cm	10 cm = 1 dm	1 m = 100 cm
Decimetre	dm	10 dm = 1 m	1 m = 10 dm
Metre	m	10 m = 1 dam	1 m = 0.1 dam
Decametre	dam	10 dam = 1 hm	1 m = 0.01 hm
Hectometre	hm	10 hm = 1km	1 m = 0.001 km
Kilometre	km		

Conversion of units of length

Suppose 5 people were asked to measure the distance from Dar es Salaam to Morogoro. The first person measured the distance from Dar es Salaam to Kibaha, the second person from Kibaha to Mlandizi, the third person from Mlandizi to Chalinze, the fourth person from Chalinze to Bwawani, and the last person from Bwawani to Morogoro. If all of them used different units in measuring the distances, then the conversion of the units should be done in order to get the required distance. That is, the measurements obtained should be converted into the same unit, and then added to get the total distance from Dar es Salaam to Morogoro.

Example 1

Convert 3 075 metres into kilometres.

Solution

Using the comparison between metres and kilometres, we have

1 000 m = 1 km

1 m = 1 km ÷ 1 000

That is,

1 m = 0.001 km

3 075 m = x

Thus, x = 0.001 km × 3 075

= 3.075 km.

Therefore, 3 075 metres = 3.075 kilometres.

Example 2

Convert 9.17 kilometres into decametres.

Solution

Using the comparison between decametres and kilometres, we have

$$1 \text{ km} = 100 \text{ dam}$$
$$9.17 \text{ km} = x$$

That is,

$$x = \frac{100 \text{ dam} \times 9.17 \text{ km}}{1 \text{ km}}$$
$$= 917 \text{ dam}.$$

Therefore, 9.17 kilometres = 917 decametres.

Example 3

Convert 879 centimetres into metres.

Solution

Using the comparison between centimetres and metres, we have

$$1 \text{ m} = 100 \text{ cm}$$
$$1 \text{ cm} = (1 \div 100) \text{ m}$$

That is,

$$1 \text{ cm} = 0.01 \text{ m}$$

Thus,

$$879 \text{ cm} = 879 \times 0.01 \text{ m}$$
$$= 8.79 \text{ m}.$$

Therefore, 879 centimetres = 8.79 metres.

Example 4

Convert 3 567 metres into:
(a) Decametres (b) Kilometres

Solution

(a) Using the comparison between metres and decametres, we have

$$10 \text{ m} = 1 \text{ dam}$$
$$3\,567 \text{ m} = (3\,567 \div 10) \text{ dam}$$
$$= 356.7 \text{ dam.}$$

Therefore, 3 567 metres = 356.7 decametres.

(b) Using the comparison between metres and kilometres, we have

$$1 \text{ km} = 1\,000 \text{ m}$$
$$3\,567 \text{ m} = (3\,567 \div 1\,000) \text{ km}$$
$$= 3.567 \text{ km.}$$

Therefore, 3 567 metres = 3.567 kilometres.

Example 5

Convert 0.5 milimetres into decimetres.

Solution

$$1 \text{ dm} = 100 \text{ mm}$$
$$x = 0.5 \text{ mm}$$

Thus, $x = (0.5 \div 100)$ dm

$$= 0.005 \text{ dm.}$$

Therefore, 0.5 milimetres = 0.005 decimetres.

Altenatively,

$$1 \text{ mm} = \frac{1}{100} \text{ dm}$$
$$= 0.01 \text{ dm.}$$

Thus, $0.5 \text{ mm} = 0.01 \text{ dm} \times 0.5$

$$= 0.005 \text{ dm.}$$

Therefore, 0.5 milimetres = 0.005 decimetres.

Exercise 1

1. Convert 350 millimetres into the following units of measurements:
   (a) Centimetres
   (b) Decimetres
2. Convert 0.0042 kilometres into the following units of measurements:
   (a) Decametres
   (b) Metres
   (c) Decimetres
3. How many metres are there in a decimetre?
4. How many decimetres are there in a kilometre?
5. The length of a matchstick is 37.5 millimetres. Convert this measurement into the following units:
   (a) Decimetres (b) Metres
6. The length of an office file is 37.2 centimetres. Convert this measurement into the following units:
   (a) Metres (b) Decametres
7. Convert 968.5 decimetres into the following units of measurement:
   (a) Hectometres (b) Kilometres (c) Centimetres
8. The diameter of a nichrome wire is 0.862 millimetres. Convert this measurement into the following units:
   (a) Metres (b) Hectometres
9. Convert each of the following measurements into metres and state which is shorter than the other:
   (a) 80 000 millimetres (b) 80 decimetres
10. How many millimetres are there in 3 decimetres?
11. Convert each of the following measurements into metres:
    (a) 6 875 decimetres
    (b) 628 decametres
    (c) 286 hectometres
    (d) 0.862 kilometres
    (e) 68 750 millimetres
    (f) 0.0862 hectometres
    (g) 21.008 millimetres
    (h) 10 500 centimetres
12. Arrange the following units of measurements in ascending order after converting them into metres:
    68 hectometres, 0.68 kilometres, 16 800 centimetres, 750 decametres.

13. Arrange the following units of measurements in descending order after converting them into metres: 0.1875 hectometres, 87 500 decimetres, 8 750 000 000 millimetres, 0.00875 decimetres.
14. Given that, the length of a ruler is 30 centimetres, the length of a pen is 0.09 metres, the length of a pencil is 0.00095 hectometres, convert each dimension into kilometres.
15. Which one is greater between 970 metres and 9.7 centimetres?
16. Convert $\frac{1}{5}$ millimetres into metres.
17. Mwajuma runs 3.5 kilometres in two days. How many metres does she run in the two days?

Operations on units of length

The units of measurement of length can be added or subtracted. They can also be multiplied or divided by a constant number.

Addition of units of length

When adding different units of measurement of length, the corresponding units need to be placed together and added as required.

Example 1

Find the value of each of the following units of measurement:

(a)

	m	dm	cm
	6	7	8
	2	5	9
+	3	9	7

(b)

	km	hm	dam
	6	9	9
	9	9	9
+	8	8	8

(c)

	km	m	mm
	9	6	9
	8	8	8
+	6	9	9

(d)

	m	cm
	6	17
+	4	13

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Solution

Add the corresponding units as follows:

(a)

	m	dm	cm
	6	7	8
	2	5	9
+	3	9	7
	13	3	4

(b)

	km	hm	dam
	6	9	9
	9	9	9
+	8	8	8
	25	8	6

(c)

	km	m	mm
	9	6	9
	8	8	8
+	6	9	9
	23	23	26

(d)

	m	cm
	6	17
+	4	13
	10	30

Example 2

Three pieces of a string measure 96 decametres, 2.8 metres, and 1 250 millimetres, respectively. Find the total length of the three pieces in kilometres.

Solution

Using the comparison of units

$$100 \text{ dam} = 1 \text{ km}$$
$$1 \text{ dam} = 1 \text{ km} \div 100$$

That is, $96 \text{ dam} = \frac{1}{100} \times 96 \text{ km}$.

Thus, $96 \text{ dam} = 0.96 \text{ km}$.

Similarly,

$$1\ 000 \text{ m} = 1 \text{ km}$$
$$1 \text{ m} = 1 \text{ km} \div 1\ 000$$
$$2.8 \text{ m} = \frac{1}{1000} \times 2.8 \text{ km}.$$

Thus, $2.8 \text{ m} = 0.0028 \text{ km}$.

$$1 \text{ m} = 1\ 000 \text{ mm}.$$

Also, $1\ 250 \text{ mm} = \frac{1}{1000} \times 1\ 250 \text{ m}$

$$= 1.25 \text{ m}$$
$$1.25 \text{ m} = 0.00125 \text{ km}.$$

Thus, $1\ 250 \text{ mm} = 0.00125 \text{ km}$.

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Adding the three lengths in kilometres gives
0.96 km + 0.0028 km + 0.00125km = 0.96405 km.

Therefore, the total length is 0.96405 kilometres.

Exercise 2

Answer the following questions:

1.

	m	dm	cm
	8	9	6
	8	7	7
+	5	6	3

2.

	km	hm	dam
	8	9	5
	9	4	6
+	5	6	8

3.

	km	dam	m
	8	9	9
	8	7	7
+	5	6	3

4.

	dm	cm	mm
	5	6	7
	4	9	8
+	3	3	1

5.

	m	dm	cm
	7	7	7
	8	8	8
+	9	9	9

6.

	km	hm	dam
	8	8	8
	7	7	7
+	6	6	6

7.

	km	dam	m
	7	8	7
	8	8	9
+	9	7	8

8.

	dm	cm	mm
	7	8	0
	8	2	7
+	1	7	8

9.

	km	m	mm
	7	8	9
	8	9	7
+	9	7	8

10.

	km	dam	m
	18	9	9
	21	8	8
+	11	2	3

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11. Add each of the following units of measurements:
    (a) 785 m, 97 m, 605 m.
    (b) 24.37 km, 187.5 km, 21.13 km.
    (c) 8 dam, 9 m, 2 cm (give your answer in metres).

12. Find the sum of 0.002 decametres, 6 metres, and 0.564 hectometres (give your answer in metres).

Subtraction of units of length

When subtracting different units of measurement of length, the corresponding units need to be placed together and subtracted as required.

Example 1

Subtract the following units of measurement:

(a)

	km	dm	cm
	5	4	3
−	2	9	8

(b)

	km	hm	dam
	4	0	0
−	2	9	8

(c)

	km	m	mm
	4	0	0
−	2	9	8

Solution

Subtract the corresponding units as follows:

(a)

	km	dm	cm
	5	4	3
−	2	9	8
	2	9994	5

(b)

	km	hm	dam
	4	0	0
−	2	9	8
	1	0	2

(c)

	km	m	mm
	4	0	0
−	2	9	8
	1	990	992

Example 2

The length of a basketball ground is 800 centimetres, while that of a volleyball pitch is 7.5 metres. Find the difference in their lengths in metres.

Solution

Convert 800 cm into metres as follows:

Since 1 m = 100 cm, then 8 m = 800 cm.

Thus, the length of the basketball ground is 8 m.

The length of the volleyball pitch is 7.5 m.

Thus, the difference is 8 m − 7.5 m = 0.5 m.

Therefore, the difference in their lengths is 0.5 metres.

Exercise 3

Answer the following questions:

1.

	km	hm	dam
	21	8	8
−	11	2	3

2.

	m	dm
	8	6
−	6	7

3.

	m	dm	cm
	6	7	8
−	5	7	9

4.

	km	hm	cm
	7	8	8
−	6	9	8

5.

dam	m	dm
6	7	8
− 5	8	9

6.

km	m	dm
2	0	6
− 1	98	9

7.

km	m	dm
6	6	6
− 4	87	7

8.

dam	dm	mm
6	82	0
− 4	98	4

9.

km	hm	dam
160	9	7
− 150	9	9

10. Calculate the following, giving your answers in metres:
    (a) 7 dm − 7 cm
    (b) 2 dm − 10 cm
    (c) 0.85 km − 168 m
    (d) 86 m − 48 m

11. A stick of 71 centimetres is divided into two pieces. Find the length of the second piece in metres if the first piece has the length of 0.52 metres.

Multiplication of units of length

The units of measurement of length can be multiplied by a constant number. The resulting product will be in the given units.

Example

Compute the following:

(a)

km	dm	cm
6	8	9
×		5

(b)

km	hm	dam
6	8	4
×		28

(c)

km	m	mm
6	80	4
×		100

Solution

Multiply the given units of measurement as follows:

(a)

km	dm	cm
6	8	9
×		5
30	44	5

(b)

km	hm	dam
6	8	4
×		28
191	5	2

(c)

km	m	mm
6	80	4
×		100
608	0	400

Exercise 4

Answer the following questions:

1.

m	cm
6	9
×	28

2.

km	dam
9	6
×	30

3.

dam	m
5	5
×	25

4.

hm	dam	m
8	7	6
×		20

5.

dm	cm	mm
5	8	9
×		10

6.

hm	dam	m
15	6	9
×		100

Division of units of length

The units of measurement of length can be divided by a constant number. The resulting quotient will be in the given units, and can be converted into other units.

Example 1

Compute each of the following:

(a) 5 km ÷ 10 (give your answer in metres).

(b) 38 m ÷ 50 (give your answer in centimetres).

Solution

(a) Using the comparison of units

1 km = 1 000 m

5 km = 5 000 m.

Thus,

5 km ÷ 10 = 5 000 m ÷ 10

= 500 m.

Therefore, 5 km ÷ 10 = 500 m.

(b) Using the comparison of units

1 m = 100 cm

38 m = 3 800 cm.

Thus,

38 m ÷ 50 = 3 800 cm ÷ 50

= 76 cm.

Therefore, 38 m ÷ 50 = 76 cm.

Example 2

Compute each of the following:

(a) 56 km ÷ 7 (give your answer in metres).

(b) 48 m ÷ 60 (give your answer in centimetres).

Solution

(a) Using the comparison of units

1 km = 1 000 m

56 km = 56 × 1 000 m

= 56 000 m.

Thus,

56 km ÷ 7 = 56 000 m ÷ 7

= 8 000 m.

Therefore, 56 km ÷ 7 = 8 000 m.

(b) 1 m = 100 cm

48 m = (48 × 100) cm

= 4 800 cm.

Thus,

48 m ÷ 60 = (4 800 ÷ 60) cm

= 80 cm.

Therefore, 48 m ÷ 60 = 80 cm.

Exercise 5

1. Compute each of the following, giving your answer in metres:
   (a) (50 m 5 dm) ÷ 5
   (b) (18 m 9 cm) ÷ 9
   (c) 15 m ÷ 6
   (d) (28 hm 1 dm) ÷ 10
   (e) (18 km 5 hm) ÷ 5
   (f) (11 km 4 m) ÷ 8

2. A roll of string is 13.5 metres long. If four strings of lengths 2.30 metres, 1.8 metres, 2.37 metres, and 0.95 metres are removed from the roll, and the remaining part is divided into equal pieces of lengths 32 centimetres, how many equal pieces are there?
3. How many pieces of length 8.9 centimetres each can be obtained from a thread of length 1.75 metres? What is the length of the remaining part?
4. A piece of wire is 5 decimetres long. If the wire is divided into 10 equal pieces, what will be the length of each piece?
5. Juma planted trees on each side of the road to his house. The road is $\frac{3}{4}$ km long, and the trees are 5 m apart. How many trees are there?
6. A piece of wire is 0.5 m long. If the wire is divided into equal pieces of length 25 cm each, how many pieces are there?
7. A rope of length 1 250 centimetres was equally divided to five students. Calculate the length of the rope received by each student.
8. Compute $3\overline{)22\text{ km}\quad 6\text{ m}\quad 2\text{ mm}}$.

Metric units of mass

Activity 2: Recognising the metric unit of mass

Perform the following tasks individually or in groups:
1. List down the instruments used to measure the mass of different objects.
2. Use a beam balance to measure the mass of the following items in your school: basic mathematics textbook, counter book, mathematical set, and a ruler.
3. Display the results of your measurements on a manila sheet.
4. Share your results with the rest of the class through presentations and discussions.

The basic unit of mass is kilogram, denoted by kg. Other units of mass include; tonne, hectogram, decagram, gram, decigram, centigram, and milligram. Normally, heavy objects are measured in tonnes (t).

The conversion of the metric units of mass can be done as shown in the following table.

Unit	Abbreviations	Conversion	Conversion into kilogram
Milligram	mg	10 mg = 1 cg	1 kg = 1 000 000 mg
Centigram	cg	10 cg = 1 dg	1 kg = 100 000 cg
Decigram	dg	10 dg = 1 g	1 kg = 10 000 dg
Gram	g	10 g = 1 dag	1 kg = 1 000 g
Decagram	dag	10 dag = 1 hg	1 kg = 100 dag
Hectogram	hg	10 hg = 1kg	1 kg = 10 hg
Kilogram	kg	1 000kg = 1t	
Tonne	t		

Conversion of units of mass

Mathematical operations on measurement of mass can be done only when the units used are the same. If the measurements contain different units of mass, conversion should be done to obtain the same units of measurement.

Example 1

Convert 1 256 grams into kilograms.

Solution

Using the comparison of units, we have

$$1\ 000 \text{ g} = 1 \text{ kg}$$

$$1 \text{ g} = 1 \text{ kg} \div 1\ 000$$

Thus, $1\ 256 \text{ g} = \frac{1}{1000} \times 1\ 256 \text{ g}$

$= 1.256 \text{ kg}.$

Therefore, 1 256 grams = 1. 256 kilograms.

Example 2

How many grams are there in 0.086 kilograms?

Solution

Using the comparison of units, we have

$$1 \text{ kg} = 1\ 000 \text{ g}$$

$$\text{Thus, } 0.086 \text{ kg} = 1\ 000 \times 0.086 \text{ g}$$

$$= 86 \text{ g.}$$

Therefore, there are 86 grams in 0.086 kilograms.

Exercise 6

1. Convert 30 kilograms into the following units of measurements:
   (a) Grams
   (b) Milligrams
2. How many kilograms are there in the following measurements?
   (a) 70 000 000 milligrams
   (b) 0.008 tonnes
3. Convert 2 kilograms into grams.
4. Convert 98 630 milligrams into tonnes.
5. Convert 6 000 milligrams into kilograms.
6. If the mass of a bag of rice is 60 kilograms, what is this mass in grams?
7. Convert 36 kilograms into the following units of measurements:
   (a) Hectograms
   (b) Decigrams
   (c) Decagrams
8. Convert the following measurements into grams:
   (a) 0.86 kilograms
   (b) 0.075 tonnes
   (c) 3 decigrams

9. The mass of a sack of wheat flour is 0.08 tonnes. Convert this mass into kilograms.

10. Arrange the following measurements in descending order:
0.02 kilograms; 0.02 hectograms; 700 grams; 70 milligrams; 100 centigrams.

Operations on units of mass

The units of measurement of mass can be added or subtracted and multiplied or divided by a constant number.

Addition of units of mass

When adding different units of measurement of mass, the corresponding units have to be placed together and added as required.

Example 1

Compute the following:

(a)

	kg	g
	60	33
+	19	969

(b)

kg	hg	g
60	9	960
+	11	45

Solution

Add the corresponding units as follows:

(a)

	kg	g
	60	33
+	19	969
	80	2

(b)

kg	hg	g
60	9	960
+	11	45
63	0	5

Example 2

The mass of a car is 958 kilograms. Two bags of masses of 62 kilograms and 77 kilograms, are loaded in the car. What is the total mass of the loaded car in:

(a) kilograms?

(b) tonnes?

Solution

(a) Mass of the car = 958 kg

Mass of the first bag = 62 kg

Mass of the second bag = 77 kg

Add the masses as follows:

958 kg + 62 kg + 77 kg = 1 097 kg.

Therefore, the total mass of the loaded car is 1 097 kilograms.

(b)1 tonne = 1 000 kg

$x = 1\,097$ kg

$$x = \frac{1 \text{ tonne} \times 1\,097 \text{ kg}}{1\,000 \text{ kg}}$$

$$x = \frac{1\,097 \text{ tonnes}}{1\,000}$$

$= 1.097$ tonnes.

Therefore, the total mass of the loaded car is 1.097 tonnes.

Exercise 7

Answer each of the following questions:

1.

	g	dg	cg
	8	9	6
	5	6	3
+	8	7	7

2.

	kg	hg	dag
	8	9	7
	9	7	6
+	5	6	3

3.

	g	dg	cg
	7	7	7
	8	8	8
+	9	9	9

7.

	kg	dag	g
	7	9	7
	8	8	9
+	9	7	8

4.

	kg	dag	g
	7	9	6
	8	7	7
+	5	6	3

8.

	dg	cg	mg
	7	8	9
	8	9	7
+	9	7	8

5.

	hg	dag	g
	8	9	3
	8	7	2
+	5	6	6

9.

	kg	g	mg
	7	8	9
	8	9	7
+	9	7	8

6.

	kg	hg	mg
	5	6	7
	4	9	8
+	3	3	1

10. In each of the following, add the given measurements (give your answers in kilograms).
(a) 785 grams, 97 grams, 605 grams
(b) 24.37 kilograms, 187.5 kilograms, 21.13 kilograms
(c) 8 decagrams, 9 grams, 2 centigrams

Subtraction of units of mass

When subtracting different units of measurement of mass, the corresponding units have to be placed together and subtraction is done as required.

Example 1

Compute the following:

(a)

	kg	g
	60	39
−	28	940

(b)

	kg	hg	dag
	2	9	8
−	1	9	9

Solution

Subtract the corresponding units as follows:

(a)

	kg	g
	60	39
−	28	940
	31	99

(b)

	kg	hg	dag
	2	9	8
−	1	9	9
		9	9

Example 2

A school lorry has a mass of 10 tonnes 500 kilograms when loaded with beans. If the mass of the beans is 4 tonnes 90 kilograms, find the mass of the lorry.

Solution

Mass of the lorry and beans = 10 tonnes 500 kg

Mass of beans = 4 tonnes 90 kg

Mass of unloaded lorry is obtained by subtracting the masses as follows:

	t	kg
	10	500
−	4	90
	6	410

Therefore, the mass of the lorry is 6 tonnes 410 kilograms.

Exercise 8

Answer each of the following questions:

1.

	g	dg
	8	7
–	6	9

4.

kg	g	dg
6	7	8
– 5	8	9

7.

dag	dg	mg
6	89	2
– 4	98	9

2.

g	dg	cg
6	7	8
– 5	7	9

5.

kg	g	dg
2	0	6
– 1	98	9

8.

kg	hg	dag
160	9	7
– 150	9	9

3.

kg	hg	dag
7	8	9
– 6	9	8

6.

kg	hg	dg
6	6	6
– 4	87	7

9.

kg	hg	dag
6	4	7
– 5	9	8

10. Compute the following:
   (a) 7 dg – 7 cg
   (b) 2 g – 10 dg
   (c) 0.85 kg – 168 g
   (d) 86 mg – 68 mg

Multiplication of units of mass

The metric units of mass can be multiplied by a constant number. The product retains the given units. The following examples explain how multiplication is done.

Example 1

Compute the following:

(a)

kg	g
3	81
×	50

(b)

t	kg
5	90
×	50

Solution

Compute the given units as follows:

(a)

kg	g
3	81
×	50
154	50

(b)

t	kg
5	90
×	50
254	500

Example 2

Eight trucks are loaded with 12 tonnes of crude oil each. Find the total mass of the crude oil in kilograms.

Solution

The total mass is 8×12 tonnes $= 96$ tonnes

1 tonne $= 1\ 000$ kg

Thus, 96 tonnes $= 96\ 000$ kg.

Therefore, the total mass of the crude oil is 96 000 kilograms.

Exercise 9

Answer the following questions:

1.

t	kg
4	200
×	50

2.

g	cg
6	9
×	28

3.

dag	g
5	5
×	25

4.

hg	dag	g
8	7	6
×		20

5.

dg	cg	mg
5	8	9
×		10

6.

hg	dag	g
15	6	9
×		100

7. A Form One class has 42 students. If each student has a mass of 22 kilograms and 900 grams, what is their total mass?
8. Three boys have a mass of 24 kilograms 520 grams each. What is their total mass?
9. A certain school bought 22 bags of rice for a Form Four graduation ceremony. If each bag contained 29 kilograms and 420 grams, what was the total mass of the rice in kilograms?

Division of units of mass

Division of metric units of mass is done by dividing the measurements with the same units, or dividing by a constant number.

Example 1

Compute the following:

(a) (40 kg 200 g) ÷ 4

(b) (5 kg 400 g) ÷ 3

Solution

Divide the units as follows:

(a) (40 kg 200 g) ÷ 4 = 10 kg 50 g

(b) (5 kg 400 g) ÷ 3 = 1 kg 800 g

Example 2

If 60 tonnes of fertilizer is to be shared equally among 12 villages, how many kilograms will each village get?

Solution

Amount of fertilizer to be shared is 60 tonnes

Number of villages is 12

Divide the units by the number of villages.

That is, each village will get 60 tonnes ÷ 12 villages = 5 tonnes per village

Converting 5 tonnes into kilograms, we have

1 tonne = 1 000 kg

Thus, 5 tonnes = 5 × 1 000 kg

= 5 000 kg.

Therefore, each village will get 5 000 kilograms.

Exercise 10

1. Compute the following:

   (a) (50 g 5 dg) ÷ 5

   (b) (18 g 9 cg) ÷ 9

2. Compute each of the following:
   (a) (15 g) ÷ 6
   (b) (28 hg 1 dag) ÷ 10
3. If one tea spoonful of sugar is added to every 120 g of juice, how many spoons of sugar should be added to 1 kg 800 g of juice?
4. The mass of a bottle full of mercury is 1 kg and that of an empty bottle is 184 g. What is the mass of mercury?
5. A student carried 11 exercise books of the same mass. If the total mass of the exercise books was 1 kg 320 g, what was the mass of each exercise book?
6. There are 230 ice cubes in Jasson's fridge. If the mass of an empty fridge and the ice cubes is 665 000 grams, what is the mass of each ice-cube?

Metric units of time

Activity 3: Recognising the unit of time

Perform the following tasks individually or in groups:
1. Identify a specific time at which all students arrive at school. What kind of instruments have you used to obtain the answers? Explain how you read the measurements.
2. Share your answers with other students in a class through discussion.

The basic unit of time is second, denoted by *s*. Other units of time include; minutes, hours, days, weeks, months, years, decades, centuries, and millennia. These units of time are related as follows:
60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
52 weeks = 1 year and
12 months = 1 year
The months in a year have different numbers of days. The months of January, March, May, July, August, October, and December have 31 days each. The months of April, June, September, and November have 30 days each. The month of February has 28 days for an ordinary year and 29 days for a leap (long) year.

The following table shows the conversion and relationship of units of time:

Unit	Abbreviation	Conversion
Second	s	
Minute	min	60 s = 1 min
Hour	hr	60 min = 1 h
Day		24 hr = 1 day
Week		7 days = 1 week
Month		28, 29, 30, or 31 days = 1 month
Year		365 or 366 days = 1 year

Reading time and conversion of units of time

In Standard Six, you learnt about reading and writing time in both 12-hour format and 24-hour format. The same knowledge will be applied in this section. The two ways of reading time are as shown in Figure 5.1:

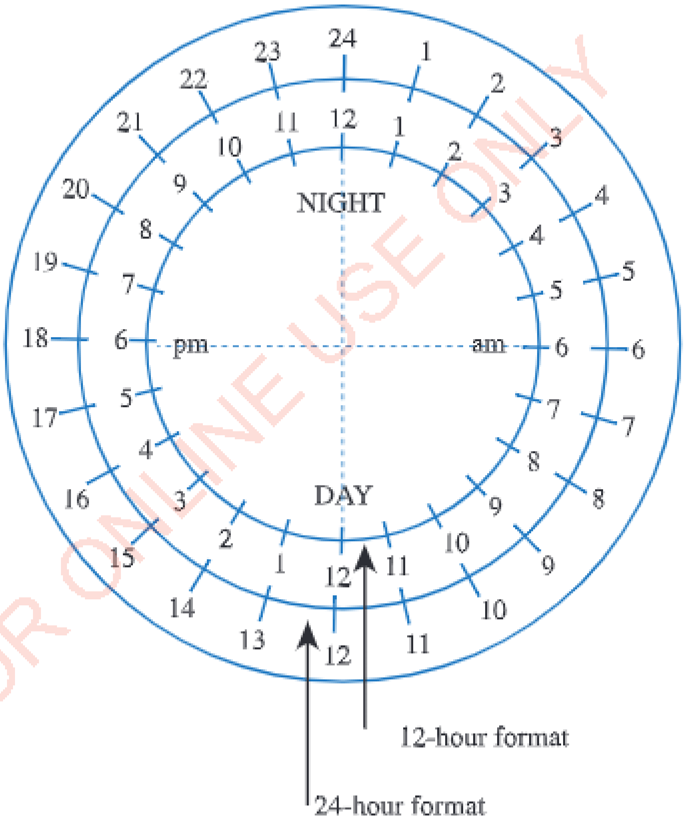

Figure 5.1: *12-hour format and 24-hour format*

The following table shows two ways of reading time:

12-hour format	24-hour format
12:00 midnight	2400 or 0000
1:00 am	0100
2:00 am	0200
3:00 am	0300
4:00 am	0400
.	.
.	.
.	.
12:00 noon	1200
1:00 pm	1300
2:00 pm	1400
3:00 pm	1500
4:00 pm	1600
.	.
.	.
.	.
12:00 midnight	2400 or 0000

Example 1

Write 25 minutes past 10 in the morning using the following time formats:

(a) 12-hour format

(b) 24-hour format

Solution

(a) 25 minutes past 10 in the morning in 12-hour format is 10:25 am.

(b) 25 minutes past 10 in the morning in 24-hour format is 1025 hours.

Example 2

(a) Draw a clock face of 12-hours showing ten minutes to eight in the evening.

(b) Write the time in (a) into a 12-hour format.

(c) Convert the time in (b) into a 24-hour format.

Solution

(a) The 12 hour format clock face is as follows:

(b) In a 12-hour format, it is written as 7:50 pm.

(c) In a 24-hour format, it is written as 1950 hours.

When writing time in a 12-hour format, a colon is placed between the hours and minutes. Minutes appear to the right side of the colon. For example; 10:25 am is read as twenty-five minutes past ten am. When using a 24-hour format, always express time in four figures numeral, where the first two digits represent hours and the last two digits represent minutes. For example; 1025 is read as ten twenty five hours.

Exercise 11

Answer the following questions:

1. Write the following times in a 24-hour format:
   (a) 6:30 am
   (b) 5:37 pm
   (c) 3:20 pm
   (d) 10:53 am
   (e) 10:50 pm
   (f) 9:45 pm
2. Write the following times in 12-hour and 24-hour formats:
   (a) Five minutes to ten in the evening
   (b) Sixteen minutes to twelve in the morning
   (c) Eighteen minutes past three in the morning
   (d) Twenty-seven minutes past seven in the evening

3. Write the following hours in words using a 12-hour format:

(a) 2040 (d) 1836
(b) 2349 (e) 0100
(c) 1745

4. Carefully, study the following table and answer the questions that follow.

Departing time (hrs)	Station
0430	Tabora
0547	Nzubuka
0649	Ipala
0759	Bukene
0839	Mahene
0935	Isaka
1018	Luhumbo
1121	Usule
1225	Shinyanga
1325	Songwa
1410	Seke
1500	Malampaka
1545	Malya
1640	Bukwimba
1755	Mantale
1855	Fela
1955	Mwanza South
2010	Mwanza

At which stations will the train be at the following times?

(a) 3:00 pm (b) 0839 (c) 1325 (d) 7:55 pm

A leap year has 366 days and occurs if the year is exactly divisible by 4, or in the case of the final year of a century, it is divisible by 400. That is, the final year of the century will be a leap year if and only if it is exactly divisible by 400. For example, 1964 and 2000 were leap years, while 1700, 1982, and 1991 were not leap years. The year 1700 was the last year of the century, and it is not exactly divisible by 400, so it was not a leap year.

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Example 1

How many hours are there in a week?

Solution

Using the comparison of units of time, we have

1 week = 7 days

1 day = 24 hours

Thus, 1 week = 24×7 hours

= 168 hours.

Therefore, there are 168 hours in a week.

Example 2

Convert 70 leap years into seconds.

Solution

Using comparison of units of time, we have

1 year = 366 days

70 years = 70×366 days

1day = $24 \times 60 \times 60$ s

Thus, 70 years = $70 \times 366 \times 24 \times 60 \times 60$ s

= 2 213 568 000 seconds.

Therefore, 70 leap years = 2 213 568 000 seconds.

Exercise 12

In question 1 to 10, convert the following times into hours:

1. Two weeks
2. All the days of January
3. All the days of the year 1968
4. Forty years (ordinary years)

5. 67 824 000 seconds
6. Eighty-four days
7. The days of the years 1964 and 1965
8. Forty-eight minutes
9. All the days of February, 1961
10. All the days of September, 1983
11. Convert the following into seconds:
    (a) All the mondays of January, 1984 given that January 1st, 1984 was a Sunday
    (b) One week
    (c) 12 hours
12. How many days are there from the first day of the leap year to 11th May of the same year?

Addition and subtraction of units of time

When adding or subtracting different units of time, the corresponding units are placed together and added or subtracted as required.

Example 1

Compute the following:

(a)

	hr	min	sec
	2	12	43
	4	20	32
+		65	15

(b)

	hr	min
	10	45
+	17	20

(c)

	hr	sec
	3	260
	5	180
+	2	9

Solution

Compute the corresponding units as follows:

(a)

	hr	min	sec
	2	12	43
	4	20	32
+		65	15
	7	38	30

(b)

	hr	min
	10	45
+	17	20
	28	5

(c)

	hr	sec
	3	260
	5	180
+	2	9
	10	449

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Example 2

Compute the following:

(a)

	hr	min	sec
	7	42	18
−	4	17	23

(b)

	hr	min
	12	39
−	4	21

Solution

Compute the corresponding units as follows:

(a)

	hr	min	sec
	7	42	18
−	4	17	23
	3	24	55

(b)

	hr	min
	12	39
−	4	21
	8	18

Exercise 13

Answer each of the following questions:

1.

	hr	min
	5	24
+	3	52

2.

	hr	min
	8	16
−		9

3.

	hr	min	sec
	10	24	40
	5	31	5
+	2	2	39

4.

	hr	min	sec
	9	23	37
−	2	11	41

5.

	hr	min
	10	24
	5	31
+	2	2

6.

	min	sec
	36	45
−	24	59

Metric units of capacity

Activity 4 : Recognising the metric units of capacity

Perform the following tasks individually or in groups:

1. In everyday life, there are some units used to measure capacity. Identify and briefly explain them.
2. Share your answers to the rest of the class. Do you have similar measurements? If not why?

When we buy a bottle of drinking water or cooking oil, our interest is mainly on how much water or oil is contained in the bottle. That is, we want to know the amount of water or oil the container holds or the capacity.

The basic unit of capacity in the metric system is the litre. The most common units for capacity are litre and millilitre. One litre of water weighs one kilogram. Other units of capacity include centilitre, decilitre, decalitre, hectolitre, and kilolitre.

The following table shows the conversion of the units of capacity:

Unit	Abbreviation	Conversion
Millilitre	ml	10 ml = 1 cl
Centilitre	cl	10 cl = 1 dl
Decilitre	dl	10 dl = 1 l
Litre	l	10 l = 1 dal
Decalitre	dal	10 dal = 1 hl
Hectolitre	hl	10 hl = 1 kl
Kilolitre	kl	

Note: 1 litre = 1 000 cm^3

Uses of units of capacity in our daily life

In hospitals, doctors prescribe liquid medications in millilitres. Manufacturers of baby milk powders give a feeding table with the quantities written in millilitres. Fuels for motor vehicles are always sold in litres. There are so many other uses of units of capacity in our daily life activities.

Problems involving units of capacity

In our daily life activities, we normally encounter tasks which require the use of units of capacity.

Example 1

One litre of cooking oil costs 2 500 Tanzanian shillings. Find the cost of 15 litres of the cooking oil.

Solution

Cost of 1 litre is Tsh 2 500.

Thus,

Cost of 15 litres = Tsh 2 500 × 15 litres

= Tsh 37 500.

Therefore, the cost of 15 litres of the cooking oil is 37 500 Tanzanian shillings.

Example 2

Dotto sells 1 litre of milk for 1 000 Tanzanian shillings. How many litres of milk does he need to sell to get 34 800 Tanzanian shillings?

Solution

Cost of 1 litre is Tsh 1 000

Dotto needs to get Tsh 34 800

Divide 34 800 Tanzanian shillings by 1 000 Tanzanian shillings to get the number of litres required.

That is,

$$\begin{array}{r} 34.8 \\ 1000 \overline{)\ 34800} \\ -3000 \\ \hline 4800 \\ -\ 4000 \\ \hline 8000 \\ -\ 8000 \\ \hline --- \end{array}$$

Therefore, Dotto needs to sell 34.8 litres of milk.

Example 3

Kulwa bought 60 bottles of water containing 350 millilitres each. Write the amount of water that Kulwa bought in litres.

Solution

Capacity of 1 bottle is 350 millilitres

Number of bottles is 60

Thus,

Amount of water bought is given by

Capacity = 350 millilitres × 60

= 21 000 millilitres

Convert 21 000 millilitres into litres.

1 litre = 1 000 millilitres

Thus,

21 000 litres ÷ 1 000 = 21 litres

Therefore, Kulwa bought 21 litres of water.

Exercise 14

1. Anna bought 3 bottles of juice of 350 millilitres each, and John bought 1 bottle of juice of 1 litre. Who had more juice to drink? What is the difference in capacity of their juices?
2. Mosi used $\frac{1}{4}$ of 52 litres of water for washing clothes. How many litres of water were left?
3. How many bottles of 400 millilitres each will be filled from a bucket of water of capacity 20 litres?
4. Mgeni bought 13 bottles of Fanta and 9 bottles of Cocacola. If each bottle has a capacity of 350 millilitres, find the total number of litres of Fanta and Cocacola bought all together.
5. Mashaka's cow produces 18 litres of milk every day:
   (a) How many cows of the same type should Mashaka keep to get 126 litres every day?
   (b) How much money does he get every day if 1 litre is sold at 650 Tanzanian shillings?
6. What is $\frac{3}{5}$ of 30 litres in millilitres?

Conversion of metric units of capacity

Example 1

Convert the following measurements into cm^3:

(a) 6.5 litres (b) 35 litres

Solution

Using the comparison of units

1 litre = 1 000 cm^3

Thus,

(a) 6.5 litres = $6.5 \times 1\,000\ cm^3$
= 6 500 cm^3.

Therefore, 6.5 litres = 6 500 cm^3.

(b) 35 litres = $35 \times 1\,000\ cm^3$
= 35 000 cm^3.

Therefore, 35 litres = 35 000 cm^3.

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Example 2

Convert the following measurements into litres:

(a) 5 600 cm^3 (b) 24 000 cm^3 (c) 4.2 kl

Solution

(a) Using the comparison of units

1 litre = 1 000 cm^3

Thus,

5 600 cm^3 = 5 600 litres ÷ 1 000

= 5.6 litres.

Therefore, 5 600 cm^3 = 5.6 litres.

(b) 24 000 cm^3 = 24 000 litres ÷ 1 000

= 24 litres.

Therefore, 24 000 cm^3 = 24 litres.

(c) 1 kl = 1 000 litres

Thus,

4.2 kl = 4.2 × 1 000 litres

= 4 200 litres.

Therefore, 4.2 kl = 4 200 litres.

Exercise 15

1. Which of the following are units of capacity?
   (a) centimetre (d) litre
   (b) millilitre (e) kilolitre
   (c) metre
2. Convert the following measurements into kilolitres:
   (a) 2 580 litres (b) 5 070 litres (c) 1 854 litres
3. Convert the following measurements into litres:
   (a) 4 800 cm^3 (b) 3.6 kilolitres (c) 5 640 millilitres

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4. How many litres are there in a kilolitre?
5. How many centilitres are there in a hectolitre?
6. Which of the following units of capacity are less than a litre?
centilitre, millilitre, kilolitre, decilitre, and hectolitre.

Chapter summary

1. The basic unit of length is metre (m).
2. The largest unit of length commonly used is kilometre (km).
3. The smallest unit of length commonly used is millimetre (mm).
4. Instruments which are commonly used to measure length are; ruler, measuring tape, and metre stick.
5. The standard unit of mass is kilogram (kg).
6. A tonne is the highest unit of measurement of mass (1t = 1 000 kg).
7. The basic unit of time is second and is denoted by s. Other units of time are; minute, hour, week, month, year, decade, century, and millennia.
8. Time is measured accurately using watches and clocks.
9. A calendar is used to display time in days, weeks, months, and years.

Revision exercise 4

Answer each of the following questions:

1.

	kg	g
	3	81
×		3

2.

	t	kg
	3	81
×		30

3.

	t	kg
	4	2
−	2	9

4.

	kg	g
	2	50
+	3	65

5.

	m	cm
	2	50
+	3	65

6.

	km	m
	4	2
−	2	9

7.

	dam	m	dm
	186	6	5
+	812	3	5

8. Convert the following measurements into the given units:
   (a) 15 hours into minutes
   (b) 7 250 minutes into hours
   (c) 480 seconds into minutes
9. Compute the following:
   (a) (4 kg 200 g) ÷ 4
   (b) (4 km 20 m) ÷ 5
   (c) (56 min 20 s) ÷ 13
10. What is the total number of days of the months of June, July, August, and September?
11. How many ordinary years were there between 1898 and 1967?
12. What were the leap years between 1953 and 1969?
13. What is the total number of days from January 17th to April 17th, inclusive in a leap year?

Study the following table and use it to answer questions 14 to 16.

Departing time (hrs)	Station	Distance in kilometres
0430	Tabora	0.0
0547	Nzubuka	30.4
0649	Ipala	60.8
0759	Bukene	91.2
0839	Mahene	110.4
0935	Isaka	131.2
1018	Luhumbo	148.8
1121	Usule	176.0
1225	Shinyanga	196.8
1325	Songwa	217.6
1410	Seke	238.4
1500	Malampaka	260.8
1545	Malya	278.4
1640	Bukwimba	300.8
1755	Mantale	324.8
1855	Fela	353.8
1955	Mwanza South	374.4
2010	Mwanza	377.6

14. Compare the distances from Mahene to Songwa, and from Seke to Mwanza. Which distance is longer? By how many kilometres?
15. Madata travelled from Malampaka to Mwanza, and then travelled to Tabora. How many kilometres did he travel altogether?
16. What is the distance from Tabora to:
    (a) Shinyanga? (c) Fela? (e) Mwanza?
    (b) Malya? (d) Luhumbo?
17. A boy has a pile of four books on his desk. The thicknesses of the books are 22 millimetres, 13 millimetres, 39 millimetres, and 18 millimetres, respectively. Can the pile fit in a shelf of a height of 10 centimetres? If so, calculate the length of the space left.
18. A lorry weighs 7.5 tonnes. If the total mass of the bricks carried by the lorry is 5 tonnes 400 kilograms 50 grams, what is the total weight of the lorry and bricks?

Project 4

In groups, collect 20 empty bottles of one litre capacity each and then, do the following:

1. Fill in each bottle with water up to its topmost part.
2. Pour all the water from each bottle into a bucket.
3. Determine the capacity of water in the bucket.

Chapter Six

Approximations

Introduction

Approximation involves expressing a number into a higher value or a lower value which is close to the exact value. Approximation is also termed as estimation, that is, the process of finding a number that is close enough to the exact answer. Estimation is not done to obtain the exact answer. It is used because some numbers can never be expressed in exact decimals. The symbol used for approximation is '≈.' In this chapter, you will learn about approximation of whole numbers and decimals, rounding off numbers, and writing numbers in significant figures. The competencies developed will help you to estimate quantities such as money, time, volume, and distance, among many other quantities. Also, you will be able to calculate and present approximated numbers and figures from different real life situations such as the population of a certain country, number of domestic animals in certain regions, data from experimental sciences, engineering, and construction, among many others.

Rounding off numbers

Activity

Individually or in groups, perform the following tasks:

1. Measure the heights of your group members by using a tape measure.
2. Write the heights to the nearest centimetres.
3. Write the answers obtained in task 2 to the nearest tens of centimetres.

4. Compare the answers in tasks 2 and 3. Which one is easier to remember? Does it change the rank of heights of your group members?
5. Share your answers with other members of the class through discussion.

Rounding off numbers is a process of making a number simpler, but keeping its value closer to what it was. The result is less accurate, but easier to use. Rounding off is done for whole numbers and decimals at various places like thousands, hundreds, tens, tenths, hundredths and so on. In order to round off a number, first check the digit to the right of the digit in the required place value to be rounded off.

(a) If the digit to the right is either 0, 1, 2, 3, or 4, then the digit at the required place value remains unchanged, and all digits to the right of it become zeros.
For example;
(i) 35.4 is rounded off to ones as 35.
(ii) 274 is rounded off to tens as 270.
(iii) 327 is rounded off to hundreds as 300.
(iv) 856 145 is rounded off to thousands as 856 000.

(b) If the digit to the right is either 5, 6, 7, 8, or 9, then 1 is added to the digit at the required place value and all the digits to the right of it become zeros.
For example;
(i) 0.267 is rounded off to tenths as 0.3.
(ii) 17.82 is rounded off to ones as 18.
(iii) 63 504 is rounded off to thousands as 64 000.

(c) If the digit to the right is 5, then the digit to the required place value is considered as follows:
(i) Add 1 to that digit if it is odd.
For example;
2.635 is rounded off as 2.64 to hundredths.
42.245568 is rounded off as 42.246 to thousandths.
(ii) The digit is left unchanged if it is even and the number to be rounded off has no digits after 5.
For example;
2.645 is rounded off as 2.64 to hundredths.
6.65 is rounded off as 6.6 to tenths.

(iii) Add 1 to that digit if there are other digits that follow 5 which are not all zeros.
For example;
7.4503 is rounded off as 7.5 to tenths.
81.26584 is rounded off as 81.27 to hundredths.

(iv) The digit remains unchanged if all the digits following 5 are zeros or if there are no digits after 5 and the digit before 5 is even.
For example;
3.62500 is rounded off as 3.62 to hundredths.
21.465 is rounded off as 21.46 to hundredths.

Example 1

The population of Tanzania Mainland in the 1967 census showed that there were 5 834 875 men and 6 111 188 women. Round off the figures to the nearest:
(a) Millions (b) Thousands

Solution

(a) 5 834 875 men is 6 000 000 men rounded off to the nearest millions (1 is added to 5 because the digit after 5 is 8).
6 111 188 women is 6 000 000 women rounded off to the nearest millions because the digit after 6 is 1.

(b) 5 834 875 men is 5 835 000 men rounded off to the nearest thousands because the digit after 4 is 8.
6 111 188 is 6 111 000 women rounded off to the nearest thousands because the digit after 1 is 1.

Example 2

A factory got a profit of 67 459 853 Tanzanian shillings after selling its products last year. How much profit did the factory get to the nearest thousands?

Solution

To round off the profit to the nearest thousands, a digit to the right of 9 is considered. Since the digit to the right of 9 is 8, add 1 to 9 to get 67 460 000 Tanzanian shillings.

Therefore, 67 459 853 Tanzanian shillings is rounded off to 67 460 000 Tanzanian shillings to the nearest thousands.

Example 3

Round off 0.046 to the nearest:

(a) Tenths.
(b) Hundredths.
(c) Thousandths.

Solution

(a) Since 4 is less than 5, then 0.046 becomes 0.0.
Therefore, 0.046 to the nearest tenths is 0.0.
(b) Since 6 is greater than 5, then 0.046 becomes 0.05.
Therefore, 0.046 to the nearest hundredths is 0.05.
(c) Since 0 is less than 5, then 0.0460 becomes 0.046.
Therefore, 0.046 to the nearest thousandths is 0.046.

Exercise 1

1. Round off each of the following numbers to the nearest thousands:
   (a) 8 259 (d) 100 998 (g) 60 500
   (b) 12 222 (e) 17 501 (h) 9 999
   (c) 13 709 (f) 2 349 673 (i) 1 234 567

2. Round off each of the following numbers to the nearest ones:
   (a) 41.4 (c) 2.613 (e) 0.379
   (b) 0.49 (d) 0.8 (f) 2.55

3. Round off each of the following numbers to the nearest tens:
   (a) 25.12 (d) 10.0089 (g) 20.17
   (b) 16.15 (e) 0.408 (h) 311.114
   (c) 28.929 (f) 33.456

4. The total mass of cotton harvested in Maswa district was 17 816 273 kg. Round off this mass to the nearest:
   (a) Millions of kilograms (b) Thousands of kilograms

5. In 1983, the number of primary school pupils in Kilimanjaro region was 237 268. Round off the number of pupils to the nearest thousands.

6. Find the value of each of the following, and round off the answers to the nearest hundredths:
   (a) 38.5×4.1 (c) 2.5×43.642 (e) 72×0.98
   (b) 9.9×9.9 (d) 10.3×4.4 (f) 0.048×20.08

7. Find the value of each of the following, and round off the answers to the nearest tenths:
   (a) $2\,684 \div 17$ (c) $0.33 \div 1.1$
   (b) $4\,314 \div 430$ (d) $5\,301 \div 18$

8. Convert each of the following fractions to decimals by rounding off the answers to the nearest tenths:
   (a) $\frac{2}{9}$ (c) $\frac{17}{3}$
   (b) $\frac{3}{7}$ (d) $\frac{1}{4}$

9. Consider the following numbers: 9, 2.1, 42.045, and 635.7891.
   (a) Add all the given numbers.
   (b) Determine the place value of each digit in the number obtained in (a).
   (c) Round off the number obtained in (a) to the nearest:
      (i) Hundredths (ii) Tenths (iii) Hundreds

Approximations in calculations

When working out the expressions or equations, it is useful to find a rough estimate of an answer. To find an estimated answer, take a suitable approximation by rounding off the numbers involved.

Example 1

Estimate the value of 38×71.

Solution

For easy estimation of the product, round off 38 and 71 to the nearest tens.
That is, $38 \times 71 \approx 40 \times 70 = 2\,800$.
Therefore, $38 \times 71 \approx 2\,800$.

Example 2

Estimate the value of 4.1×0.082.

Solution

Round off 4.1 to ones and 0.082 to the nearest hundredths.

$4.1 \times 0.082 \approx 4.0 \times 0.08 = 0.32$.

Therefore, $4.1 \times 0.082 \approx 0.32$.

Example 3

Estimate the value of $256.5 \div 63.5$.

Solution

Round off both numbers to the nearest ones.

$256.5 \approx 256$

$63.5 \approx 64$

$256.5 \div 63.5 \approx 256 \div 64 = 4$.

Therefore, $256.5 \div 63.5 \approx 4$.

Exercise 2

In question 1 to 10, estimate the value of each of the given expressions:

1. 43×28
2. $2\,912 \times 32$
3. 82×61
4. 868×31
5. 2.94×248
6. $171\,220 \div 79$
7. 35.164×23.04
8. $1.029 \div 0.021$
9. $4\,981 \div 6\,438$
10. $9\,110\,218\,800 \div 4\,081$
11. A shopkeeper sold 192 T-shirts at a price of 5 950 Tanzanian shillings each. How much money did the shopkeeper get by estimation?
12. A village received 40 376 bags of fertilizer to be distributed to 392 farmers. Estimate the number of bags each farmer got.

13. Estimate the answers of each of the given expressions:
   (a) $25.45 \div 268$
   (b) $1.0031 + 56.241$
   (c) $528 \times 3\,902$
   (d) $61 \div 24$
   (e) $824 - 325$

Significant figures

Any digit from 1 to 9 appearing in a number is a significant figure. Each zero appearing in a number between digits from 1 to 9 is a significant figure. For example; the 0 in 602 is a significant figure, that is, 602 has 3 significant figures.

When the zeros are written to the right of the last non-zero digit of an exact number, the zeros are not significant figures. For example; 72 000 has only 2 significant figures.

In decimals, any zero to the left of the first non-zero digit is not a significant figure. For example; the zeros in 0.025 are not significant figures. Thus, 0.025 has 2 significant figures.

When zero is written at the end of an approximate decimal or number, it is a significant figure. By an approximate decimal or number, it means a decimal or number has been rounded off to the tenth, hundredth, and so on. For example; $2.74 \approx 3.0$, in this case 0 is a significant figure. Also, $45\,961 \approx 46\,000$, the zero at the third place value is a significant figure.

A number can be written in a required number of significant figures by rounding off. For example; to write 38 176 (which has 5 significant figures), correct to 1 significant figure, the number is rounded off to ten thousands. The first digit, 3 is increased by one to become 4 and all other digits become zeros.

Thus, $38\,176 \approx 40\,000$, correct to 1 significant figure.

Note:

(i) Significant figures are always identified from the left side to the right side.
(ii) Significant figures are also known as significant digits or precision or resolution.

Example 1

Determine the number of significant figures in each of the following numbers:

(a) 2.3004 (c) 0.00002

(b) 0.0804 (d) 4 002 000 000

Solution

(a) 2.3004 has 5 significant figures

(b) 0.0804 has 3 significant figures

(c) 0.00002 has 1 significant figure

(d) 4 002 000 000 has 4 significant figures

Example 2

Find the value of 0.3143×6.06, giving the answer correct to 3 significant figures.

Solution

$0.3143 \times 6.06 = 1.904658$

$1.904658 \approx 1.90$ correct to 3 significant figures.

Therefore, $0.3143 \times 6.06 \approx 1.90$ correct to 3 significant figures.

Exercise 3

In questions 1 to 6, approximate the numbers correct to the required number of significant figures.

1. 0.285173 (3 significant figures)
2. 88092.7 (4 significant figures)
3. 2.007138 (3 significant figures)
4. 74.471 (1 significant figure)
5. 10.6987 (3 significant figures)
6. 126.306 (2 significant figures)
7. Write:
   (a) $\frac{25}{13}$ in decimal form, correct to 4 significant figures
   (b) $\frac{5}{7}$ in decimal form, correct to 3 significant figures

8. How many significant figures does each of the following numbers have?
   (a) 5. 0372
   (b) 97168.90000
   (c) 0.0000678
   (d) 292.00044
   (e) 1.0000678
   (f) 0.101

9. Find the sum of the following numbers, and give your answers correct to 3 significant figures:
   (a) 8 917, 201 392, 500 099, 1 983
   (b) 0.5021, 21.1108, 0.00472, 3.079
   (c) 98.528, 8.888, 0.337, 24.82
   (d) 0.07637, 72.0976, 4.7135, 25.4556

10. Find the value of each of the following expressions. Write your answers correct to the given number of significant figures.
   (a) 0.83×0.737 (3 significant figures)
   (b) 5.178×20 (2 significant figures)
   (c) 7.351×4.83 (4 significant figures)
   (d) $7.31 \div 0.983$ (3 significant figures)
   (e) $149.001 \div 8.99$ (3 significant figures)
   (f) 6.814×72.35 (4 significant figures)

11. (a) Write $\frac{22}{3}$ in a decimal correct to 4 significant figures.
   (b) Find the value of the following expression in a decimal correct to 2 significant figures: $\frac{1}{2} + 3\frac{1}{3} + 4$.
   (c) Write the number 0.48965 correct to 4 significant figures.

Decimal places

All positions occupied by digits to the right of the decimal point are decimal places of a number. For example;

(a) 7.2 has 1 decimal place
(b) 6.03 has 2 decimal places
(c) 6.161 has 3 decimal places
(d) 305 has 0 decimal places
(e) 0.0004 has 4 decimal places
(f) 6.20 has 2 decimal places

Writing numbers correct to the specified number of decimal places

Take the case of writing a number correct to 3 decimal places. Consider the fourth digit after the decimal point. If the digit at the fourth decimal place is less than 5, then the digit at the third decimal place will remain unchanged, and all the digits after that will be dropped.

For example; $8.126347 \approx 8.126$ correct to 3 decimal places.

If the digit at the fourth decimal place is equal to or greater than 5, and there are some non-zero digits after the digit at the fourth decimal place, then the digit at the third decimal place should be increased by one.

If the digit at the third decimal place is even, and there are zero digits or no any other digit after the digit at the fourth decimal place, then the digit at the third decimal place should remain unchanged.

For example:

1. $0.97381 \approx 0.974$ to 3 decimal places.
2. $0.2465 \approx 0.246$ to 3 decimal places.
3. $0.2475 \approx 0.248$ to 3 decimal places.

Exercise 4

1. Write each of the following numbers correct to 2 decimal places:

(a) 0.0817
(b) 5.0744
(c) 1.70007
(d) 0.7153
(e) 12.047
(f) 3.3456
(g) 3.6149
(h) 0.00825
(i) 72.7946

2. Change each of the following fractions into decimals correct to 3 decimal places:

(a) $\frac{10}{17}$
(b) $\frac{4}{9}$
(c) $\frac{2}{3}$
(d) $\frac{4}{11}$
(e) $\frac{11}{7}$
(f) $2\frac{11}{29}$

3. Write each of the following numbers correct to 1 decimal place:

(a) 3.142
(b) 0.6667
(c) 250.707
(d) 0.09178
(e) 0.7159
(f) 10.445

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4. Write each of the following numbers correct to 3 decimal places:
   (a) 0.7526 (d) 8.4999
   (b) 34.7007 (e) 5.5555
   (c) 3.14159 (f) 2.66732

5. Divide 8.51 by 0.472, and give the answer correct to 2 decimal places.

6. Multiply 9.0017 by 0.0987, and give your answer correct to 2 decimal places.

7. Round off the following numbers correct to the given number of decimal places:
   (a) 0.002752 (4 decimal places)
   (b) 20.04416 (2 decimal places)
   (c) 0.17244 (3 decimal places)
   (d) 6.0097 (3 decimal places)
   (e) 2.14678 (1 decimal places)

8. Determine the number of decimal places in each of the following numbers:
   (a) 4000.001 (d) 25.3
   (b) 1 986 (e) 0.01001
   (c) 0.0075 (f) 10.0100165

9. Determine the number of decimal places in the values obtained from the following expressions:
   (a) $3.142 \times 15.25 \times 15$
   (b) 7.9909×0.011
   (c) 0.0073×0.73

10. Determine the number of decimal places in the values obtained from the following expressions:
   (a) $153.4 \div 6.5$
   (b) $44.0561 \div 11$
   (c) $0.001144 \div 0.056$

Chapter summary

1. Approximating or rounding off a number is a process of writing a number not exactly, but close enough to the correct number. Both integers and decimals can be rounded off.
2. During approximation, if the digit to the right is 5, the digit to required place value is:
   (a) increased by 1 if it is odd.
   (b) left unchanged if it is even and has zero digits after 5 or has no digit after 5.
3. Any digit from 1 to 9 appearing in a number is a significant figure.
4. When zeros are written to the right of the last non-zero digit of an exact number, the zeros are not significant figures.
5. In decimals, any zero written to the left of the first non-zero digit is not a significant figure.
6. When a zero is written at the end of an approximated decimal or number, it is a significant figure.
7. All positions occupied by digits to the right of the decimal point are the decimal places of a number.

Revision exercise

1. Round off each of the following numbers to the nearest hundredths:
   (a) 8.648 (d) 31.7842
   (b) 1.0544 (e) 19.6723
   (c) 0.341 (f) 0.453

2. Round off each of the following numbers to the nearest millions and thousands:
   (a) 78 911 393 (d) 1 350 095 450
   (b) 1 114 562 (e) 20 781 233
   (c) 22 878 130

3. From the 1967 Tanzanian Mainland census, there were 5 267 910 children under the age of fifteen. Round off this number to the nearest:
   (a) Millions
   (b) Thousands
   (c) Hundreds

In questions 4 to 13, estimate the values obtained from each of the given expressions:

4. 8.7×410
5. 4.17×730
6. 28×0.83
7. 54.5×1.96
8. $430 \div 31.2$
9. $24.4 \div 0.673$
10. $9.05 \div 18.2$
11. 0.633×425
12. $0.136 \div 8.45$
13. 7.40×36.4

14. Express each of the following fractions as a decimal, correct to 4 significant figures:
    (a) $\frac{5}{11}$ (b) $\frac{3}{70}$ (c) $\frac{1}{6}$

15. Write 86.463 correct to:
    (a) 1 decimal place
    (b) 2 decimal places
    (c) 1 significant figure
    (d) 2 significant figures

16. Write 0.00607049 correct to:
    (a) 3 decimal places
    (b) 4 decimal places
    (c) 5 decimal places
    (d) 6 decimal places

17. (a) Write each of the following numbers correct to 1 decimal place:
    (i) 17.84 (ii) 17.084 (iii) 2.045 (iv) 0.048
    (b) Write each of the following numbers correct to 2 decimal places:
    (i) 23.748 (ii) 23.0845 (iii) 0.0485 (iv) 0.0803

18. For each of the following numbers, determine the number of significant figures, and the number of decimal places:
    (a) 8 (b) 0.47 (c) 0.070
    (d) 0.8 (e) 1.0271 (f) 6.0470
    (g) 0.008 (h) 0.307 (i) 0.3079

19. Convert each of the following fractions into decimals and write your answer correct to 2 decimal places:

(a) $\frac{9}{13}$ (b) $\frac{18}{19}$ (c) $\frac{17}{180}$

20. Determine the number of significant figures in each of the following numbers:

(a) 2.73
(b) 400 780
(c) 0.006
(d) 0.1089

21. Write:

(a) 34.996 correct to 2 decimal places.
(b) 35.0482 correct to 3 significant figures.

22. Write each of the following numbers correct to three decimal places:

(a) 0.00606
(b) 3.199281
(c) 8.27491
(d) 2.6047
(e) 72.87247

Project 5

Visit any nearby authority responsible for land transport. Ask the authority what are the exact distances in kilometres of five different roads from one municipality, town, or city to others. Record the information obtained, and complete the following table:

Road	Exact distance (km)	Distance to the nearest 10 km
1		
2		
3		
4		
5		

From the table, which two roads (if any) have the same distance to the nearest 10 km?

Chapter Seven

Introduction to geometry

Introduction

Geometry is a branch of mathematics that deals with the properties of points, lines, angles, and different regular shapes. It was first introduced by a Greek mathematician known as Euclid, who is regarded as the father of geometry, around the year 300 BC. The word "geometry" originates from the Greek word 'geo' which means 'earth' and 'metron' which means measurement. Geometry therefore, deals with measurement of lines, angles, and other properties of regular objects. Objects such as knives, needles, pencils, or pens have sharp tips as shown in Figure 7.1. In geometry, sharp tips are represented by points. In this chapter, you will learn the properties and relations of points and lines, angles, and their construction, regions, polygons, and circles. The competencies developed can be applied in daily life activities such as carpentry, engineering, tailoring, architecture, making balls from skin, and many other applications.

Figure 7.1: *Objects with sharp tips*

Points, lines, rays, line segments, and planes

Activity 1

Individually or in groups, perform the following tasks:

1. Draw a straight line.
2. Draw a number line for all integers less than –2.
3. Draw a number line for all integers greater or equal to 1.
4. Draw a number line for all integers from –4 to 5.

Points

A point is a mark of position, and has an exact location. It has no length, width or thickness. A point is represented by a dot made by the tip of a sharp pencil or pen. It is denoted by a capital letter. In Figure 7.2, A, B, and C are examples of points.

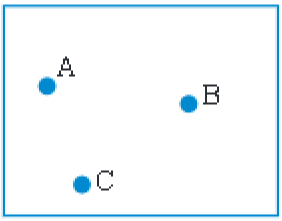


Figure 7.2: *Examples of points*

Lines

A line is a set of points which extends in both directions without an end. A line can also be considered as a straight path which can be extended indefinitely in both directions. It is shown by two arrow-heads in opposite directions as shown in Figure 7.3. A line does not have any fixed length, thus, it has no end points.

Figure 7.3: *Line*

Rays

A ray is part of a line which extends without an end in one direction only while the other end is fixed.

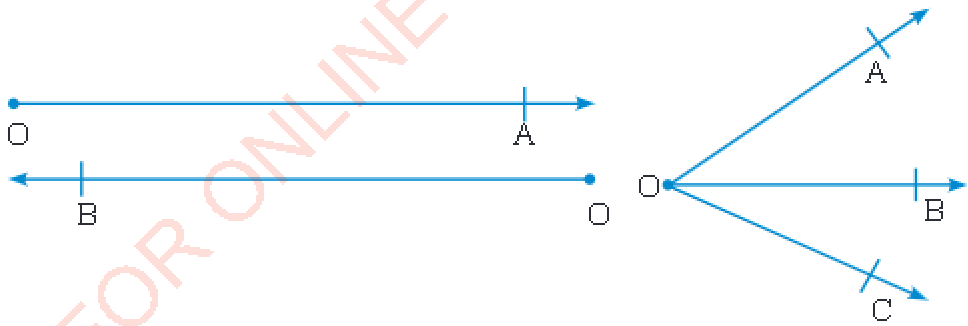


Figure 7.4: *Examples of rays*

A ray has no fixed length. However, it has one end point called the initial point. In Figure 7.4, O is the initial point of the rays OA and OB. The rays OA and OB in Figure 7.4 are different because they extend in different directions. A ray OA is denoted as $\overrightarrow{OA}$.

Line segments

A line segment is a straight path which has a definite length. It is a part of a line with two end points. A line segment with the end points A and B is denoted as $\overline{AB}$ or $\overline{BA}$, and is read as line segment AB or line segment BA. Figure 7.5 shows an example of a line segment AB.

Figure 7.5: *Line segment AB*

Planes

A smooth and flat surface gives an idea of a plane. The surface of a table, a wall, and a blackboard are examples of planes. A plane extends in all directions with no end. It has no length, width, or thickness. Figures like square, rectangle, triangle, and circle can be drawn on the plane. Hence, these figures can also be called plane figures.

Figure 7.6: *Examples of plane figures*

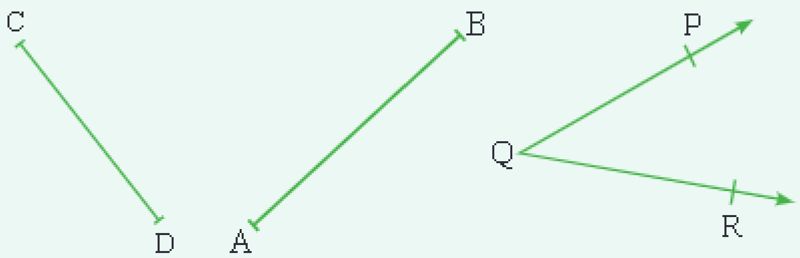
Exercise 1

Answer the following questions:

1. Mark any three points on a plane such that no end points lie on the same line. How many line segments can you draw to connect them?

2. (a) Name the line segments and rays in the following figures:

(i) (ii) (iii)

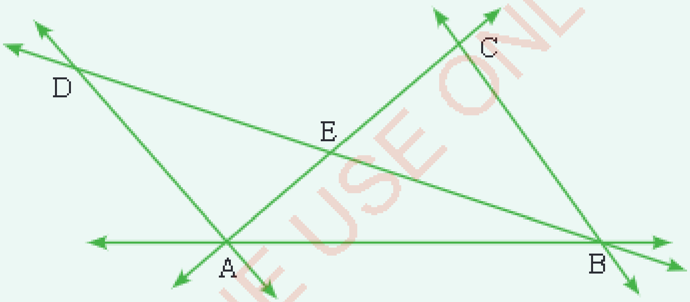


(b) Measure the length of each line segment in (a) above (give your answer to the nearest centimetres).

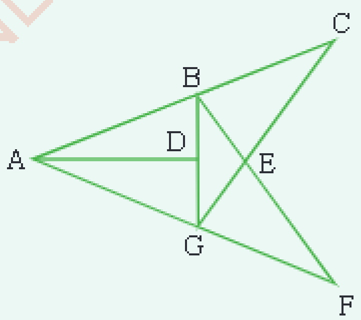
3. Identify and name all rays in each of the following figures:

(a) (b)

4. Identify and name all lines in the following figure.



5. Identify and name all line segments in the following figure.



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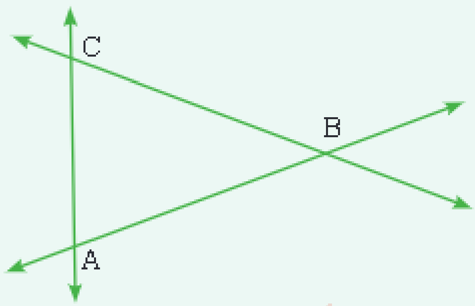
6. Use the figures in question 5 to identify the number of line segments with end points at the following points:
   (a) A (b) D (c) E

7. How many line segments can be drawn in the following manner?
   (a) Three points, but they are not on the same straight line.
   (b) Four points, but no three points lie on the same straight line.
   (c) Five points, but no three points lie on the same straight line.
   (d) Six points, but no three points lie on the same straight line.

8. Use the following figure to identify and name the given geometrical figures:
   (a) Three straight lines
   (b) Three line segments
   (c) Three rays



9. Match the definition of a term from column A with a statement from column B.

Column A	Column B
(a) A ray	(i) Has no end points
(b) A line	(ii) Has one end point
(c) A line segment	(iii) Has two end points
	(iv) Has many end points
	(v) Has infinite end points
	(vi) Has three end points

10. Draw a line segment with the following length:
   (a) 5 cm (b) 6.5 cm (c) 3.4 cm

11. Which of the following surfaces are flat?
   (a) A table top
   (b) A surface of a ball
   (c) A chalkboard
   (d) A surface of a bottle
   (e) A pane of glass
   (f) The surface of the earth

Activity 2

Individually or in groups, perform the following tasks:

1. Draw four rays marked $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, and $\overrightarrow{AE}$ on the same plane.
2. How many angles are formed in task 1 ?

When two rays have a common starting point, they form an angle. Angles are measured by an instrument called a protractor. The units of measure for angles are degrees, radians, and revolutions. Degrees and radians are commonly used, and the conversion between any of the two units can be performed. A point at which two lines meet or intersect to form an angle is called a vertex. An angle is named using any point on one ray, followed by the vertex, and then any point on the other ray. The symbol used to denote an angle is "$\angle$ or $\wedge$".

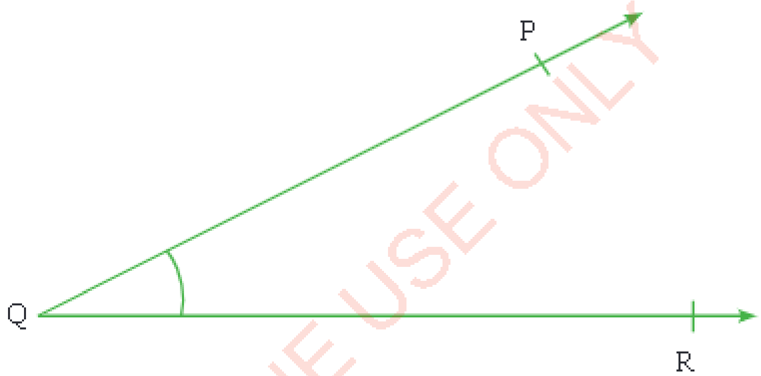


Figure 7.7: *Intersection of two rays*

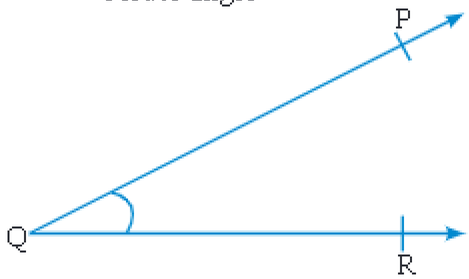
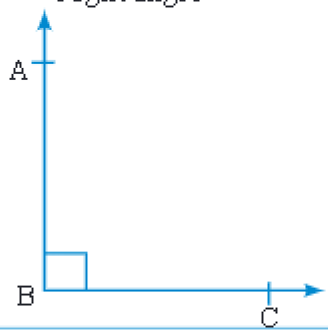
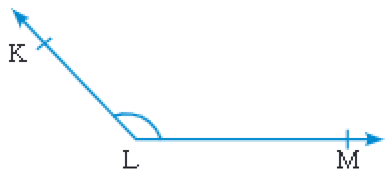
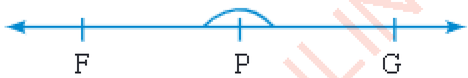
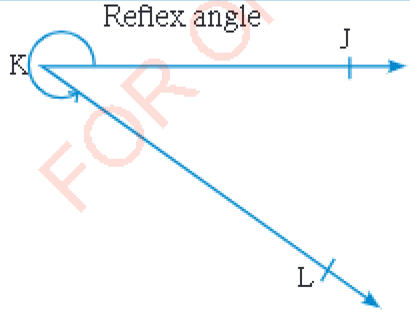
Figure 7.7 shows that, two rays QP and QR meet at point Q to form an angle. The formed angle can be named as follows:

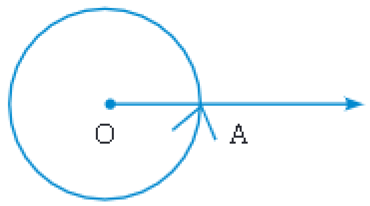
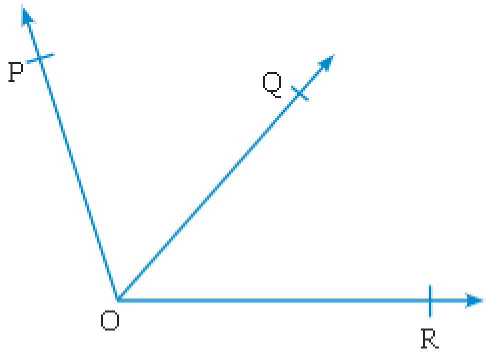
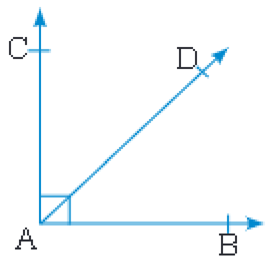
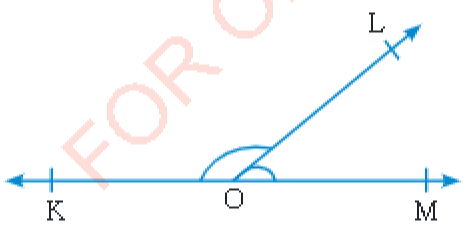
(a) $P\hat{Q}R$ or $\angle PQR$

(b) $R\hat{Q}P$ or $\angle RQP$

Types of angles

Angles are classified into different types basing on their degree of measurement.

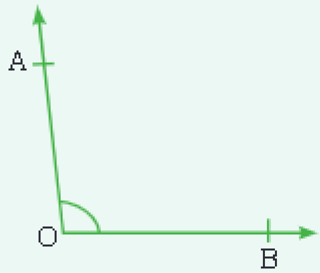
Type	Description
Acute angle  	An angle whose measure is greater than 0° but less than 90° is called an acute angle. In the given figure, $P\hat{Q}R$ is an acute angle.
Right angle  	An angle whose measure is exactly 90° is a right angle. The small square indicates that the measure is 90°. In the figure, $A\hat{B}C$ is a right angle. Any right angle is denoted by the symbol ∟.
Obtuse angle  	An angle whose measure is greater than 90° but less than 180° is called an obtuse angle. In the given figure, $K\hat{L}M$ is an obtuse angle.
Straight angle  	An angle whose measure is exactly 180° or two right angles or half turn is a straight angle. In the given figure, $F\hat{P}G = 180°$ is a straight angle.
Reflex angle  	An angle whose measure is greater than 180° but less than 360° is a reflex angle. In the given figure, the labeled angle JKL is a reflex angle.

Full angle 	If a ray OA rotates about O and after a complete rotation takes the final position OA, then it traces out an angle of 360°, which is called a full angle.
Adjacent angles 	The angles are called adjacent angles, if; (a) they have a common vertex, (b) they have a common line segment, and (c) their non-common line segments are on either side of the common line segment. In the given figure, $P\hat{O}Q$ and $Q\hat{O}R$ are adjacent angles since they have a common vertex O and a common line segment $\overline{OQ}$, and the other line segments $\overline{OP}$ and $\overline{OR}$ are on the opposite sides of $\overline{OQ}$.
Complementary angles 	Complementary angles are two angles whose measures add up to the right angle. This means, each angle is the complement of the other. In the given figure, $C\hat{A}B = 90°$. So, $C\hat{A}D + D\hat{A}B = 90°$. Therefore, $C\hat{A}D$ and $D\hat{A}B$ are complementary angles.
Supplementary angles 	Two angles are said to be supplementary if the sum of their measures is 180° and each of them is called the supplement of the other. In the given figure, $K\hat{O}M = 180°$ and $K\hat{O}L + M\hat{O}L = 180°$. Therefore, $K\hat{O}L$ and $M\hat{O}L$ are supplementary angles.

Exercise 2

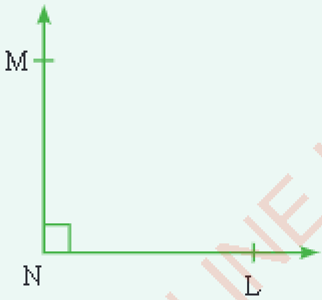
Determine the type of angles in each of the following figures:

1.

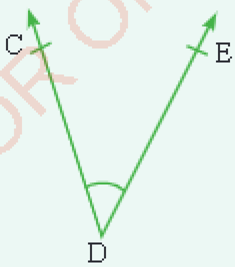


2.

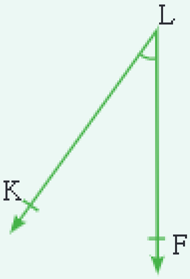
3.



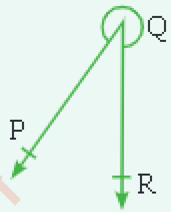
4.



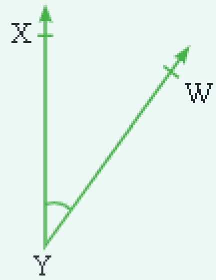
5.



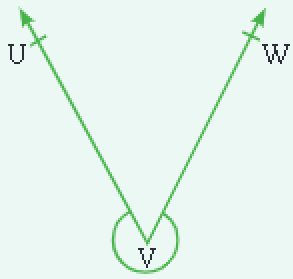
6.



7.



8.



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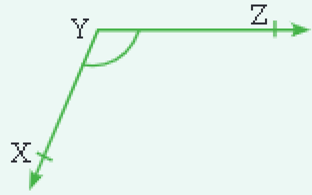
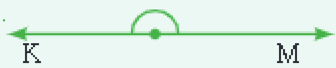
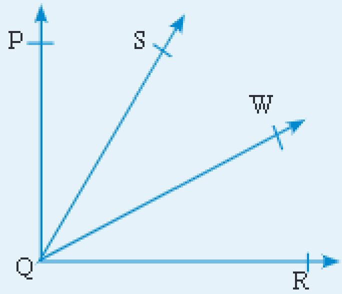


10.



Activity 3

Individually or in groups, perform the following tasks:



1. Identify and write all the angles formed in the figure.
2. Is it possible to get supplementary angles from the given figure? Give reasons.
3. If $P\hat{Q}R$ is a right angle, does the figure contain complementary angles? If yes, name all the complementary angles in it.

Measuring angles using a protractor

The size of an angle is measured using a protractor. Figure 7.8 shows a picture of the protractor. Most protractors are designed to measure angles in degrees. Radian-scale protractors measure angles in radians.

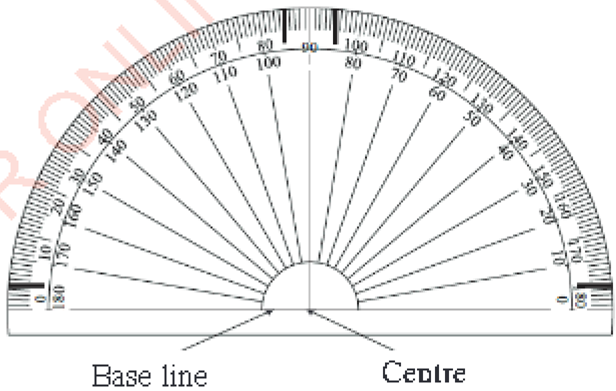


Figure 7.8: *A protractor*

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A protractor has a straight line at or near the straight edge and has a semicircular shape. This line is called the base line. The mid-point of the base line is called a central point marked centre. The protractor is divided into 180 equal sections which show measurements from 0° to 180°. Each section measures 1°.

The protractor has two scales namely, inner anticlockwise scale and outer clockwise scale. The marking on the inner scale is 0° to 180° and runs anticlockwise. The outer scale marking is 0° to 180° and runs clockwise. Figure 7.8 shows a protractor which measures angles in degrees.

The following are the steps taken when measuring angle $P\hat{O}Q$ using a protractor:

Step 1: Place a protractor over the angle $P\hat{O}Q$ so that its centre lies on the vertex O of the angle and the base line lies on $\overrightarrow{OQ}$.

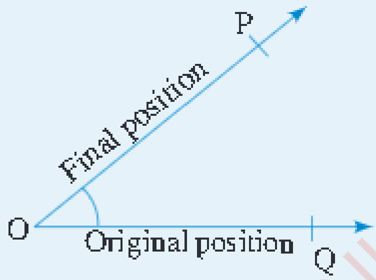


Figure 7.9: *Angle POQ*

Step 2: Read the angle on the mark through which $\overrightarrow{OP}$ passes, starting from 0° on the side of Q as shown in Figure 7.10.

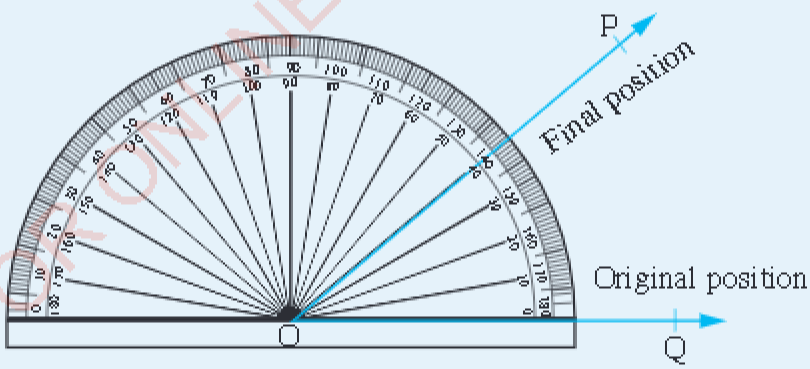


Figure 7.10: *Measuring angle POQ*

Figure 7.10 shows that $\overline{OP}$ passes through the 40° mark. Therefore, $P\hat{O}Q = 40°$.

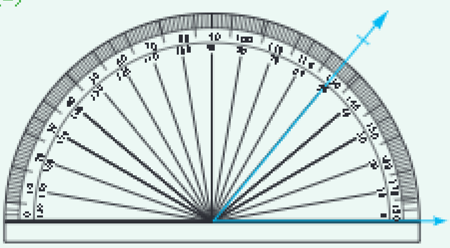
Exercise 3

Answer the following questions:

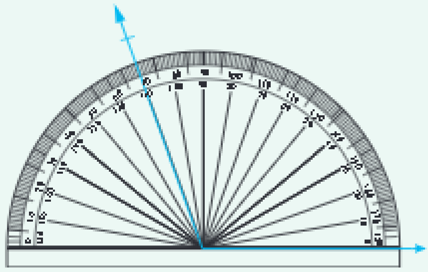
1. Draw the following angles:
   (a) Reflex angle
   (b) Obtuse angle
   (c) Acute angle
   (d) Right angle

2. Read the size of the angle shown in each of the following figures:

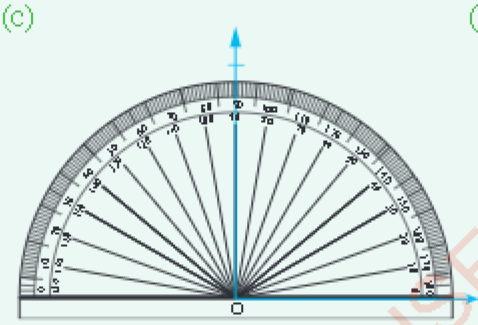
(a)



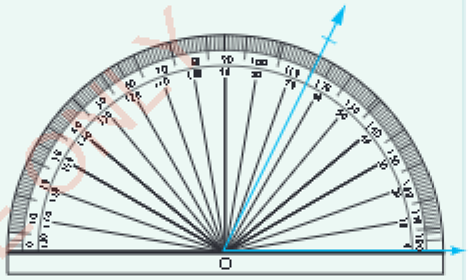
(b)



(c)

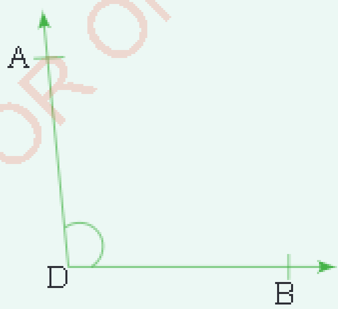


(d)

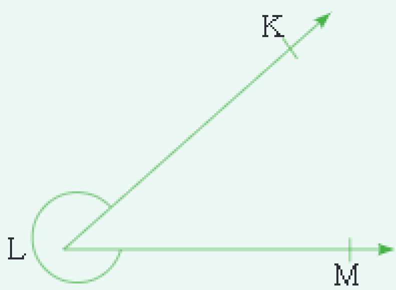


3. Use a protractor to measure each of the following angles:

(a)



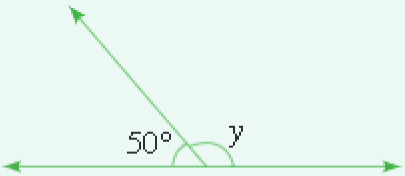
(b)



4. Classify each of the following angles on the basis of their degree of measurement:
   (a) 115° (c) 180° (e) 25°
   (b) 90° (d) 360°

In question 5 to 9, find the value of the angle marked y.

5.



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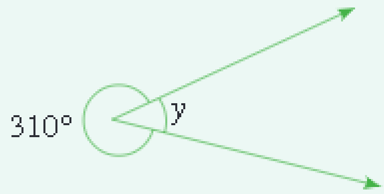

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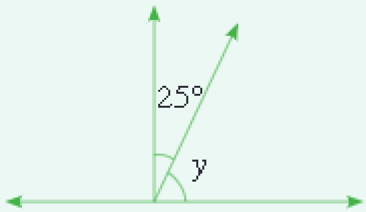
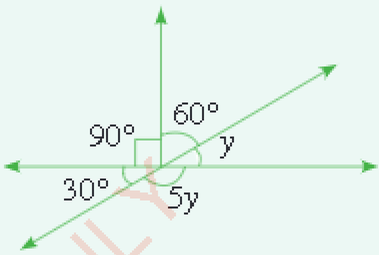


9.



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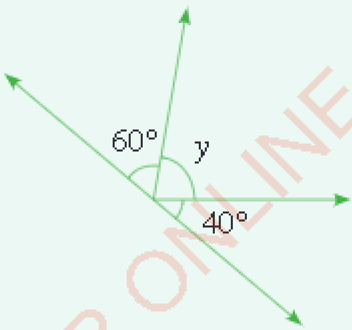
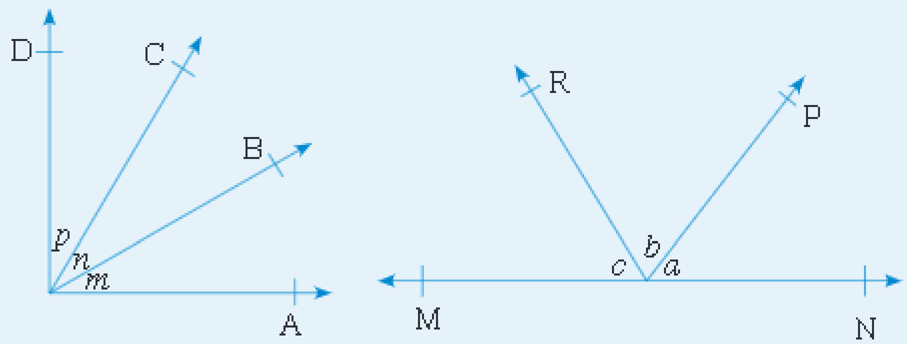


Activity 4

Individually or in groups, perform the following tasks:

1. Use a protractor to measure the angles marked with letters *m, n, p, a, b,* and *c*.

(a) (b)



2. Find the results of each of the following:
   (a) Obtuse angle – Acute angle
   (b) Obtuse angle + Acute angle

Activity 5

Individually or in groups, perform the following tasks:

Follow the steps outlined below to draw an angle that measures 50°.

1. Draw a ray and name it.
2. Keep the protractor with its central point at O and the horizontal edge along the ray.
3. Look at the scale with 0° mark of the protractor lying on the ray.
4. On this scale, mark a point against the 50° angle and remove the protractor.
5. Join the point O and the point in step 4.
6. Join the angle formed.

Share your findings with other groups.

Exercise 4

Draw each of the following angles using a protractor:

1. 45°   2. 82°   3. 155°   4. 49°   5. 150°
6. 75°   7. 120°   8. 25°   9. 67°   10. 138°

Perpendicular lines

Activity 6: Recognising perpendicular lines

Individually or in groups, perform the following tasks:

1. Draw two lines, AB and CD.
2. Insert a point M between $\overleftrightarrow{AB}$ and point N between $\overleftrightarrow{CD}$. Make sure that, the two points M and N divide $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ into two equal parts, respectively.
3. Let $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ cross each other and intersect at point M and N to form a point P.
4. Measure the angles and name the types of the four angles formed.

When two lines cross each other or meet at a right angle (90°), the lines are said to be perpendicular. The symbol '⊥' is used to denote two perpendicular lines. For example, in Figure 7.11, $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, meaning that $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$.

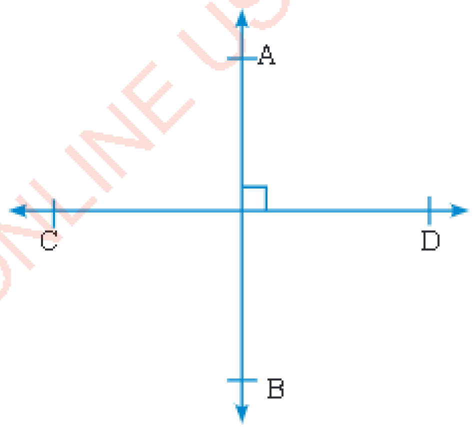
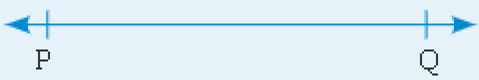


Figure 7.11: *Two perpendicular lines*

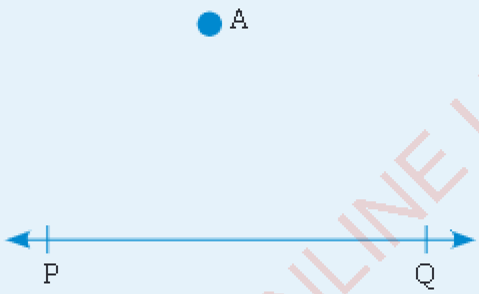
Construction of perpendicular lines

Construction of perpendicular lines can be done in two ways, that is, construction through a point outside a line and construction through a given point on a line. A perpendicular line through an external point can be constructed using a pair of compasses and a ruler. A line segment is created on the given line and then bisected. Let $\overleftrightarrow{PQ}$ be the given line and A be an external point, that is, A is the point outside the line $\overleftrightarrow{PQ}$. Construction of a perpendicular line from A to $\overleftrightarrow{PQ}$ is done using the following steps:

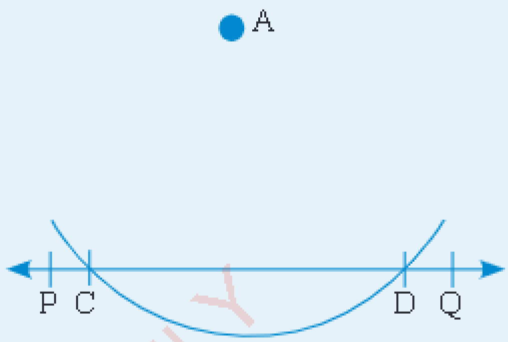
Step 1:
Construct $\overleftrightarrow{PQ}$ as shown in the following figure.



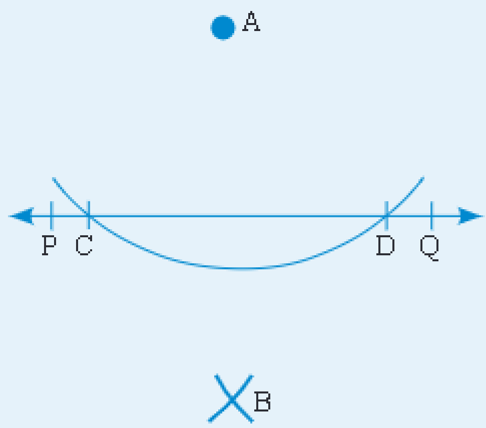
Step 2:
Put a point A outside $\overleftrightarrow{PQ}$ as shown in the following figure.



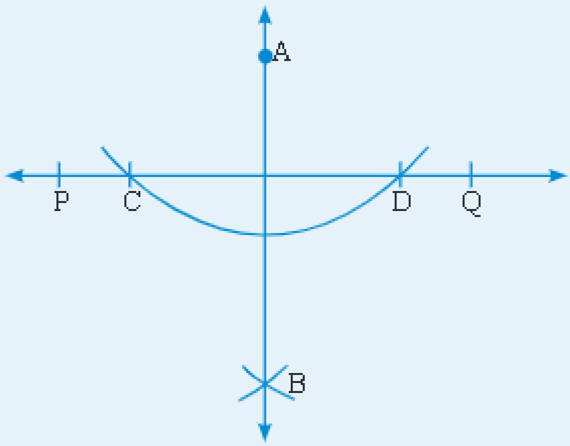
Step 3:
With point A as a centre and any convenient radius, construct an arc crossing $\overleftrightarrow{PQ}$ at C and D as shown in the following figure.



Step 4:
With points C and D as centres, construct arcs of equal radii at the next side of $\overleftrightarrow{PQ}$ to cross each other at B as shown in the following figure.



Step 5: Join the points A and B by a straight line to form perpendicular lines AB and PQ as shown in the following figure.

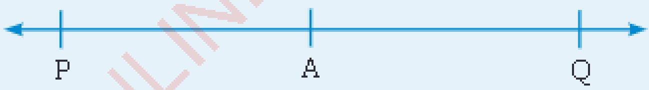


Therefore, $\overleftrightarrow{AB}$ is the required line perpendicular to $\overleftrightarrow{PQ}$.

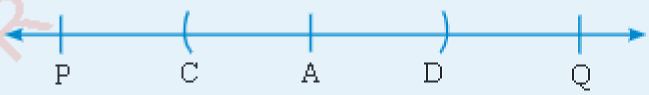
Alternatively, a perpendicular line through a given point on a line can be constructed using a pair of compasses and a ruler. Let $\overleftrightarrow{PQ}$ be a line and A the point on it. Construction of a perpendicular line to $\overleftrightarrow{PQ}$ through A is done using the following steps:

Step 1: Construct $\overleftrightarrow{PQ}$ as shown in the following figure.

Step 2: Put a point A at the mid-point of $\overleftrightarrow{PQ}$ as shown in the following figure.



Step 3: With point A as a centre and any convenient radius, construct small arcs cutting $\overleftrightarrow{PQ}$ at C and D as shown in the following figure.



Step 4: With the points C and D as centres, construct two arcs of equal radius to cut above $\overleftrightarrow{PQ}$ and below $\overleftrightarrow{PQ}$ at points E and B as shown in the following figure.

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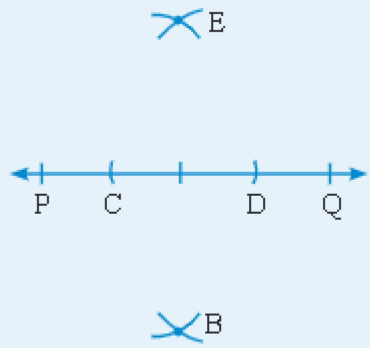
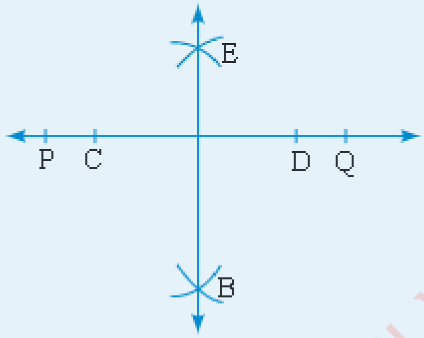


Step 5: Join the points E and B by a straight line to form perpendicular lines EB and PQ as shown in the following figure.



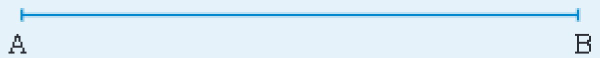
Therefore, $\overleftrightarrow{EB}$ is the required line perpendicular to $\overleftrightarrow{PQ}$.

Construction of a perpendicular bisector

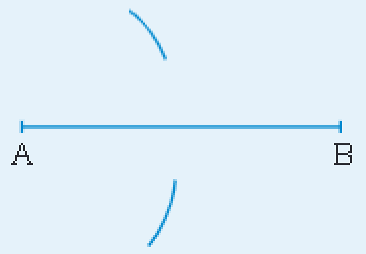
A line which divides a line segment into two equal parts is called a bisector. If this line is perpendicular to the line segment, then the line is called a perpendicular bisector.

The following are the steps for constructing a perpendicular bisector of a line segment:

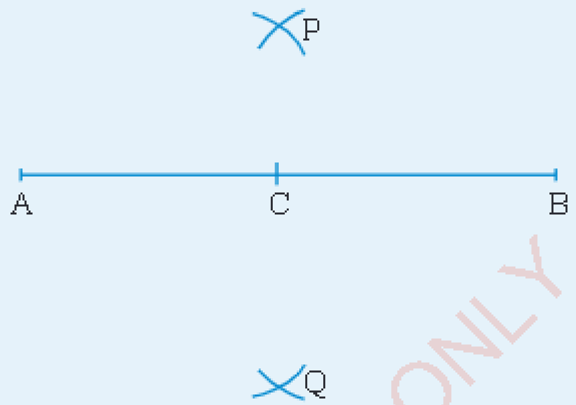
Step 1: Construct a line segment AB as shown in the following figure.



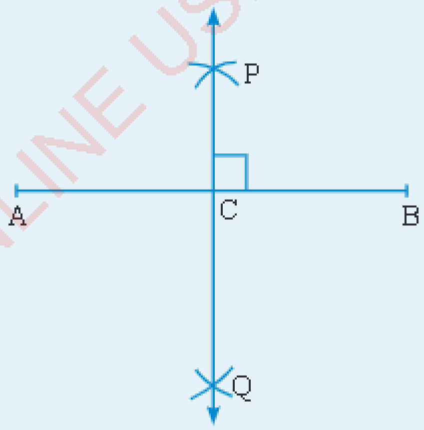
Step 2: With point A as a centre and radius more than half of $\overline{AB}$, construct arcs on both sides of $\overline{AB}$ as shown in the following figure.



Step 3: With point B as a centre and the same radius (as in step 2), construct arcs on both sides of $\overline{AB}$ to cut the previous arcs at P and Q as shown in the following figure.



Step 4: Draw a line PQ, with C as the centre, where $\overline{AB}$ and $\overleftrightarrow{PQ}$ intersect.

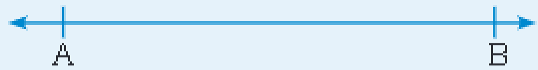


Therefore, $\overleftrightarrow{PQ}$ is the required perpendicular bisector of $\overline{AB}$. It can also be deduced that $\overline{AC} = \overline{CB}$ and $A\hat{C}P = B\hat{C}P = 90°$.

Construction of an angle 60°

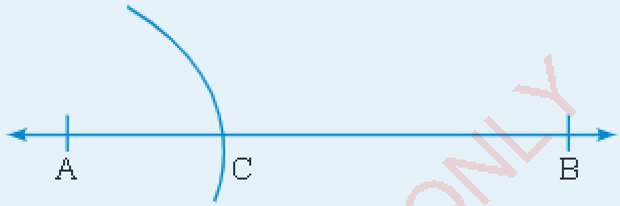
Construction of an angle that measures 60° can be done using a pair of compasses and a ruler. The following are the steps for constructing an angle measuring 60°.

Step 1: Draw $\overleftrightarrow{AB}$ as shown:

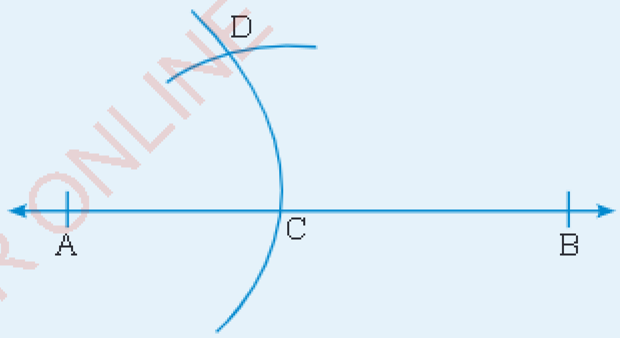


Step 2: With point A as a centre, use any convenient radius to construct an arc crossing $\overleftrightarrow{AB}$ at point C as shown in the following figure.

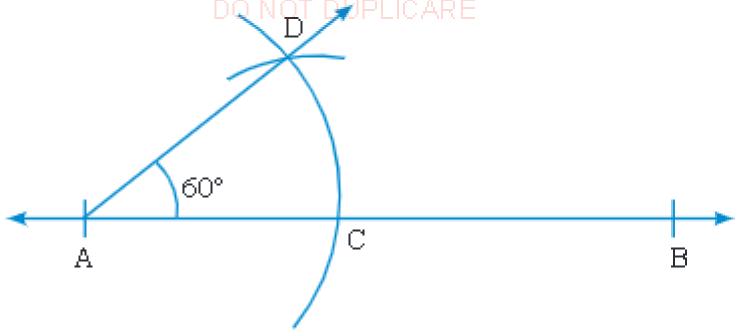
Step 3: Extend the arc above $\overleftrightarrow{AB}$ as shown in the following figure.



Step 4: Using the same radius as in step 2 at the centre C, construct an arc crossing the first arc at point D as shown in the following figure.



Join the points A and D by a straight line, and use a protractor to measure $D\hat{A}C$ as shown in the following figure.



Thus, $D\hat{A}C = 60°$.

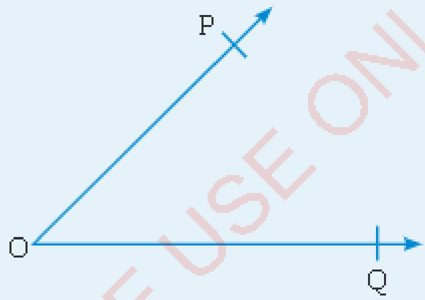
Therefore, $D\hat{A}C$ is the required angle measuring 60°.

Construction of a bisector of an angle

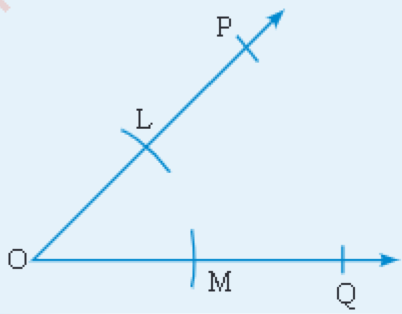
The angle bisector is a line segment or ray which bisects the angle. A pair of compasses and a ruler can be used to construct a bisector of an angle.

The following are the steps for construction of the angle bisector:

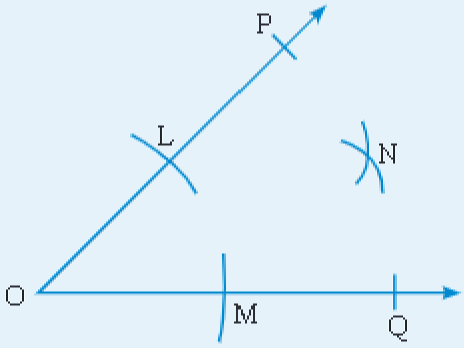
Step 1: Draw two rays with a common starting point, that is, $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ as shown in the following figure.



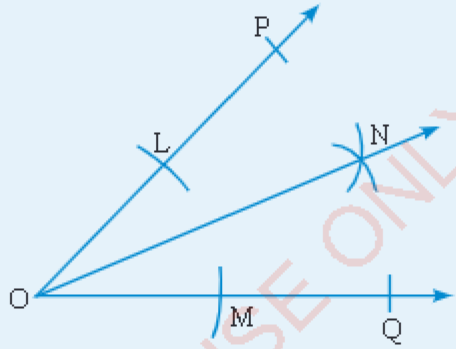
Step 2: With point O as a centre, use any convenient radius to construct arcs cutting $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ at points L and M, respectively, as shown in the following figure.



Step 3: With the same radius as in step 2, use points L and M as centres to construct arcs cutting at point N as shown in the following figure.



Step 4: Join the points O and N by a straight ray to form $\overrightarrow{ON}$ which bisects $P\hat{O}Q$.



Therefore, $\overrightarrow{ON}$ is the required bisector of $P\hat{O}Q$, that is, $P\hat{O}N = Q\hat{O}N$.

Construction of an angle 45°

Construction of an angle measuring 45° can be done using a pair of compasses and a ruler. The construction of this angle starts with a right angle and then bisecting it as shown in the following figure.

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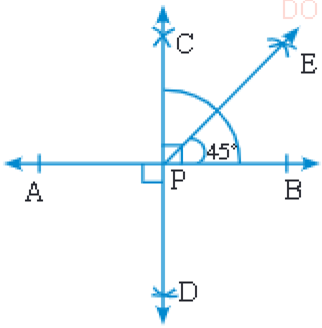
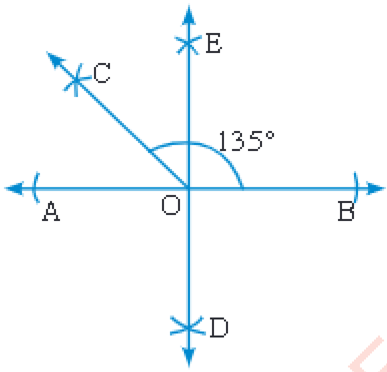


Therefore, $B\hat{P}E = 45°$.

Construction of an angle 135°

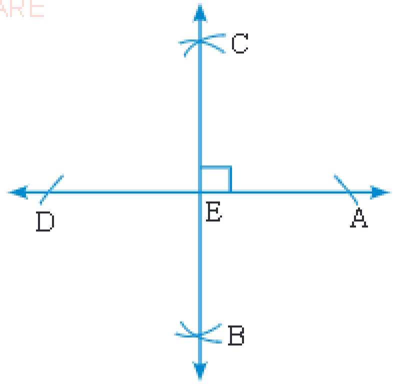
The construction of an angle 135° is done by constructing two right angles and bisecting one of them as shown in the following figure.



Therefore, $B\hat{O}C = 135°$.

Construction of an angle 90°

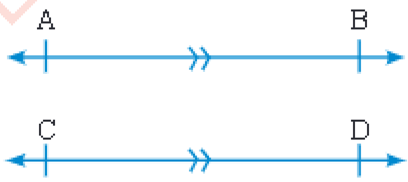
The construction of an angle 90° (right angle) is done by drawing a perpendicular line to a given line as shown in the following figure.



Therefore,
$A\hat{E}C = D\hat{E}C = A\hat{E}B = B\hat{E}D = 90°$.

Construction of parallel lines

When two or more lines on a plane do not meet, they are said to be parallel. Parallel lines are always at the same distance apart, called equidistant. The parallel lines are marked by arrows as shown in the following figure.



The symbol // represents 'parallel'. That is, $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$ is read as $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$.

Parallel lines can be constructed in different ways:

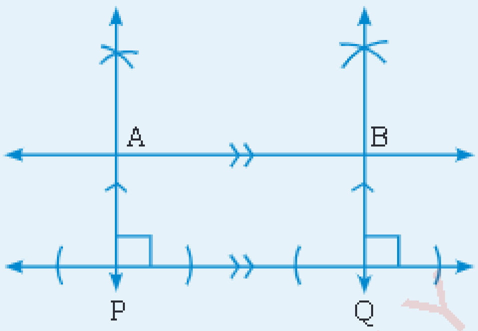
(a) Using a ruler and a pair of compasses only

Let $\overleftrightarrow{PQ}$ be a given line. Construct $\overleftrightarrow{AB}$ parallel to $\overleftrightarrow{PQ}$ using the following steps:

Step 1: Draw perpendicular lines through points P and Q, respectively.

Step 2: With a convenient length of a pair of compasses, use equal lengths to mark the points A and B.

Step 3: Draw a line through $\overleftrightarrow{AB}$.



Therefore, the construction shows that $\overleftrightarrow{PA} // \overleftrightarrow{QB}$ and $\overleftrightarrow{AB} // \overleftrightarrow{PQ}$.

(b) Using equal angles

Let $\overleftrightarrow{PQ}$ be a given line. Construct $\overleftrightarrow{AB}$ parallel to $\overleftrightarrow{PQ}$ using the following steps:

Step 1: Draw $\overleftrightarrow{AP}$ crossing $\overleftrightarrow{PQ}$ at point P.

Step 2: Copy angle APQ as follows:

(i) Using a pair of compasses with any convenient radius, construct an arc from P cutting $\overleftrightarrow{AP}$ at point R and $\overleftrightarrow{PQ}$ at point S.

(ii) With the same radius and point A as the centre, construct an arc cutting $\overleftrightarrow{PA}$ at point T.

(iii) With point R as the centre, adjust the compasses to fit $\overleftrightarrow{RS}$.

(iv) With the same radius ($\overleftrightarrow{RS}$) and point T as the centre, construct an arc to cross the first arc at point U.

(v) Draw a line through $\overleftrightarrow{AU}$. The resulting figure shows that, $\angle TAU = \angle RPS$.

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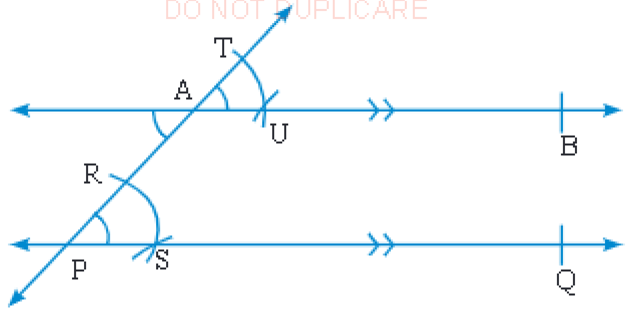
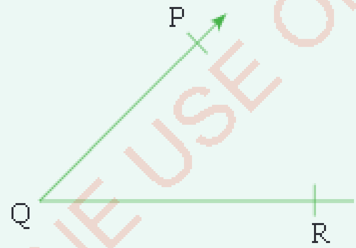


Therefore, $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{PQ}$.

Exercise 5

1. Mark four points which are not on the same straight line on a plane. How many pairs of lines crossing each other and passing through the points can be drawn?
2. A line segment AB has length 10 cm. Use the method of perpendicular bisector to bisect the segment into four equal parts.
3. Construct two parallel lines which are 3 cm apart.
4. Copy angle PQR from the following figure by using a ruler and a pair of compasses only.



5. Draw a line EF, 5.8 cm long. Choose a point D outside $\overleftrightarrow{EF}$ and construct a perpendicular line to $\overleftrightarrow{EF}$ through D.

Activity 7

Individually or in groups, perform the following tasks:

1. Use a ruler and a pair of compasses to construct an angle measuring 150°.
2. Bisect the angle 150° in task 1.
3. Measure the bisected angles formed in task 2.

Transversals

A transversal is any line which crosses two or more lines in the same plane at distinct points. For example; $\overleftrightarrow{AB}$ in Figure 7.12 is a transversal. If two parallel or non-parallel lines PQ and RS are crossed by a third line AB at distinct points as shown in Figure 7.12, then $\overleftrightarrow{AB}$ is called a transversal.

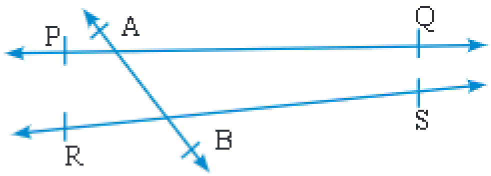


Figure 7.12: *Transversal*

Angles formed by transversals

When a transversal crosses two parallel lines, the angles formed can be indicated as in Figure 7.13. $\overleftrightarrow{AB}$ is a transversal and $\overleftrightarrow{PQ}$ is parallel to $\overleftrightarrow{RS}$.

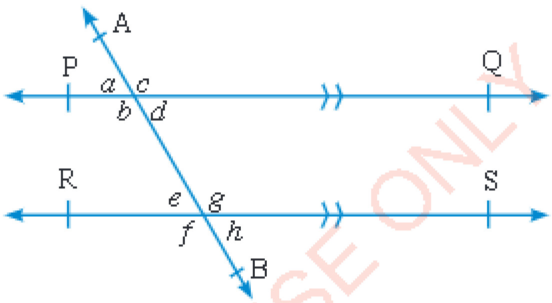


Figure 7.13: *Angles formed by a transversal*

Vertically opposite angles

Vertically opposite angles are equal. In Figure 7.13, the pairs *a* and *d*, *c* and *b*, *e* and *h*, and *f* and *g* are vertically opposite angles. Thus, the angles in each pair are equal.

Corresponding angles

Corresponding angles are equal. In Figure 7.13, the pairs *a* and *e*, *c* and *g*, *b* and *f*, *d* and *h* are corresponding angles. Thus, these pairs of angles are equal.

Alternate angles

In this case, we consider two types of alternate angles: the alternate interior angles and the alternate exterior angles.

The alternate interior angles are equal. In Figure 7.13, the pairs b and g, and d and e are alternate interior angles. Thus, these pairs of angles are equal.

The alternate exterior angles are equal. In Figure 7.13, the pairs a and h, and c and f are alternate exterior angles. Thus, these pairs of angles are equal.

The sum of the angles on the same side of a transversal and between the parallel lines is 180°. These angles are referred to as interior opposite angles.

Thus, $b + e = 180°$ and $d + g = 180°$.

Complementary angles

Two angles are complementary when their sum is 90°. In Figure 7.14, a and b are complementary angles. Similarly, e and f are complementary angles.

Thus, $a + b = 90°$ and $e + f = 90°$.

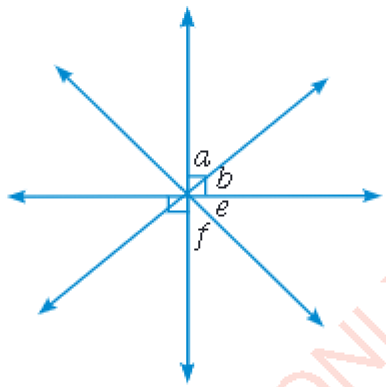


Figure 7.14: *Complementary angles*

Supplementary angles

Two angles are supplementary if their sum is 180°. In Figure 7.15, c and d are supplementary angles because $c + d = 180°$.

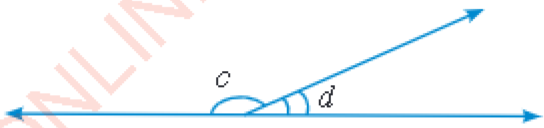


Figure 7.15: *Supplementary angles*

> **Note:**
> One way of avoiding to mix up the definitions of complementary and supplementary angles is to remember that, 's comes after c in the alphabet, while 180° is greater than 90°'.

Straight angle

When two rays in opposite directions are joined at their starting points, they form a straight angle. That is, an angle which measures exactly 180° is formed as shown in Figure 7.16.

Figure 7.16: *Straight angle*

Note that: If a transversal crosses two straight lines such that, one of the following conditions is satisfied, then the two lines are parallel.

(a) Two alternate angles are equal.

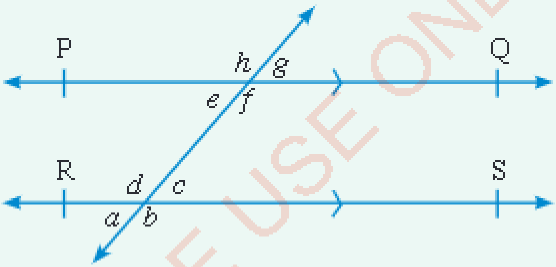
(b) Two corresponding angles are equal.

(c) Two vertically opposite angles are equal.

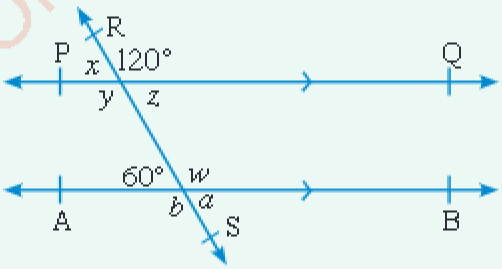
(d) The sum of a pair of interior angles on the same side of the transversal is 180°.

Exercise 6

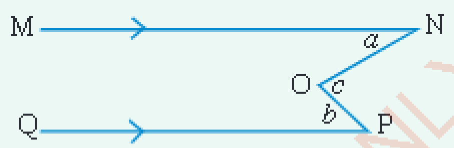
1. Use the following figure to name the angles formed by the transversal.



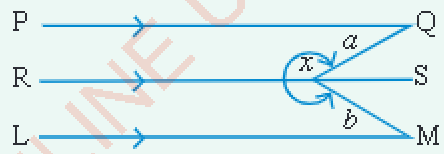
2. In the following figure, if $\overleftrightarrow{PQ}$ is parallel to $\overleftrightarrow{AB}$ and $\overleftrightarrow{RS}$ is a transversal, find the value of the angles a, b, x, y, z, and w.



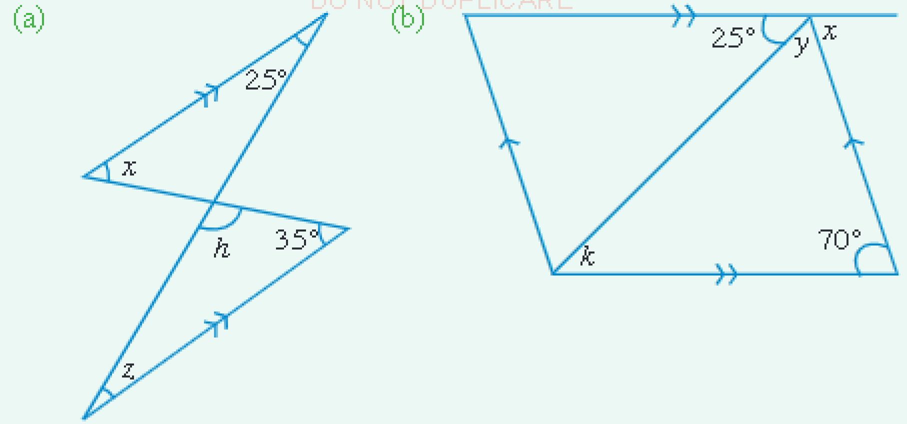
3. Construct an angle that measures 120° using a ruler and a pair of compasses only.
4. Construct an angle that measures 135° and bisect it. Measure the two formed angles.
5. Draw a line segment AB of length 8 centimetres, and then construct its perpendicular bisector using a ruler and a pair of compasses only.
6. Use a pair of compasses and a ruler only to construct a reflex angle that measures 225°.
7. Construct two line segments PQ and PR of length 5 centimetres each, making an angle that measures 45°. Join Q and R.
   (a) Measure the length of the line segment QR.
   (b) Measure the angle PQR.
8. Draw a line segment LM of length 6 cm. If a point N is 4 centimetres from point L on $\overline{LM}$, construct a perpendicular line PQ at N.
9. If $a = 30°$ and $b = 40°$, find the value of c in the following figure.



10. If $\overleftrightarrow{PQ}$, $\overleftrightarrow{RS}$, and $\overleftrightarrow{LM}$ are parallel lines, $a = 40°$ and $b = 32°$, find the value of x in the following figure.



11. Find the size of the angles marked by letters in each of the following figures:



Polygons and polygonal regions

In our surroundings, many shapes and figures have sides whose line segments lie on the same plane and not more than two sides can meet at one vertex. In this section, the types of shapes and figures will be studied.

Regions

Activity 8

Individually or in groups, perform the following tasks:

1. Draw two parallel lines named $\overleftrightarrow{PQ}$ and $\overleftrightarrow{MN}$.
2. Draw a transversal through P and N.
3. Draw a transversal through Q and M.
4. Put a point R on the intersection of $\overleftrightarrow{PN}$ and $\overleftrightarrow{QM}$.
5. Name the figure formed by the points P, R, and Q.

A closed path together with its surface is called a region. A triangular region is the surface of the triangle and its boundary line segments. Figure 7.17 describes a triangular region.

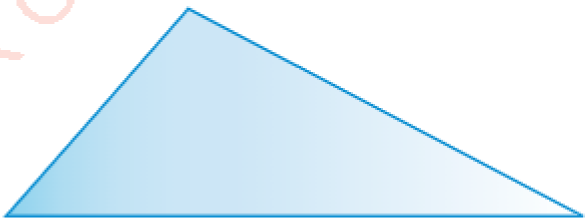


Figure 7.17: *Triangular region*

A rectangular region is the surface of a rectangle together with its boundary. Figure 7.18 shows a rectangular region.

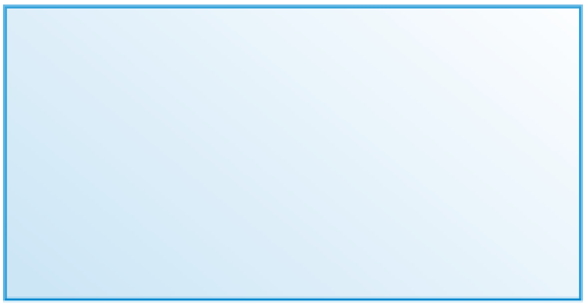
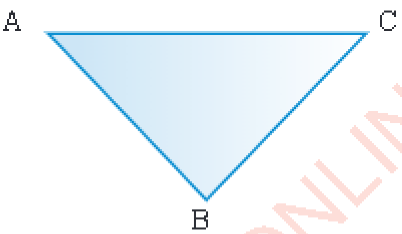


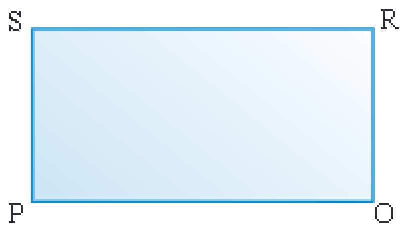
Figure 7.18: *Rectangular region*

Polygons

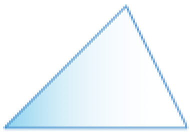
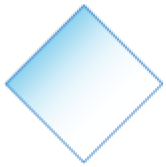
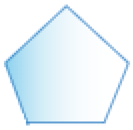
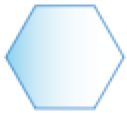
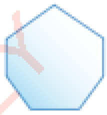
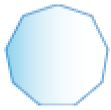
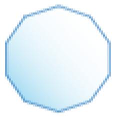
A polygon is a closed path figure bound by a finite number of line segments placed end to end successively. In a polygon, no line segments cross each other, that is, they meet at their end points. Also, a polygon has no line segments with a common end point lying on the same straight line. The point of intersection of two consecutive sides of a polygon is called a vertex (in plural, vertices). The number of vertices of a polygon is equal to the number of its sides. A polygon together with its interior is called a polygonal region. The line segments which form a polygon are called sides of the polygon. Polygons are named according to the number of their sides.

The following figures are examples of polygons:



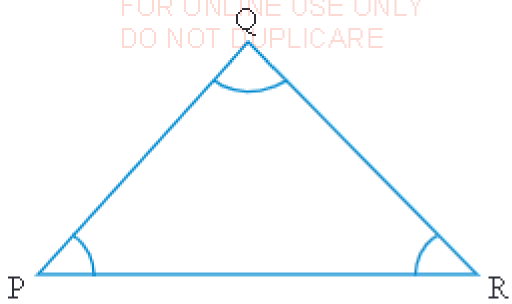


The following table shows the names of different polygons.

Name of the polygon	Number of sides	Shape of the polygon
Triangle	3	
Quadrilateral	4	
Pentagon	5	
Hexagon	6	
Heptagon	7	
Octagon	8	
Nonagon	9	
Decagon	10	

Triangles

A triangle is a polygon with three sides, three vertices, and three angles. The sum of the three interior angles of a triangle is always 180°. The sum of the lengths of two sides of a triangle is always greater than the length of the third side. The triangle PQR is written as ΔPQR, where Δ denotes 'triangle'. The common end points P, Q, and R of the triangle are called vertices (singular vertex).



Types of triangles

There are six types of triangles based on the lengths of their sides and the size of their angles. Three triangles can be classified based on their sides. These triangles are isosceles triangle, equilateral triangle, and scalene triangle. The other three triangles are classified based on their angles. These triangles are right-angled triangle, acute angled triangle, and obtuse angled triangle.

(a) Isosceles triangles

An isosceles triangle is a triangle that has two sides of equal length. Also, the angles opposite the equal sides are equal. For example; ΔABC in Figure 7.19 is an isosceles triangle, where

$\overline{AB} = \overline{AC} = 3.8$ cm, $\overline{BC} = 4.2$ cm, and $A\hat{B}C = A\hat{C}B$.

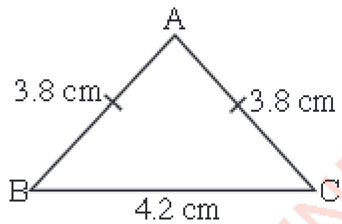


Figure 7.19: *Isosceles triangle*

Figure 7.20 is also an isosceles triangle because the base angles are equal. That is, $P\hat{Q}R = P\hat{R}Q$ and $\overline{PQ} = \overline{PR}$.

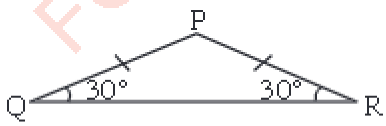


Figure 7.20 : *Isosceles triangle*

(b) Equilateral triangles

An equilateral triangle is a triangle in which all sides are equal and all angles are equal. The triangles PQR and ABC in Figure 7.21 are equilateral triangles.

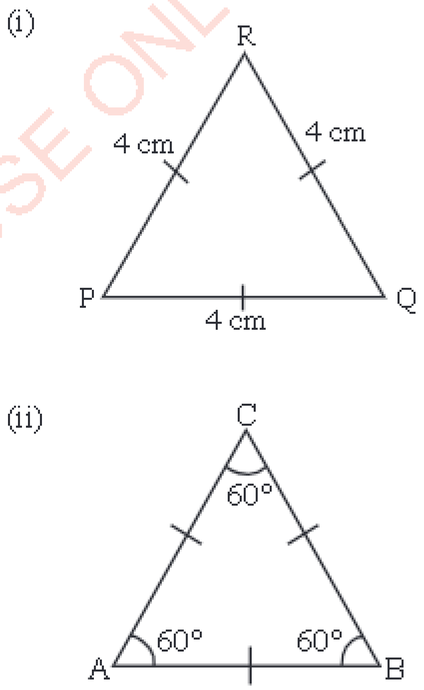


Figure 7.21: *Equilateral triangles*

Figure 7.21 (i) shows that, $\overline{PQ} = \overline{QR} = \overline{RP} =$ 4 cm and Figure 7.21 (ii) $A\hat{B}C = B\hat{C}A = C\hat{A}B = 60°$.

(c) Right-angled triangles

A right-angled triangle is a triangle in which one of its angles measures exactly 90°. The triangle QRS in Figure 7.22 is a right-angled triangle.

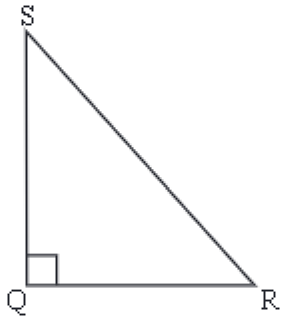


Figure 7.22: *Right-angled triangle*

In Figure 7.22, $S\hat{Q}R$ measures exactly 90°.

(d) Acute angled triangles

An acute angled triangle is a triangle in which all the three internal angles are acute, that is, they measure less than 90°.

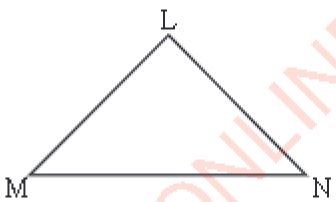


Figure 7.23: *Acute angled triangle*

Triangle LMN in Figure 7.23 has all its angles measuring less than 90°. Therefore, ΔLMN is an acute angled triangle.

(e) Obtuse angled triangles

An obtuse angled triangle is a triangle in which one of the interior angles measures more than 90°. This means that, if one angle is obtuse, then the other two angles will always be acute. The angle ABC in Figure 7.24 is an obtuse angle. Therefore, triangle ABC is an obtuse angled triangle.

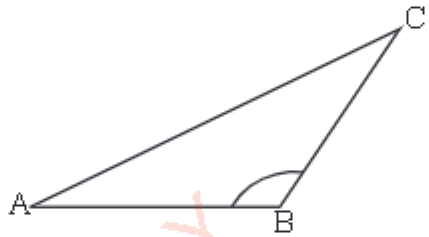


Figure 7.24: *Obtuse angled triangle*

(f) Scalene triangles

A scalene triangle is a triangle in which all of its three sides have different lengths and all its angles have different measures.

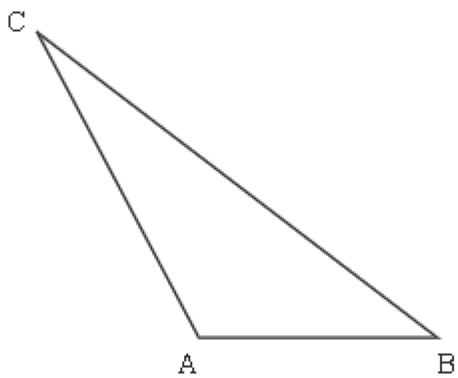


Figure 7.25: *Scalene triangle*

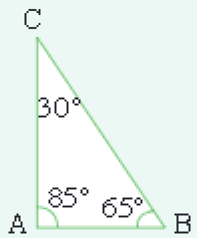
In Figure 7.25, $\overline{AB} < \overline{AC} < \overline{BC}$. Also $A\hat{C}B < A\hat{B}C < C\hat{A}B$. Hence, all the three sides and all the angles of the triangle have different measures. Therefore, the triangle ABC is a scalene triangle.

Exercise 7

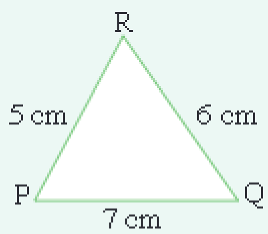
Answer the following questions:

1. Identify the types of the following triangles:

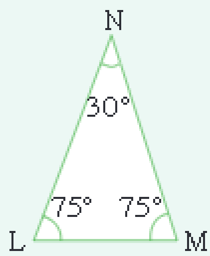
(a)



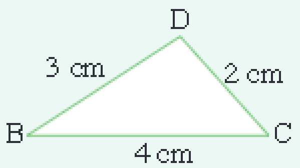
(b)



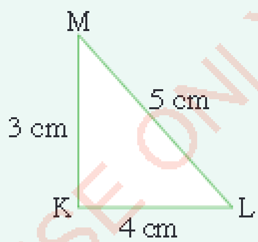
(c)



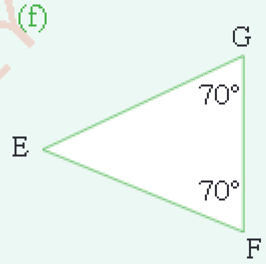
(d)



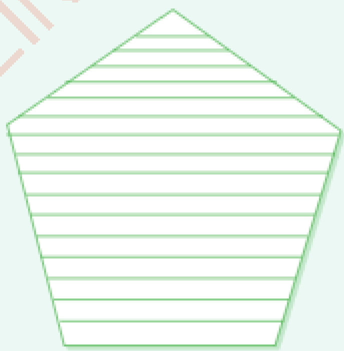
(e)



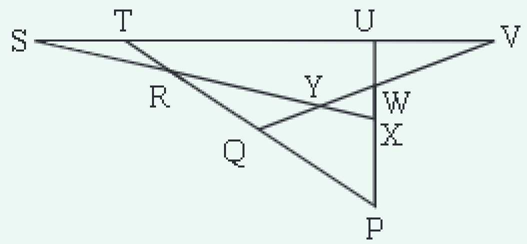
(f)



2. What type of a region is shown in the following figure?



3. How many vertices are there in the following figure? Name all obtuse angled triangles found in the figure.



4. Construct a scalene triangle in which one of its angles measures 78°.

Construction of triangles

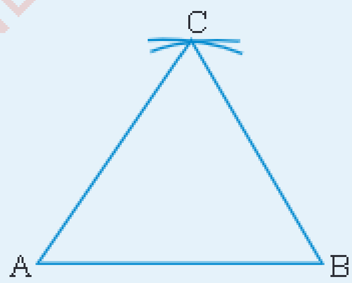
A triangle can be constructed when the sum of the lengths of any two sides is greater than the length of the third side, or the difference between the lengths of any two sides is less than the length of the third side. A triangle can be constructed using a pair of compasses, a ruler, a divider, and sometimes a protractor and a set square.

Example 1

Construct an equilateral triangle ABC whose sides have length 3.5 cm.

Construction

Step 1: Draw a line segment AB of length 3.5 cm, using a ruler as shown in the following figure.



Step 2: With point A as a centre and a pair of compasses adjusted to 3.5 cm, construct an arc on one side of $\overline{AB}$.

Step 3: Using the same length and point B as a centre, construct an arc cutting the first arc at point C.

Step 4: Draw $\overline{AC}$ and $\overline{BC}$ to form an equilateral triangle ABC.

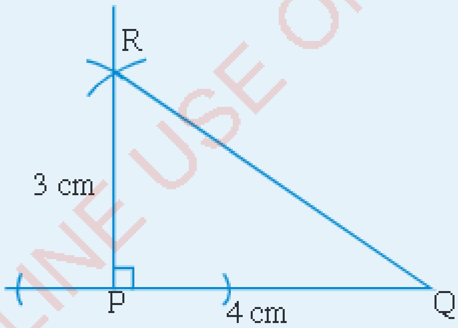
Example 2

Construct a right-angled triangle PQR, where $\overline{PQ} = 4$ cm and $\overline{PR} = 3$ cm.

Construction

Step 1: Draw $\overline{PQ}$ of length 4 cm using a ruler.

Step 2: With point P as a centre, use a pair of compasses to construct a perpendicular line segment at point P as shown in the following figure.



Step 3: Use a pair of compasses adjusted to fit a length of 3 cm to mark point R such that $\overline{PR} = 3$ cm.

Step 4: Draw $\overline{PR}$ and $\overline{RQ}$ to form a right angled triangle PQR.

Example 3

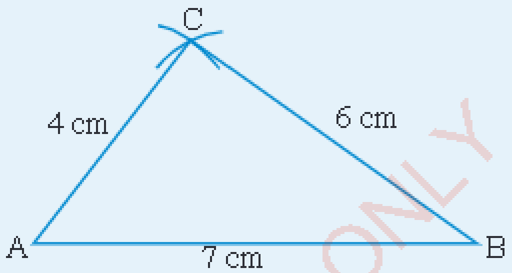
Construct triangle ABC with sides $\overline{AB} = 7$ cm, $\overline{AC} = 4$ cm, and $\overline{BC} = 6$ cm.

Construction

Step 1: Draw $\overline{AB}$ of length 7 cm using a ruler.

Step 2: With point A as centre, use a pair of compasses adjusted to fit a length of 4 cm to construct an arc on one side $\overline{AB}$ at point C.

Step 3: With point B as a centre, use a pair of compasses adjusted to fit a length of 6 cm to construct the second arc at point C as shown in the following figure:

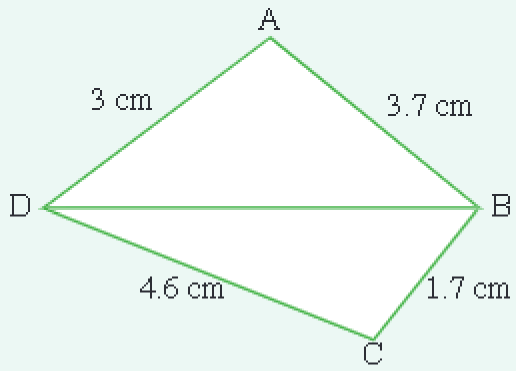


Step 4: Draw $\overline{AC}$ and $\overline{BC}$ to form triangle ABC.

Exercise 8

1. Construct an isosceles right–angled triangle with two sides each measuring 4 cm. Measure its base angles and the length of the other side.
2. Construct triangle ABC, with $\overline{AB} = 3.3$ cm, $\overline{BC} = 4.2$ cm, and $\overline{AC} = 3.3$ cm. Measure the angle ABC.
3. Construct triangle PQR, where $\overline{PQ} = 5$ cm, $\overline{QR} = 5$ cm, and $\overline{PR} = 5$ cm. Measure all its angles.

4. Construct triangle PQR, where $\overline{PQ}$ = 6 m, ∠RPQ= 67°, and ∠PQR = 38°. Construct a perpendicular line from Q to cross $\overline{PR}$ at N. Measure:
   (a) $\overline{PR}$ (b) $\overline{QN}$ (c) ∠PNQ
5. Construct ΔABC, where $\overline{AB}$ = 5 cm, $\overline{BC}$ = 7 cm, and $\overline{CA}$ = 6 cm. What is the type of the triangle formed?
6. Consider the following figure, and then answer the questions that follow.



   (a) Construct the figure using the given dimensions.
   (b) Draw and measure $\overline{AC}$.
   (c) Measure $D\hat{B}C$.

7. (a) Construct ΔXYZ, where $\overline{XY}$ = 4.5 cm, $\overline{XZ}$ = 6 cm, and $Z\hat{X}Y$ = 50°.
   (b) Measure the size of $X\hat{Y}Z$ and length of $\overline{YZ}$.

8. (a) Draw triangle MNO, where $\overline{MN}$ = 11cm, $\overline{NO}$ = 5cm, and $\overline{OM}$ = 12 cm.
   (b) On $\overline{MN}$, mark point P such that $\overline{MP}$ = 4 cm.
   (c) Draw $\overline{OP}$ and measure its length.

Quadrilaterals

A quadrilateral is a polygon with four sides; consequently, it has four edges and four vertices. A quadrilateral is sometimes called a quadrangle, tetragon or a 4-gon. A diagonal of a quadrilateral is a line segment which connects its vertices. The word 'quadrilateral' originated from the Latin words 'quadri', which means four, and 'latus' meaning 'side'.

For example, Figure 7.26 is a quadrilateral, where $\overline{HF}$ and $\overline{EG}$ are the diagonals.

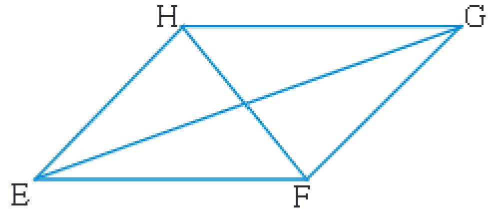


Figure 7.26: *Quadrilateral*

The most common quadrilaterals are trapeziums, parallelograms, rhombuses, rectangles, squares, and kites.

(a) Trapeziums

A trapezium is a quadrilateral with one pair of opposite sides which are parallel. In Figure 7.27, PQRS is a trapezium, where $\overline{SR}$ // $\overline{PQ}$. The plural of a trapezium is trapeziums or trapezia.

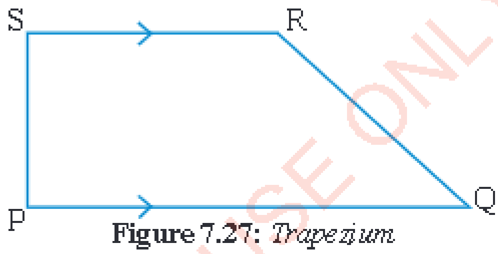


Figure 7.27: *Trapezium*

Properties of trapeziums

A trapezium has the following properties:

(i) The bases are parallel to each other, and

(ii) The sides adjacent to parallel sides are not parallel.

(b) Parallelograms

A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. In Figure 7.28, PQRS is a parallelogram such that $\overline{PS}$ // $\overline{QR}$ and $\overline{SR}$ // $\overline{PQ}$.

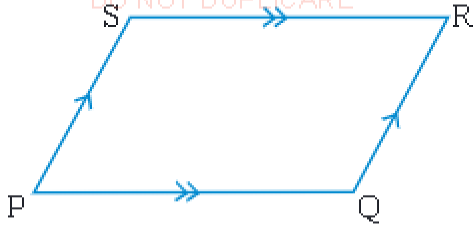


Figure 7.28: *Parallelogram*

Properties of parallelograms

A parallelogram has the following properties:

(i) Pairs of opposite sides are parallel.

(ii) Pairs of opposite angles are equal.

(iii) Pairs of opposite sides are equal.

(iv) Diagonals bisect each other.

(v) Adjacent angles are supplementary.

(vi) Each diagonal separates the parallelogram into two equal triangles.

(c) **Rhombuses**

A rhombus is a parallelogram whose all sides have equal length. It is sometimes called an equilateral parallelogram, since equilateral means equal length. Figure 7.29 shows a rhombus PQRS. Thus, $\overline{PQ} = \overline{QR} = \overline{RS} = \overline{SP}$.

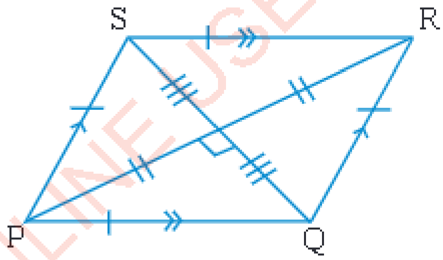


Figure 7.29: *Rhombus*

Properties of rhombuses

A rhombus has the following properties:

(i) Opposite sides are parallel.

(ii) All sides are equal.

(iii) Pairs of opposite angles are equal.

(iv) Diagonals bisect each other at right angles.
(v) Adjacent angles are supplementary.
(vi) Each diagonal divides the rhombus into two equal triangles.

(d) Rectangles

A rectangle is a quadrilateral with four right angles. It is also defined as a parallelogram with all its angles measuring 90°. In Figure 7.30, ABCD is a rectangle, where $\overline{AB}$ // $\overline{DC}$ and $\overline{AD}$ // $\overline{BC}$.

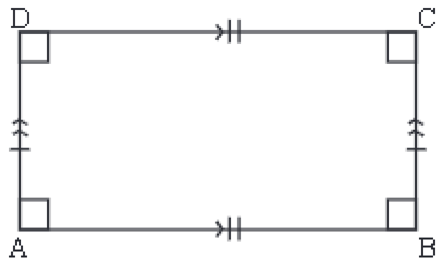


Figure 7.30: *Rectangle*

Properties of rectangles

A rectangle has the following properties:

(i) All angles are right angles.
(ii) Diagonals bisect each other.
(iii) Opposite sides are parallel.
(iv) Opposite sides are equal.
(v) The opposite parallel sides are equidistant from each other.

(e) Squares

A square is a rectangle in which all of the four sides are equal. In Figure 7.31, PQRS is a square, where $\overline{PS}$ // $\overline{QR}$ and $\overline{PQ}$ // $\overline{SR}$.

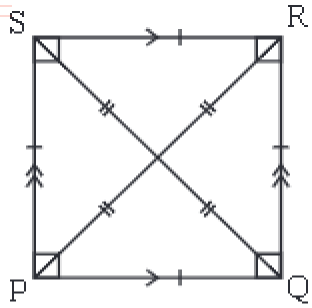


Figure 7.31: *Square*

Properties of squares

A square has the following properties:

(i) All sides are equal.
(ii) All angles are right angles.
(iii) Opposite sides are parallel.
(iv) The diagonals are equal.
(v) Diagonals bisect each other at right angles.
(vi) The opposite parallel sides are equidistant from each other.

(f) Kites

A kite is a quadrilateral with two pairs of equal adjacent sides and a pair of equal opposite angles. In Figure 7.32, ABCD is a kite, where $\overline{AB} = \overline{AD}$ and $\overline{BC} = \overline{DC}$ and $A\hat{B}C = A\hat{D}C$.

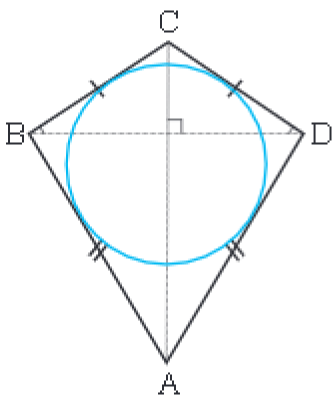


Figure 7.32: *Kite*

Properties of kites

A kite has the following properties:

(i) Two pairs of adjacent sides are equal.

(ii) Diagonals bisect each other at right angles.

(iii) The shorter diagonal divides the kite into two isosceles triangles.

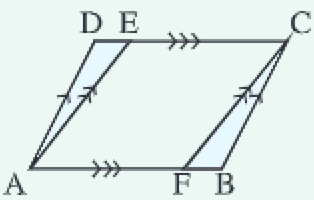
(iv) The longer diagonal is a perpendicular bisector of the shorter diagonal.

(v) It has one pair of equal opposite angles.

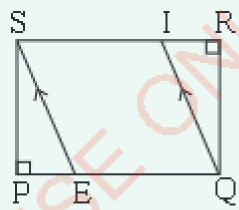
Exercise 9

1. Construct a parallelogram ABCD, where $\overline{AB} = 5$ cm, $\overline{BC} = 4$ cm and $A\hat{B}C = 70°$.
2. Construct a rhombus of length 4 cm and an angle measuring 65°.
3. Construct trapezium ABCD, where $\overline{BC} = 8$ cm, $\overline{AB} = 4$ cm, $A\hat{B}C = 80°$, $B\hat{C}D = 70°$, and $\overline{AD} // \overline{BC}$. Measure the length of $\overline{AD}$.
4. Name all the parallelograms in each of the following figures:

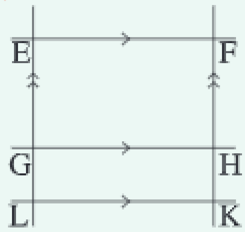
(a)



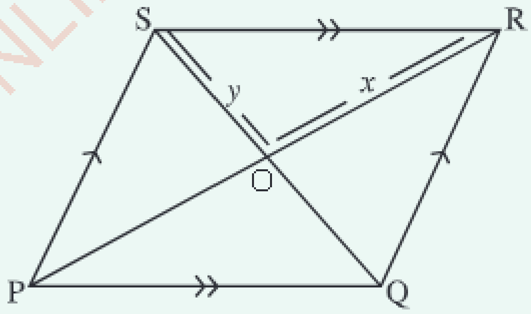
(b)



(c)



5. If PQRS is a rhombus, find the values of x, y, and $S\hat{O}R$, where $\overline{PO} = 3.3$ cm and $\overline{QO} = 2$ cm.



6. Construct a square EFGH of a side of length 5 cm. Draw $\overline{EG}$, and measure its length and $E\hat{G}H$.

Activity 9

Individually or in groups, perform the following tasks:

1. Draw a circle of any radius.
2. Find the diameter of the circle drawn in task 1.
3. Divide the circle drawn in task 1 into four equal parts and shade two parts. Write the fraction of the shaded region.

A circle is a closed path with all its points on the edge being at the same distance from a fixed point. Figure 7.33 shows a sketch of a circle with O as a fixed point.

The fixed point is called the centre of the circle, while the fixed distance from the centre to any point on the path is called the radius of the circle. The complete boundary (perimeter) of the circle is called the circumference.

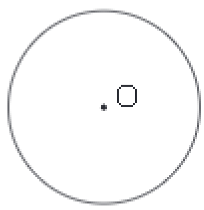


Figure 7.33: *Circle*

Arcs

A part of a circumference is called an arc of a circle. An arc which forms half of a circumference is called a semi-circular arc (see Figure 7.34(a)). An arc which is less than the semi-circular arc is called a minor arc. In Figure 7.34(b) a solid arc AB is a minor arc, while a solid line AB in Figure 7.34 (c) which is greater than a semi-circular arc is called a major arc.

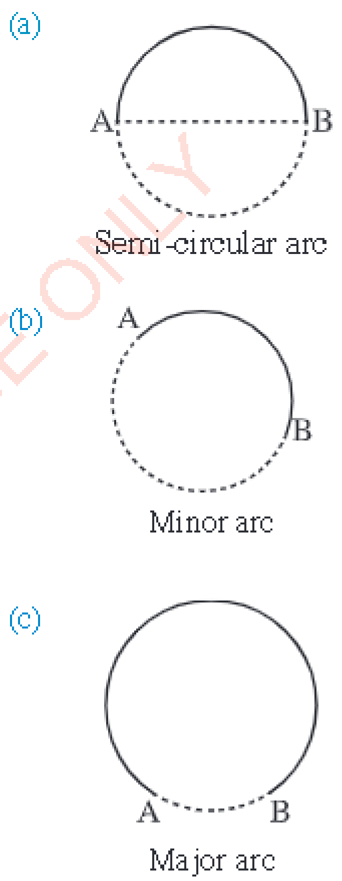


Figure 7.34: *Types of arcs*

Symbolically, an arc AB is denoted as $\overset{\frown}{AB}$. Three points are used to name an arc as shown in Figure 7.35. The arc from P to Q through A is $\overset{\frown}{PAQ}$, while the arc from P to Q through C is $\overset{\frown}{PCQ}$.

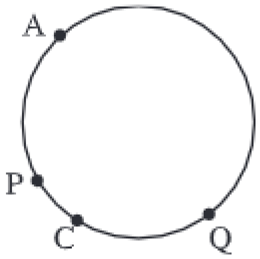


Figure 7.35: *Names of arcs*

Chord

A line segment connecting two points on the circumference is called a chord of a circle. In Figure 7.36, $\overline{AB}$ is a chord.

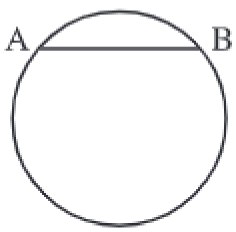


Figure 7.36: *Chord*

Diameter

A chord of a circle which passes through the centre of a circle is called a diameter. In Figure 7.37, $\overline{AB}$ passes through the centre. Thus, $\overline{AB}$ is the diameter of the circle.

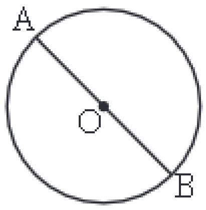


Figure 7.37: *Diameter*

Radius

A line segment joining any point on the circumference of a circle to its centre is called a radius. The plural of radius is radii. The length of a diameter is twice the length of the radius. In Figure 7.37, $\overline{AO} = \overline{OB}$ and both are radii.

Thus, $\overline{AB} = \overline{AO} + \overline{OB}$.

Therefore, $\overline{AB} = 2\overline{AO}$ or $\overline{AB} = 2\overline{OB}$.

Circular region

A surface bounded by a circle and the circle itself is called a circular region. In other words, a circular region consists of the circle and its interior.

Sector

A part of a circular region bounded by two radii of a circle and an arc is called a sector of a circle. In Figure 7.38, OED is a sector. The sector which contains a minor arc is called the minor sector, whereas the major sector contains a major arc. In Figure 7.38, the shaded region is the minor sector, while the unshaded part is the major sector.

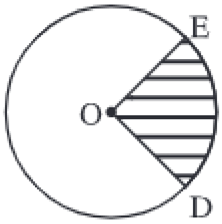


Figure 7.38: *Sector*

Segment

A segment of a circle is a region bounded by a chord of a circle and the intercepted arcs. The segment bounded by the minor arc is the minor segment, whereas; the segment bounded by the major arc is called the major segment. In Figure 7.39, O is the centre of the circle, ACBA is the minor segment, and ADBA is the major segment.

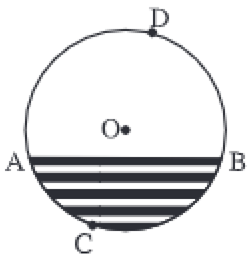


Figure 7.39: *Segment*

Exercise 10

1. Draw circles of the following radii using a pair of compasses:
   (a) 4 cm (c) 5.2 cm
   (b) 3.8 cm (d) 0.7 cm
2. What are the lengths of the diameters if the radii of the circles are:
   (a) 0.8 cm?
   (b) 6 cm?
   (c) 7.8 cm?
3. Draw a circle of radius 5 cm and shade any two sectors.
4. What is the difference between a segment of a circle and a sector?
5. Draw a circle of radius 4 cm and mark any minor and major arcs.
6. Draw a line segment CD = 8 cm, and then draw a circle of diameter CD. In the circle, draw a chord of length 6 cm and shade the minor segment of the circle.

Chapter summary

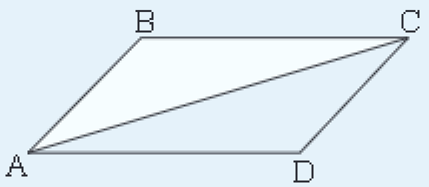
1. A point is denoted by a full stop.
2. A line segment is formed by joining two points.
3. The shortest distance between any two points is through a straight line segment.
4. A ray is a straight line segment extended in one direction.

5. A line is a set of points which extends in both directions without an end.
6. An angle is the amount of turning.
7. The following are types of angles:
   (a) An acute angle is an angle whose measure is greater than 0° but less than 90°.
   (b) An obtuse angle is an angle whose measure is greater than 90° but less than 180°.
   (c) A reflex angle is an angle whose measure is greater than 180° but less than 360°.
   (d) A right angle is an angle measuring exactly 90°.
   (e) A straight angle is an angle measuring exactly 180°.
8. Perpendicular lines intersect at 90°.
9. Parallel lines are always at the same distance apart. Arrows are used to denote parallel lines.
10. A polygon is a closed plane figure bounded by a finite number of line segments placed end to end successively.
11. An isosceles triangle has two equal sides and two equal base angles.
12. An equilateral triangle has all sides equal and all angles equal.
13. A scalene triangle is a triangle in which all of its sides have different lengths and all of its angles are different.
14. A trapezium is a quadrilateral with one pair of parallel opposite sides.
15. A parallelogram is a quadrilateral in which both of its pairs of opposite sides are parallel.
16. A rhombus is a parallelogram whose sides are equal.
17. A rectangle is a parallelogram in which all its angles measure 90°.
18. A square is a rectangle with four equal sides and each angle measures 90°.
19. A circle is a closed path which is at equal distance from its fixed point called a centre.
20. An arc is a part of a circumference of a circle.
21. A chord is a line segment connecting two points on the circumference.
22. A diameter is a chord of a circle which passes through the centre of the circle.
23. A circular region is a surface bounded by a circle itself.
24. A sector is a part of a circular region bounded by two radii of a circle and an arc.
25. A segment is a plane bounded by a chord of a circle and the intercepted arcs.

Revision exercise

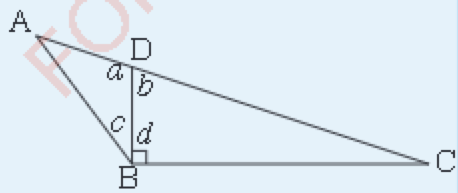
1. Which of the following illustrates a point, a ray, a line segment, or a plane?
   (a) A ray of light
   (b) A ruler's edge
   (c) A top of a table
   (d) A tip of a pin

2. Study the following figure and answer the questions that follow.



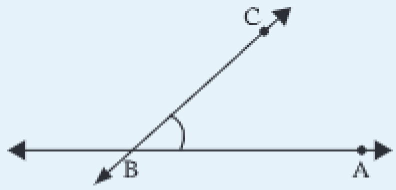
Use the given figure to name the following:
   (a) All line segments.
   (b) The line segments which intersect at A.
   (c) Other line segments that can be drawn using the given points.
   (d) The point of intersection of $\overline{CD}$ and $\overline{AD}$.
   (e) The point of intersection of $\overline{BC}$, $\overline{AC}$, and $\overline{CD}$.

3. Study the following figure and answer the questions that follow.



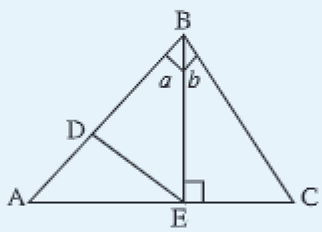
Use the figure to name the following:
   (a) Two obtuse angles.
   (b) A right angle.
   (c) A straight angle.
   (d) An acute angle at D.
   (e) An acute angle at B.

4. Study the following figure and answer the questions that follow.



Use a ruler and a pair of compasses to do the following:
   (a) Copy the angle ABC.
   (b) Construct a line parallel to $\overleftrightarrow{BC}$ at A.

5. Study the following figure and answer the questions that follow.



   (a) Name any two pairs of perpendicular line segments.
   (b) If $b = 42°$, find the value of a.
   (c) Find the size of angle AEB.

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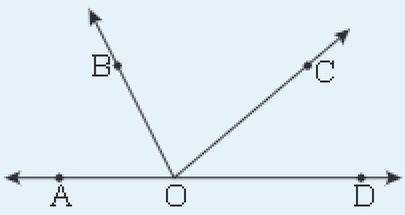
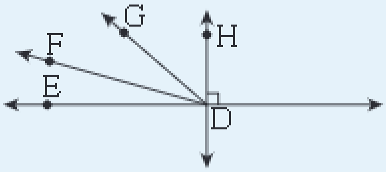
6. Name two pairs of supplementary angles in the following figure.



7. Name two pairs of complementary angles in the following figure.

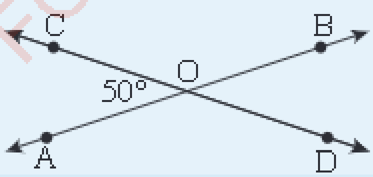


8. Using a pair of compasses and a ruler, bisect a line segment of length 10 cm.

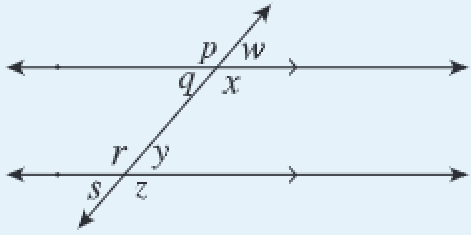
9. Use a protractor to draw an angle measuring 240° and bisect it using a pair of compasses.

10. Construct the following angles using a pair of compasses and a ruler:
    (a) 15° (c) 60°
    (b) 30° (d) 120°

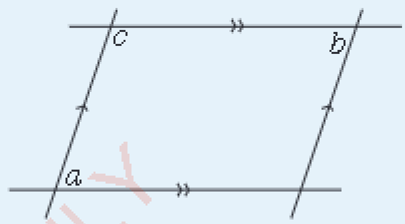
11. In the following figure, $A\hat{O}C = 50°$. Find the values of $A\hat{O}D$, $C\hat{O}B$, and $B\hat{O}D$.



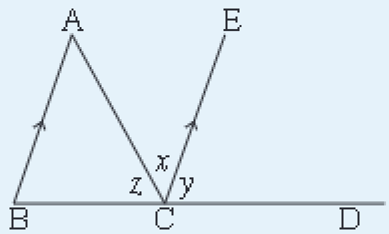
12. Using the following figure, name the pairs of equal angles and give reasons to justify your answer.



13. If $a = 75°$, find the values of b and c in the following figure.

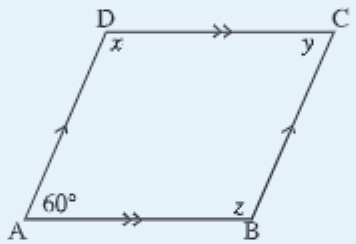


14. In the following figure, find the values of:
    (a) x, y and z.
    (b) $z + y + x$.

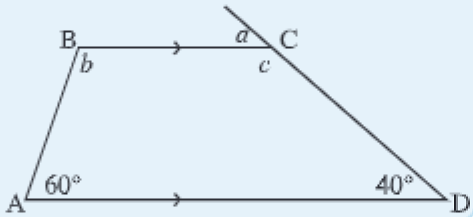


15. What is the name of an arc of a circle formed by a chord passing through the centre?

16. Given a rhombus ABCD, find the values of the angles x, y, and z.



17. Are all parallelograms trapezia?
18. Are all trapezia parallelograms?
19. What is the difference between a rhombus and a square?
20. What is the difference between a square and a rectangle?
21. If ABCD is a trapezium, find the values of the angles *a*, *b*, and *c* in the following figure:



Chapter Eight

Algebra

Introduction

Algebra deals with the representation of numbers and quantities using letters or symbols that are connected by addition, subtraction, multiplication, and division operations. In this chapter, you will learn about algebraic expressions, equations with one unknown, equations with two unknowns, and inequalities with one unknown. The competencies developed will help you to solve daily life problems such as estimating profit and loss in business, determining interests in financial institutions, and currency conversions. For example; learning simultaneous equations will help you to determine the best loan choice to make when buying a car or a house by considering the duration of the loan, the interest rate, and the monthly payment of the loan. The knowledge of simultaneous equations can also be used to determine cost and demand by considering the relationship between the price of a commodity and the quantity of the commodity people want to buy at a certain price. With this knowledge, you will also be able to determine the speed, distance, and duration when travelling by a car, a plane, and a train among many others.

Algebraic expressions

In algebra, letters are used to represent numbers. For example; it is common to represent the length of an object by a letter l, a radius by r, a diameter by d, and any other unknown by x, y or z. These letters are known as variables.

An algebraic expression is a combination of letters and numbers connected by one or more mathematical operations, that is, $+$, $-$, $\times$, $\div$. An algebraic expression is made up of terms. A term can be a number, a variable, a number multiplied by a variable, or a variable multiplied by a variable or variables.

For example; in the algebraic expression $5x + 3y + 8$, the terms are: $5x$, $3y$, and 8.

Terms with the same variables or exponents are called like terms. For example;

$8y$ and $19y$ or $8yx$ and $3xy$ or $6x^2y$ and $2x^2y$ are like terms.

Terms with different variables or exponents are called unlike terms. For example; $6x$ and $6z$ or $3mn$ and $4vy$ or $5u^2y$ and $6y^2v$ are unlike terms.

When a term is made up of a number multiplied by a variable or variables, that number is called a coefficient. A term which is not multiplied by any variable is called a constant term. For example; in the algebraic expression $3x - 5y + 4$; 3 is a coefficient of x, -5 is a coefficient of y, and 4 is a constant term.

Addition and subtraction of algebraic expressions

The basic principle of addition and subtraction of algebraic expressions is that, only like terms can be added or subtracted.

Example 1

Simplify the expression $3n - 7n + 12n$.

Solution

$$\begin{aligned}3n - 7n + 12n &= 3n + 12n - 7n\\ &= 15n - 7n\\ &= 8n.\end{aligned}$$

Therefore, $3n - 7n + 12n = 8n$.

Example 2

Simplify the expression $6m - 4 - 2m + 15$ and state the coefficient of m.

Solution

$$\begin{aligned}6m - 4 - 2m + 15 &= 6m - 2m - 4 + 15\\ &= 4m + 11.\end{aligned}$$

Therefore, $6m - 4 - 2m + 15 = 4m + 11$ and the coefficient of m is 4.

Example 3

If $m = 2$, $n = 4$, and $p = -5$, find the value of each of the following expressions:

(a) $3mn - 6mp + 2n^2p^3$

(b) $m^2n^2 - p^2n^3 + 7nmp$

Solution

(a) $3mn - 6mp + 2n^2p^3$

$= 3 \times 2 \times 4 - 6 \times 2 \times (-5) + 2(4)^2(-5)^3$

$= 24 + 60 - 4\ 000$

$= -3\ 916.$

Therefore, the value of $3mn - 6mp + 2n^2p^3$ is $-3\ 916$.

(b) $m^2n^2 - p^2n^3 + 7nmp$

$= (2)^2(4)^2 - (-5)^2(4)^3 + 7 \times 4 \times 2 \times -5$

$= 64 - 1\ 600 - 280$

$= -1\ 816.$

Therefore, the value of $m^2n^2 - p^2n^3 + 7nmp$ is $-1\ 816$.

Example 4

Simplify $8m^2n - 6a^2c + 14 - 4m^2n + 3a^2c - 8$ and state:

(a) The coefficient of m^2n

(b) The number of terms

(c) The constant term

Solution

$8m^2n - 6a^2c + 14 - 4m^2n + 3a^2c - 8$

$= 8m^2n - 4m^2n + 3a^2c - 6a^2c + 14 - 8$

$= 4m^2n - 3a^2c + 6.$

Therefore, $8m^2n - 6a^2c + 14 - 4m^2n + 3a^2c - 8 = 4m^2n - 3a^2c + 6.$

(a) The coefficient of m^2n is 4.

(b) There are 3 terms.

(c) The constant term is 6.

Exercise 1

1. Simplify each of the following algebraic expressions and then, state:

   (i) The number of terms

(ii) The coefficient of each of the terms

(a) $n + n + n + n + n + k + k + k + x + x$

(b) $3x + 4y - 7z + 3x - 7y + 2z$

(c) $3\frac{1}{2}x + 7x - \frac{1}{4}x$

Simplify each of the following algebraic expressions:

2. $12m + 13m$
3. $5y + 7y - 4y$
4. $9z + 6z - 8z + 5z$
5. $24w - 28w$
6. $3k - 18k + 15k$
7. $15n - 9n$
8. $x + 4 - 5x$
9. $4k - k + 3k$
10. $r + 2r + 3r + 4$
11. $8y - 3 - 7y + 4$

Find the value of each of the following algebraic expressions:

12. $6n + 8m + 4n - 5m$, if $m = -3$ and $n = 4$
13. $14x + 8 - 3x + 2$, if $x = -2$
14. $26 + 2b - 6c - 3b + 6c$, if $c = 5$ and $b = -4$
15. $3a - 5b - 7a + 6c + 7a$, if $a = 3$, $b = 2$, and $c = 5$
16. $5a - b - a + 9b + 3a$, if $a = 4$ and $b = -2$
17. $4x - 6y + 7x + 2y$, if $x = 10$ and $y = -2$
18. $9y + 5 - 4y + 3x + 2$, if $x = 4$ and $y = -4$
19. $13x + 14 + 8x - 4 - 11x$, if $x = -6$
20. $7x + 3y - 4 - 3y$, if $x = 5$ and $y = -3$
21. $8m + 0.4m - 2 - 6m + 8$, if $m = -4$
22. $4x^2y^3 - 6k^3l + 10xy^3 - 4kl^3$, given that $x = 1$, $y = 2$, $k = -2$, and $l = -3$

Multiplication and division of algebraic expressions

Just as $2 \times 3 = 3 \times 2$, so is $n \times m = m \times n$ and $a \times b = b \times a = ab = ba$.

Therefore, to multiply $3n$ by 5 is the same as to multiply 5 by $3n$.

That is, $3n \times 5 = 5 \times 3n = 15 \times n = 15n$.

Similarly, $3n \times 5m = 3 \times n \times 5 \times m$
$= 3 \times 5 \times n \times m$
$= 15 \times n \times m$
$= 15mn$.

Thus, $3n \times 5m = 15mn$.

Also, $(2n + m) \times 3 = (2n + m) + (2n + m) + (2n + m)$
$= (2n + 2n + 2n) + (m + m + m)$
$= 6n + 3m.$

Therefore, $(2n + m) \times 3 = 6n + 3m.$

The multiplication sign between a number and a bracket is normally dropped but the meaning remains the same. For example;
$3 \times (2n + m) = 3(2n + m) = 6n + 3m$ and $3 \times (2n - m) = 3(2n - m) = 6n - 3m.$
In general, $a(b + c) = ab + ac.$

When multiplying an algebraic expression by a number, each term must be multiplied by the number. Similarly, when dividing an algebraic expression by a number, each term must be divided by the number.

Example 1

Multiply $5a + 2b - 3c$ by 4.

Solution

$(5a + 2b - 3c) \times 4 = 4(5a + 2b - 3c)$
$= 4 \times 5a + 4 \times 2b + 4 \times (-3c)$
$= 20a + 8b - 12c.$

Therefore, $5a + 2b - 3c$ multiplied by 4 gives $20a + 8b - 12c.$

Example 2

Multiply $2x - 3y$ by $-2a$.

Solution

$(2x - 3y) \times (-2a) = (-2a)\ (2x - 3y)$
$= (-2a)\ (2x) + (-2a)\ (-3y)$
$= -4ax + 6ay.$

Therefore, when $2x - 3y$ is multiplied by $-2a$, the result is $-4ax + 6ay.$

Example 3

Re-write $-5a(m+2n-2)$ without brackets.

Solution

$-5a(m+2n-2) = (-5a)m + (-5a \times 2n) + (-5a \times -2)$

$= -5am - 10an + 10a.$

Therefore, $-5a(m+2n-2) = -5am - 10an + 10a.$

Example 4

Divide $4ax + 6ay - 10az$ by $2a$.

Solution

$$(4ax + 6ay - 10az) \div 2a = \left(\frac{4ax + 6ay - 10az}{2a}\right)$$

$$= \left(\frac{4ax}{2a} + \frac{6ay}{2a} - \frac{10az}{2a}\right)$$

$$= 2x + 3y - 5z.$$

Therefore, $4ax + 6ay - 10az$ divided by $2a$ is $2x + 3y - 5z$.

Exercise 2

Multiply each of the following algebraic expressions:

1. $5x + 6m + 4y$ by 5
2. $3m + 7n - r$ by 2
3. $2p - 5q + 3$ by 2
4. $4m - 8n - 5p$ by -4
5. $a - 7b + 9c$ by r
6. $3x - 6y - 2z$ by a
7. $5x + 3y - 3z$ by $3a$
8. $7d + 10e - 5f$ by $-3a$
9. $6x - 2y - 7z$ by $2ab$
10. $-13x + y - 6z$ by $-2m$

Simplify each of the following algebraic expressions where possible:

11. $5x(2a + b + 3c)$
12. $3m(4x + 3y - 1)$
13. $-2n(5p - 8q - 3)$
14. $9x(-2a - 3b - c)$
15. $8mn(a - 2b + 5c)$
16. $-5hk(2r - 3d - 1)$
17. $4uv(3a - 7)$
18. $a(6m - n)$
19. $(3x - b)(-2a)$
20. $0.5m(3a - 2b)$

Divide each of the following algebraic expressions by the given numbers:

21. $2ax + 2nm - 4cz$ by 2
22. $5ax - 15ay + 12.5az$ by $5a$

Complete each of the following algebraic statements:

23. $9a - 6b + 3c = 3(\quad)$
24. $12x - 8y + 16z = 4(\quad)$
25. $5a + 20b - 10c = 5(\quad)$
26. $12x - 6y - 18z = 6(\quad)$
27. $4at - 6ar + 8am = 2a(\quad)$
28. $10am - 5bm - 15cm = 5m(\quad)$
29. $12ap - 2aq + 8ar - 6as = 2a(\quad)$
30. $ab - ac + ad = a(\quad)$

Algebraic expressions involving fractions and brackets

When dealing with algebraic expressions involving fractions and brackets, the brackets are opened by multiplying each term inside the brackets by the fraction, and simplifying the expression.

Example 1

Simplify the following algebraic expressions:

(a) $\frac{1}{4}(8b + 12b - b)$

(b) $\frac{1}{5}(15g + 5g)$

Solution

(a) $\frac{1}{4}(8b + 12b - b) = \frac{8b}{4} + \frac{12b}{4} - \frac{b}{4}$

$= \frac{8b + 12b - b}{4}$

$= \frac{19b}{4}.$

Therefore, $\frac{1}{4}(8b + 12b - b) = \frac{19b}{4}.$

(b) $\frac{1}{5}(15g + 5g) = \frac{15g}{5} + \frac{5g}{5}$
$= 3g + g$
$= 4g.$

or $\frac{1}{5}(15g + 5g) = \frac{15g + 5g}{5}$
$= \frac{20g}{5}$
$= 4g.$

Therefore, $\frac{1}{5}(15g + 5g) = 4g.$

Example 2

Simplify each of the following algebraic expressions:

(a) $\frac{5t + 7t}{9}$

(b) $\frac{17c - 3c}{4}$

Solution

(a) $\frac{5t + 7t}{9} = \frac{12t}{9}$
$= \frac{4t}{3}.$

Therefore, $\frac{5t + 7t}{9} = \frac{4}{3}t.$

Solution

(b) $\frac{17c - 3c}{4} = \frac{14c}{4}$
$= \frac{7}{2}c.$

Therefore, $\frac{17c - 3c}{4} = \frac{7}{2}c.$

Example 3

Express each of the following algebraic expressions as a single fraction:

(a) $\frac{h - 1}{3} + \frac{h + 1}{2}$

(b) $\frac{b - 2a}{4b} - \frac{b + 3a}{20b}$

Solution

(a) $\frac{h - 1}{3} + \frac{h + 1}{2}$ (The LCM of 3 and 2 is 6)

$= \frac{2(h - 1) + 3(h + 1)}{6}$ (Open the brackets)

$= \frac{2h - 2 + 3h + 3}{6}$ (Collect the like terms)

$= \frac{5h + 1}{6}.$

Therefore, $\frac{h - 1}{3} + \frac{h + 1}{2} = \frac{5h + 1}{6}.$

(b) $\frac{b - 2a}{4b} - \frac{b + 3a}{20b}$ (The LCM of $4b$ and $20b$ is $20b$)

$= \frac{5(b - 2a) - (b + 3a)}{20b}$ (Open the brackets)

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$= \frac{5b - 10a - b - 3a}{20b}$ (Collect the like terms)

$= \frac{4b - 13a}{20b}$.

Therefore, $\frac{b - 2a}{4b} - \frac{b + 3a}{20b} = \frac{4b - 13a}{20b}$.

Exercise 3

Simplify each of the following algebraic expressions where possible:

1. $4(3h - 5)$
2. $k(s - t)$
3. $\frac{1}{2}(8p - 6)$
4. $\frac{1}{q}(qz + 4qz)$
5. $\frac{1}{5}(10x - 15y)$
6. $\frac{1}{3}(6et - 4h)$
7. $\frac{1}{2}(b - 8c)$
8. $\frac{1}{3}(2b + 3) - \frac{1}{2}(4b - 1)$
9. $\frac{a + b}{4} - \frac{a - b}{8}$
10. $\frac{m - 2}{4n} - \frac{m - 1}{6n}$
11. $\frac{5(1 + y)}{3} + \frac{3y + 1}{5}$
12. $6 + \frac{3(d - 7)}{4}$

Algebraic equations

An algebraic equation is a statement connecting two mathematical expressions with an equal sign (=). For example; $2x = 16$ is an algebraic equation in which $2x$ is related to 16 by the equal sign '='. Therefore, an equation has two sides, the left hand side and the right hand side. An algebraic equation expresses a statement or a problem in a clear and short way. For example; to find a number which when multiplied by two gives 16 is the same as to find the value of x in $2x = 16$.

Linear equations

A linear equation is an algebraic equation in which each variable has an exponent of one. For example; $2x = 16$ and $x + y = 3$ are linear equations. A linear equation may involve one or more variables. For example; $3x - 2 = 7$ is a linear equation with one variable x, whereas $5x + 2y = 9$ is a linear equation with two variables, x and y.

Note that: writing $3x - 2 = 7$ is the same as writing $7 = 3x - 2$. Generally, if $a = b$, then $b = a$.

Forming linear equations

When you are given some statements, the following steps can be used to form linear equations.

Step 1: Understand the question, that is, identify the unknowns.

Step 2: Let the unknowns be represented by letters (variables).

Step 3: Write the equation using the given conditions.

Linear equations with one unknown

A linear equation with one unknown is the equation which involves only one variable.

Example 1

The sum of two numbers is 20. If one of the numbers is 8, form a linear equation connecting the two numbers.

Solution

Let the unknown number be x.

Since the known number is 8, then the algebraic equation connecting the two numbers is $8 + x = 20$.

Therefore, $8 + x = 20$.

Example 2

The difference between 24 and another number is 8. Form a linear equation connecting the numbers.

Solution

Let the unknown number be y.

Since the known number is 24, then the algebraic equation connecting the two numbers is $24 - y = 8$ or $y - 24 = 8$.

Therefore, $24 - y = 8$ or $y - 24 = 8$.

Example 3

The product of two numbers is 24. If one of the numbers is 8, form a linear equation.

Solution

Let the unknown number be z.

Since the known number is 8, then the algebraic equation is $8z = 24$.

Therefore, $8z = 24$.

It is useful to relate different English words with the corresponding arithmetic operation.

For example:

Signs	Corresponding words or phrases
+	Addition, sum of, increased by, plus, total
–	Difference, subtract, decreased, reduced by, minus
×	Multiplication, times, product
÷	Division, divided by, quotient, ratio
=	Equals, result is, is equal to, is, gives, the same as

Exercise 4

Formulate a linear equation for each of the following statements:

1. Five times a number gives twenty.
2. The difference between 123 and another number is 150.
3. The sum of 21 and another number is 125.
4. When a certain number is increased by 15, the result is 88.
5. When 99 is decreased by a certain number, the result is 63.
6. The product of 12 and another number is the same as two times the sum of 12 and the number.
7. A number is such that, when it is doubled and 8 is added to it, the result is the same as multiplying the number by 3, and subtracting 7.

8. When 36 is added to a certain number, the result is the same as multiplying the number by 5.
9. Janeth's mother is five times as old as Janeth. After 8 years, mother will be 4 times as old as Janeth.
10. When the sum of n and $(n + 3)$ is multiplied by 5, the result is half the product of the two numbers.

Solving linear equations with one unknown

To solve an equation with one unknown is to find the value of the unknown in the equation. For example; to solve the equation $x + 8 = 24$ is to find the value of x which makes the mathematical statement or sentence true. In solving linear equations, whatever operation performed on one side of the equation must be performed on the other side. After solving the equation, it is advised to check if the solution satisfies the equation.

Example 1

Solve the equation $x + 5 = 8$.

Solution

Given $x + 5 = 8$;

Collect the like terms by subtracting 5 from both sides.

$$x + 5 - 5 = 8 - 5$$
$$x = 3.$$

Therefore, $x = 3$.

Check: $\begin{pmatrix} x + 5 = 8 \\ 3 + 5 = 8 \\ 8 = 8 \end{pmatrix}$

Therefore, the solution satisfies the equation.

Example 2

Find the value of x that satisfies the equation $x - 8 = 16 - 2x$.

Solution

Given $x - 8 = 16 - 2x$;

collect the like terms by performing the following:

$x - 8 + 8 = 16 - 2x + 8$ (Add 8 on both sides)

$x + 2x = 24 - 2x + 2x$ (Add $2x$ on both sides)

$3x = 24$

Divide by 3 on both sides:

$\frac{3x}{3} = \frac{24}{3}$

$x = 8.$

Therefore, $x = 8$.

(Check: $x - 8 = 16 - 2x$
$8 - 8 = 16 - 16$
$0 = 0$.)

Therefore, the solution satisfies the equation.

Example 3

Find the value of x if $3x - 5 = 7$.

Solution

Given $3x - 5 = 7$;

Add 5 on both sides

$3x - 5 + 5 = 7 + 5$

$3x = 12$

Divide by 3 on both sides

$\frac{3x}{3} = \frac{12}{3}$

$x = 4.$

Therefore, $x = 4$.

Example 4

Solve the for x in the equation $\frac{3x}{2} + 3 = 12$.

Solution

Given $\frac{3x}{2} + 3 = 12$;

Subtract 3 from both sides

$\frac{3x}{2} + 3 - 3 = 12 - 3$

Multiply both sides by 2

$\frac{3x}{2} \times 2 = 9 \times 2$

$3x = 18$

$\frac{3x}{3} = \frac{18}{3}$ (Divide by 3 on both sides)

$x = 6.$

Therefore, $x = 6$.

Example 5

Solve for x, if $\frac{5}{3}(3x - 2) = 10$.

Solution

Given $\frac{5}{3}(3x - 2) = 10$;

Multiply by 3 on both sides

$3 \times \frac{5}{3}(3x - 2) = 3 \times 10$

$5(3x - 2) = 30$ (Open the brackets)

$$15x - 10 = 30$$

$$15x - 10 + 10 = 30 + 10 \quad \text{(Add 10 on both sides)}$$

$$15x = 40$$

$$\frac{15x}{15} = \frac{40}{15} \quad \text{(Divide by 15 on both sides)}$$

$$x = \frac{8}{3}$$

$$x = 2\frac{2}{3}.$$

Therefore, $x = 2\frac{2}{3}$.

Example 6

Solve for x, if $\frac{8}{3x-2} = 2$.

Solution

Given $\frac{8}{3x-2} = 2$;

$\frac{8}{(3x-2)} \times (3x-2) = 2(3x-2)$ Multiply by $(3x - 2)$ on both sides

$$8 = 2(3x-2) \quad \text{(Open brackets)}$$

$$8 = 6x - 4$$

$$8 + 4 = 6x - 4 + 4 \quad \text{(Add 4 on both sides)}$$

$$12 = 6x$$

$$\frac{6x}{6} = \frac{12}{6} \quad \text{(Divide by 6 on both sides)}$$

$$x = 2.$$

Therefore, $x = 2$.

$$\left(\begin{aligned} \text{Check: } \frac{8}{3x-2} &= 2 \\ \frac{8}{3\times 2 - 2} &= 2 \\ \frac{8}{4} &= 2 \\ 2 &= 2. \end{aligned}\right)$$

Therefore, the solution satisfies the equation.

Exercise 5

Find the value of the unknown variable x in each of the following equations:

1. $x + 12 = 25$
2. $x - 8 = 8$
3. $3x = 15$
4. $\frac{x}{5} = 5$
5. $2x + 12 = 25$
6. $2x - 8 = 8$
7. $3x - 3 = 15$
8. $\frac{x}{5} - 3 = 5$
9. $0.2x + 7 = 9$
10. $0.6x - 5 = 7$
11. $\frac{x}{5} + 3 = 5$
12. $\frac{x-6}{7} - \frac{2x+3}{4} = \frac{x-5}{2}$
13. $\frac{7x+1}{2} = 14$
14. $\frac{3x+2}{5} = 4$
15. $\frac{3x}{5} - 1 = 5$

Formulating and solving algebraic equations from word problems

Solving word problems involve formulating the corresponding equations first. The formulation of an equation requires careful interpretation of the given word problem. Once the equation has been formulated, the usual procedures of solving the equation are used.

Example 1

If John has 5 600 Tanzanian shillings, how many oranges can he buy if each orange costs 200 Tanzanian shillings?

Solution

Let the number of oranges that John can buy be x. The cost of x oranges is Tsh 200 x.

$$200x = 5\,600$$

$$\frac{200}{200}x = \frac{5600}{200}$$

$$x = 28.$$

Therefore, John can buy 28 oranges.

Example 2

The age of a father is four times the age of his son. If the sum of their ages is 50 years, find the age of the son.

Solution

Let the age of the son be x years.

Thus, the age of the father is $4x$ years.

$$x + 4x = 50$$
$$5x = 50$$
$$\frac{5}{5}x = \frac{50}{5}$$
$$x = 10.$$

Therefore, the age of the son is 10 years.

Example 3

The sum of two consecutive integers is 31. Find the smaller number.

Solution

Consecutive integers are integers that are next to each other on a number line. These are integers that follow each other in the order from the smallest to the largest. The difference between any two consecutive integers is 1.

For example; 1, 2, 3, 4, 5, … or 12, 13, 14, 15, ... are consecutive integers.

Let the smaller number be n. The other number is $(n + 1)$ and the sum of the two numbers is $n + (n + 1)$.

Thus, $n + (n + 1) = 31$

$$2n + 1 = 31$$
$$2n = 30$$
$$\frac{2n}{2} = \frac{30}{2}$$
$$n = 15.$$

Therefore, the smaller number is 15.

Example 4

A father is 32 years older than his son. After 4 years, the father's age will be twice the age of his son. Find their present ages.

Solution

Let the son's present age be x years.

Thus, the father's present age is $(x + 32)$ years. The son's age after 4 years will be $(x + 4)$ years. The father's age after 4 years will be $(x + 32 + 4)$ years $= (x + 36)$ years.

After 4 years, the father's age will be twice the age of his son.

That is,

$x + 36 = 2(x + 4)$

$x + 36 = 2x + 8$

$36 - 8 = 2x - x$

$28 = x$

$x = 28$ (The son's present age).

The father's present age $= x + 32$

$= 28 + 32$

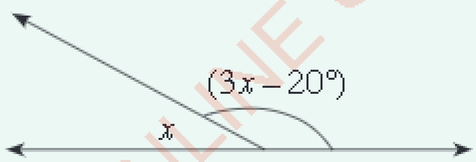
$= 60.$

Therefore, the son's present age is 28 years, and the father's present age is 60 years.

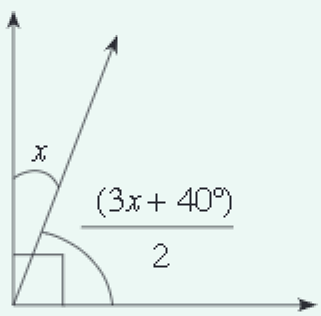
Exercise 6

1. John is six years older than Amina. If the sum of their ages is 30 years, find Amina's age.
2. Asha's age is five times the age of Martin. If the sum of their ages is 36 years, find Martin's age.
3. If the sum of two consecutive numbers is 37, find the numbers.
4. If the angles of a triangle are $2x$, $(x + 20°)$, and $(x - 30°)$, find the value of x.
5. In each of the following figures, calculate the value of x in degrees:

(a)



(b)



6. When a certain number n is added to 7, the result is $12\frac{3}{4}$. Find the value of n.
7. If the product of $2\frac{1}{2}$ and another number y is 25, find the value of y.
8. The difference between 48 and another number is 17. Find the number.

9. When the difference between 24 and m is multiplied by 5, the result is –20. Find the value of m.
10. One third of a certain number is added to three fifths of the same number to give the sum of 14. Find the number.
11. The product of half of a certain number and 6 is 48. Find the number.
12. The sum of a number and its 40 percent is 28. Find the number.
13. A certain woman is 28 years older than her daughter. After 10 years, the woman will be three times older than her daughter. Find their present ages.
14. A boy bought y balls at 500 Tanzanian shillings each. This number of balls could decrease by 4 if the price for each ball were 700 Tanzanian shillings each. Find the value of y.
15. When a number is multiplied by 8, then 9 is subtracted from the product, the result is 45 more than twice the number. Find the number.
16. Julius is four times older than Amina. Five years ago, the sum of their ages was 50. How old are they now?

Simultaneous equations

Simultaneous equations involve many equations and many unknowns requiring a solution at the same time (simultaneously). At this level, you will only learn linear simultaneous equations with two equations and two unknowns.

Consider the following example;

$x + y = 20$ (1)

This equation involves two unknowns x and y. Different values of x and y can be used to make such an equation true. The values of x and y that satisfy equation (1) in ordered pairs (x, y) are (1,19), (2,18), (3, 17), (10,10), (13.4, 6.6) etc. Therefore, to solve for x and y in (1), another equation is needed. For example; if it is also known that $x – y = 8$, (2)

it is possible to use equations (1) and (2) to find the values of x and y which satisfy both equations. These equations are such that the value of x in (1) is the same as the value of x in (2) and the same applies to y. The equations (1) and (2) must be satisfied together, that is, simultaneously.

Solving linear simultaneous equations

Three methods of solving linear simultaneous equations will be discussed. These methods are; elimination, substitution, and combination of elimination and substitution method.

(a) Solving linear simultaneous equations by elimination method

Elimination method involves eliminating one of the unknowns from the equations in order to make a single equation with one unknown.

The following are the steps for solving linear simultaneous equations by elimination method:

Step 1: Multiply the given equations by suitable constants so as to make the coefficients of one of the unknowns numerically equal.

Step 2: Add new equations, if the numerically equal coefficients are opposite in signs, otherwise subtract them.

Step 3: Solve the obtained equation. This gives the value of one of the unknowns.

Step 4: Repeat steps 1 to 3 for the value of the second unknown.

Example 1

Solve the following linear simultaneous equations by elimination method:

$$\begin{cases} 6x + y = 15 \\ 3x + y = 9 \end{cases}$$

Solution

Label the equations with numbers, say (1) and (2);

$$\begin{cases} 6x + y = 15 & (1) \\ 3x + y = 9 & (2) \end{cases}$$

Choose a variable to be eliminated, say x:

Eliminate x to get y:

The coefficient of x in (1) is 6
The coefficient of x in (2) is 3 } without considering their signs

Make the coefficients of x in (1) and (2) to be the same. Multiply (2) by 2 and (1) by 1:

$$\begin{cases} 1 \times (6x + y) = 15 \times 1 \\ 2 \times (3x + y) = 9 \times 2 \end{cases}$$

$$\begin{cases} 6x + y = 15 & (3) \\ 6x + 2y = 18. & (4) \end{cases}$$

If the signs of the variable to be eliminated are the same (i.e. '+' and '+' or '–' and '–'), then subtract (2) from (1) or (1) from (2). But, if the signs are different (i.e. '–' and '+'), then add the two equations.

Since the signs of the variable x in (3) and (4) are the same, subtract equations (4) from (3).

$$-\begin{cases} 6x + y = 15 \\ 6x + 2y = 18 \end{cases}$$

$6x - 6x + y - 2y = 15 - 18$

$-y = -3$.

Hence, $y = 3$.

Eliminate y to get x :

The coefficient of y in (1) is 1
The coefficient of y in (2) is 1 } without considering their signs

The coefficients are the same, then there is no need of multiplying by any number.

Subtract equation (2) from (1):

$$-\begin{cases} 6x + y = 15 \\ 3x + y = 9 \end{cases}$$

$6x - 3x + y - y = 15 - 9$

$3x = 6$

$\frac{3x}{3} = \frac{6}{3}$.

Hence, $x = 2$.

Therefore, $x = 2$ and $y = 3$.

Example 2

Solve the following simultaneous equations by elimination method:

$$\begin{cases} 2x + y = 10 \\ 3x - 2y = 1 \end{cases}$$

Solution

Label the equations:

$$\begin{cases} 2x + y = 10 & (1) \\ 3x - 2y = 1 & (2) \end{cases}$$

Choose the variable to be eliminated. Eliminate y to get x.

The coefficient of y in (1) is 1
The coefficient of y in (2) is 2 } without considering their signs

Make the coefficients of y in (1) and (2) to be the same.
Multiply equation (1) by 2 and equation (2) by 1.

$$\begin{cases} 2(2x + y) = 10 \times 2 \\ 1(3x - 2y) = 1 \times 1 \end{cases}$$

$$\begin{cases} 4x + 2y = 20 & (3) \\ 3x - 2y = 1 & (4) \end{cases}$$

Add equations (3) and (4):

$$+\begin{cases} 4x + 2y = 20 \\ 3x - 2y = 1 \end{cases}$$

$4x + 3x + 2y + (-2y) = 20 + 1$

$4x + 3x + 2y - 2y = 20 + 1$

$7x = 21$

$\frac{7x}{7} = \frac{21}{7}$.

Hence, $x = 3$.

Eliminate x to get y.

The coefficient of x in (1) is 2
The coefficient of x in (2) is 3 } without considering their signs

Make the coefficient of x in (1) and (2) to be the same.

Multiply equation (1) by 3 and (2) by 2:

$$\begin{cases} 3(2x + y) = 10 \times 3 \\ 2(3x - 2y) = 1 \times 2 \end{cases}$$

$$\begin{cases} 6x + 3y = 30 & (5) \\ 6x - 4y = 2 & (6) \end{cases}$$

Then, subtract (6) from (5)

$$-\begin{cases} 6x + 3y = 30 \\ 6x - 4y = 2 \end{cases}$$

$6x - 6x + 3y - (-4y) = 30 - 2$

$6x - 6x + 3y + 4y = 30 - 2$

$7y = 28$

$\frac{7y}{7} = \frac{28}{7}$

$y = 4.$

Therefore, $x = 3$ and $y = 4$.

Example 3

Solve the following linear simultaneous equations by elimination method:

$$\begin{cases} 3x + 2y = 8 \\ 2x + 3y = 12 \end{cases}$$

Solution

Label the equations:

$$\begin{cases} 3x + 2y = 8 & (1) \\ 2x + 3y = 12 & (2) \end{cases}$$

Choose a variable to be eliminated.

Eliminate x to get y.

The coefficient of x in (1) is 3
The coefficient of x in (2) is 2 } without considering their signs.

Make the coefficients of x in (1) and (2) to be the same.

Multiply equation (1) by 2 and equation (2) by 3:

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$$\begin{cases} 2(3x + 2y) = 8 \times 2 \\ 3(2x + 3y) = 12 \times 3 \end{cases}$$

$$\begin{cases} 6x + 4y = 16 & (3) \\ 6x + 9y = 36 & (4) \end{cases}$$

Then, subtract (3) from (4).

$$-\begin{cases} 6x + 9y = 36 \\ 6x + 4y = 16 \end{cases}$$

$6x - 6x + 9y - (+4y) = 36 - 16$

$6x - 6x + 9y - 4y = 36 - 16$

$\frac{5y}{5} = \frac{20}{5}$.

Hence, $y = 4$.

Eliminate y to get x:

The coefficient of y in (1) is 2
The coefficient of y in (2) is 3, } without considering their signs

Make the coefficients of y in (1) and (2) to be the same.
Multiply equation (1) by 3 and (2) by 2:

$$\begin{cases} 3(3x + 2y) = 8 \times 3 \\ 2(2x + 3y) = 12 \times 2 \end{cases}$$

$$\begin{cases} 9x + 6y = 24 & (5) \\ 4x + 6y = 24 & (6) \end{cases}$$

Then, subtract (6) from (5):

$$-\begin{cases} 9x + 6y = 24 \\ 4x + 6y = 24 \end{cases}$$

$9x - 4x + 6y - (+6x) = 24 - 24$

$9x - 4x + 6y - 6y = 24 - 24$

$\frac{5x}{5} = \frac{0}{5}$.

$x = 0$.

Therefore, $x = 0$ and $y = 4$.

Example 4

Solve the following simultaneous equations by elimination method:

$$\begin{cases} \frac{x}{2} + \frac{y}{5} = 4 \\ \frac{x}{4} + \frac{y}{2} = 6 \end{cases}$$

Solution

Label the equations:

$$\begin{cases} \frac{x}{2} + \frac{y}{5} = 4 & (1) \\ \frac{x}{4} + \frac{y}{2} = 6 & (2) \end{cases}$$

Find the LCM of the denominators of (1) and the LCM of the denominators of (2). In order to remove the denominators, multiply equation (1) by 10 and equation (2) by 4:

$$\begin{cases} 10\left(\frac{x}{2} + \frac{y}{5}\right) = 4 \times 10 \\ 4\left(\frac{x}{4} + \frac{y}{2}\right) = 6 \times 4 \end{cases}$$

$5x + 2y = 40$ (3)

$x + 2y = 24$ (4)

Eliminate y to get x :

The coefficients of y in (3) and (4) are the same.

Subtract (4) from (3):

$$-\begin{cases} 5x + 2y = 40 \\ x + 2y = 24 \end{cases}$$

$5x - x + 2y - (+2y) = 40 - 24$

$5x - x + 2y - 2y = 40 - 24$

$4x = 16$

$\frac{4x}{4} = \frac{16}{4}$

Hence, $x = 4$.

Eliminate x to get y:

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The coefficient of x in (3) is 5.

The coefficient of x in (4) is 1.

Multiply equation (3) by 1 and equation (4) by 5:

$$\begin{cases} 1(5x + 2y) = 40 \times 1 \\ 5(x + 2y) = 24 \times 5 \end{cases}$$

$$\begin{cases} 5x + 2y = 40 & (5) \\ 5x + 10y = 120 & (6) \end{cases}$$

Subtract equation (6) from (5):

$$-\begin{cases} 5x + 2y = 40 \\ 5x + 10y = 120 \end{cases}$$

$5x - 5x + 2y - (+10y) = 40 - 120$

$2y - 10y = -80$

$-8y = -80$

$\frac{-8y}{-8} = \frac{-80}{-8}$.

Hence, $y = 10$.

Therefore, $x = 4$ and $y = 10$.

(b) Solving linear simultaneous equations by substitution method

Substitution method involves substituting one of the unknowns from one equation into another equation in order to make a single equation with one unknown.

The following are the steps for solving the system of simultaneous equations by substitution:

Step 1: Express one of the variables in terms of the other variable from one of the given equations.

Step 2: Substitute the value obtained in step 1 in the second equation to obtain an equation in one variable, then solve it to obtain the value of one unknown.

Step 3: Substitute the value obtained in step 2 into the equation in step 1 to obtain the value of the second unknown.

Example 1

Solve the following simultaneous equations by the substitution method:

$$\begin{cases} 3y - 2x = -7 \\ y + 2x = 11 \end{cases}$$

Solution

Label the equations:

$$\begin{cases} 3y - 2x = -7 & (1) \\ y + 2x = 11 & (2) \end{cases}$$

Using (2), define y in terms of x as follows;

$y + 2x - 2x = 11 - 2x$

$y = 11 - 2x.$ (3)

Substituting (3) into (1) gives;

$3(11 - 2x) - 2x = -7$

$33 - 6x - 2x = -7$

$33 - 8x = -7$

$33 - 33 - 8x = -7 - 33$

$-8x = -40$

$\frac{-8x}{-8} = \frac{-40}{-8}.$

Hence, $x = 5$.

Referring to (3), $y = 11 - 2x$,

substitute $x = 5$ into equation (3), that is,

$y = 11 - (2 \times 5)$

$= 11 - 10$

$= 1.$

Hence, $y = 1$.

Therefore, $x = 5$ and $y = 1$.

Example 2

Solve the following simultaneous equations by substitution method:

$$\begin{cases} 6x + 5y = 3 \\ 7x + 8y = 10 \end{cases}$$

Solution

Label the equations:

$$\begin{cases} 6x + 5y = 3 & (1) \\ 7x + 8y = 10 & (2) \end{cases}$$

Use (1) to express x in terms of y.

$6x + 5y - 5y = 3 - 5y$ (Subtract $5y$ from both sides)

$6x = 3 - 5y$.

Divide by 6 on both sides of the equation, to get

$$x = \frac{3 - 5y}{6} \qquad (3)$$

Substitute equation (3) into (2);

$$7\left(\frac{3 - 5y}{6}\right) + 8y = 10$$

$$\frac{21 - 35y}{6} + 8y = 10$$

$$\left(\frac{21 - 35y}{6} + 8y\right) \times 6 = 10 \times 6$$

$$21 - 35y + 48y = 60$$

$$21 + 13y = 60$$

$$21 - 21 + 13y = 60 - 21$$

$$13y = 39$$

$$\frac{13}{13}y = \frac{39}{13}.$$

Hence, $y = 3$.

Substitute $y = 3$ in (1)

$$6x + (5 \times 3) = 3$$

$$6x + 15 = 3$$

$$6x + 15 - 15 = 3 - 15$$

$$6x = -12$$

$$\frac{6}{6}x = \frac{-12}{6}.$$

Hence, $x = -2$.

Therefore, $x = -2$ and $y = 3$.

(c) Solving simultaneous equations by combination of elimination and substitution methods

Simultaneous equations can also be solved by using both elimination and substitution methods.

Example 1

Solve the following simultaneous equations by combination method:

$$\begin{cases} 6x - y = 3 \\ 4x - 3y = -5 \end{cases}$$

Solution

Label the equations:

$$\begin{cases} 6x - y = 3 & (1) \\ 4x - 3y = -5 & (2) \end{cases}$$

Eliminate y to get x:

Multiply (1) by 3 and (2) by 1;

$3(6x - y) = 3 \times 3$

$1(4x - 3y) = -5 \times 1$

$$-\begin{cases} 18x - 3y = 9 & (3) \\ 4x - 3y = -5 & (4) \end{cases}$$

Then, subtract (4) from (3):

$$\begin{cases} 6x + 4y = 16 \\ 6x + 9y = 36 \end{cases}$$

$18x - 4x - 3y - (-3y) = 9 - (-5)$

$18x - 4x - 3y + 3y = 9 + 5$

$14x = 14$

$\frac{14}{14}x = \frac{14}{14}$

Hence, $x = 1$.

Substitute $x = 1$ in one of the two equations. In this case, equation (1) is used.

$6x - y = 3$

$6(1) - y = 3$

$6 - y = 3$

$6 - y + y = 3 + y$

$6 = 3 + y$

$6 - 3 = 3 - 3 + y$

$3 = y.$

Hence, $y = 3.$

Therefore, $x = 1$ and $y = 3.$

Example 2

Solve the following simultaneous equations by combination method:

$$\begin{cases} \frac{1}{3}x + \frac{1}{2}y = 0 \\ \frac{1}{2}x + \frac{1}{3}y = \frac{5}{6} \end{cases}$$

Solution

Label the equations:

$$\begin{cases} \frac{1}{3}x + \frac{1}{2}y = 0 & (1) \\ \frac{1}{2}x + \frac{1}{3}y = \frac{5}{6} & (2) \end{cases}$$

Multiply equation (1) by 6 and equation (2) by 6 to remove fractions:

$$\begin{cases} 6\left(\frac{1}{3}x + \frac{1}{2}y\right) = 0 \times 6 \\ 6\left(\frac{1}{2}x + \frac{1}{3}y\right) = \frac{5}{6} \times 6 \end{cases}$$

$$\begin{cases} 2x + 3y = 0 & (3) \\ 3x + 2y = 5 & (4) \end{cases}$$

Eliminate x to get y.

Multiply (3) by 3 and (4) by 2,

$$\begin{cases} 3(2x + 3y) = 0 \times 3 \\ 2(3x + 2y) = 5 \times 2 \end{cases}$$

$$\begin{cases} 6x + 9y = 0 & (5) \\ 6x + 4y = 10 & (6) \end{cases}$$

Subtract (6) from (5)

$$-\begin{cases} 6x + 9y = 0 \\ 6x + 4y = 10 \end{cases}$$

$6x - 6x + 9y - (+4y) = 0 - 10$

$6x - 6x + 9y - 4y = 0 - 10$

$5y = -10$

$\frac{5}{5}y = \frac{-10}{5}$.

Hence, $y = -2$.

Substitute $y = -2$ in (3) or in (4). In this case, (4) is used.

$3x + 2y = 5$

$3x + 2(-2) = 5$

$3x - 4 = 5$

$3x - 4 + 4 = 5 + 4$

$3x = 9$

$\frac{3}{3}x = \frac{9}{3}$.

Hence, $x = 3$.

Therefore, $x = 3$ and $y = -2$.

Example 3

Solve the following simultaneous equations by combination method:

$$\begin{cases} x + \frac{y}{2} = 5 \\ \frac{x}{2} - \frac{y}{3} = \frac{1}{6} \end{cases}$$

Solution

Label the equations:

$$\begin{cases} x + \frac{y}{2} = 5 & (1) \\ \frac{x}{2} - \frac{y}{3} = \frac{1}{6} & (2) \end{cases}$$

Multiply (1) by 2 and (2) by 6 to remove fractions:

$$\begin{cases} 2\left(x + \frac{y}{2}\right) = 5 \times 2 \\ 6\left(\frac{x}{2} - \frac{y}{3}\right) = \frac{1}{6} \times 6 \end{cases}$$

$2x + y = 10$ (3)
$3x - 2y = 1$ (4)

Eliminate y to get x:

Multiply (3) by 2 and (4) by 1;

$$\begin{cases} 2(2x + y) = 10 \times 2 \\ 1(3x - 2y) = 1 \times 1 \end{cases}$$

$$\begin{cases} 4x + 2y = 20 & (5) \\ 3x - 2y = 1 & (6) \end{cases}$$

Add (5) and (6):

$$+\begin{cases} 4x + 2y = 20 \\ 3x - 2y = 1 \end{cases}$$

$4x + 3x + 2y + (-2y) = 20 + 1$

$4x + 3x + 2y - 2y = 20 + 1$

$7x = 21$

$\frac{7x}{7} = \frac{21}{7}$.

Hence, $x = 3$.

Substitute $x = 3$ in (3) or in (4). In this case, equation (3) is used.

$2x + y = 10$

$2(3) + y = 10$

$6 + y = 10$

$6 - 6 + y = 10 - 6$

Hence, $y = 4$.

Therefore, $x = 3$ and $y = 4$.

Exercise 7

Solve the following simultaneous equations by using elimination method:

1. $\begin{cases} 2x + y = 5 \\ 4x - y = 7 \end{cases}$

2. $\begin{cases} 3p + q = 6 \\ 5p + q = 8 \end{cases}$

3. $\begin{cases} 5x - 2y = 16 \\ x + 2y = 8 \end{cases}$

4. $\begin{cases} 8x + 5y = 40 \\ 9x - 5y = 5 \end{cases}$

5. $\begin{cases} 7x - 4y = 17 \\ 5x - 4y = 11 \end{cases}$

6. $\begin{cases} 0.7x - 0.5y = 2.5 \\ 0.7x + 0.3y = 2.9 \end{cases}$

Solve the following simultaneous equations by using substitution method:

7. $\begin{cases} 3x - 2y = 5 \\ 2x + y = 8 \end{cases}$

8. $\begin{cases} 5a + b = 23 \\ 3a - 2b = 6 \end{cases}$

9. $\begin{cases} x - 3y = 24 \\ x + 2y = 36 \end{cases}$

10. $\begin{cases} 7x - y = 14 \\ 8x - 2y = 16 \end{cases}$

11. $\begin{cases} 7x + y = 17 \\ 8x - 2y = 10 \end{cases}$

12. $\begin{cases} 2x - y = 9 \\ x + y = 9 \end{cases}$

Solve the following simultaneous equations by using combination method:

13. $\begin{cases} 3y - x = 4 \\ y + 2x = 6 \end{cases}$

14. $\begin{cases} 8m - n = 38 \\ m - 3n = -1 \end{cases}$

15. $\begin{cases} 5x - 2y = 10 \\ -x + 3y = 24 \end{cases}$

16. $\begin{cases} 2a + b = 21 \\ a + 2b = 45 \end{cases}$

17. $\begin{cases} 0.3x - 0.2y = 2.8 \\ 1.5x - 0.4y = 7 \end{cases}$

18. $\begin{cases} 2x + y = 10 \\ x - 2y = 1 \end{cases}$

19. $\begin{cases} \frac{x}{2} - \frac{y}{3} = 5 \\ \frac{x}{3} - \frac{y}{6} = 3\frac{1}{2} \end{cases}$

20. $\begin{cases} p - q = 5 \\ 3p - q = p + 13 \end{cases}$

21. $\begin{cases} \frac{x}{2} - y = 9 \\ 3x - \frac{y}{5} = 23 \end{cases}$

Word problems leading to simultaneous equations

Activity: Recognising word problems leading to simultaneous equations

Perform the following tasks individually or in groups:
Four students from a class were asked by a teacher to stand in front of the class. The teacher instructed the students to form two groups of two students each. The teacher provided a single card to each student. Each card had a number written on it. The first card was numbered 1 and the second card was numbered 2. The teacher instructed the first group to add a variable x on each card and the second group to subtract a variable y from the respective number on their cards, so that each group had a card written with a mathematical expression. The sum of the expressions on the first cards in each group was 3 and the difference between the expressions on the second cards in the first group and the second group was 5.

(a) Describe what was observed in the student's cards when they were displayed.
(b) Find the values of the unknowns.
(c) Share your results with the whole class through presentation. Discuss and conclude your results with your teacher.

Some word problems lead to pairs of simultaneous equations. Once the simultaneous equations have been formulated from word problems, the methods for solving simultaneous equations can be applied.

Formulating and solving word problems leading to simultaneous equations involves the following steps:

Step 1: Identify the requirement of the problem, that is, the unknowns.

Step 2: Let the unknowns be represented by two variables.

Step 3: Write the two simultaneous equations according to the requirement of the problem.

Step 4: Solve the simultaneous equations by using one of the methods to get the values of the variables.

Step 5: Give the conclusion according to the demand of the problem.

Example 1

The age of a father is four times the age of his son. If the sum of their ages is 60 years, find the age of the son and that of the father.

Solution

Let the age of the son be x years and the age of his father be y years.

$y = 4x$ (1)

and, $x + y = 60$ (2)

Solving the two equations by substitution method;

Substituting equation (1) into equation (2) gives;

$x + 4x = 60$

$5x = 60$

$x = \frac{60}{5}$.

Hence, $x = 12$.

Substituting $x = 12$ into (1) gives,

$y = 4 \times 12 = 48$.

Hence, $y = 48$.

Therefore, the son's age is 12 years and the father's age is 48 years.

Example 2

The sum of two numbers is 30. The difference between the larger number and three times the smaller number is 2. Find the two numbers.

Solution

Let the large number be x.

Let the small number be y.

Thus,

$$\begin{cases} x + y = 30 & (1) \\ x - 3y = 2 & (2) \end{cases}$$

Solve (1) and (2) by any method. In this case, elimination method is used.

Eliminating the variable x to get y:

$$-\begin{cases} x + y = 30 \\ x - 3y = 2 \end{cases}$$

$x - x + y - (-3y) = 30 - 2$

$4y = 28$

$y = \frac{28}{4}$.

Hence, $y = 7$.

Eliminating the variable y to get x

Multiply (1) by 3 and (2) by 1:

$$\begin{cases} 3(x + y) = 30 \times 3 \\ 1(x - 3y) = 2 \times 1 \end{cases}$$

$$\begin{cases} 3x + 3y = 90 & (3) \\ x - 3y = 2 & (4) \end{cases}$$

Add (3) and (4):

$$+\begin{cases} 3x + 3y = 90 \\ x - 3y = 2 \end{cases}$$

$3x + x + 3y + (-3y) = 90 + 2$

$4x = 92$

$x = \frac{92}{4}$.

Hence, $x = 23$.

Therefore, the larger number is 23 and the smaller number is 7.

Example 3

Dereck has 83 000 Tanzanian shillings for shopping. If he buys 2 ties and 2 shirts, he remains with 9 000 Tanzanian shillings. If he buys 1 tie and 3 shirts he spends all the money. Find the price of each tie and each shirt.

Solution

Let the price of a tie be x Tanzanian shillings.

Let the price of a shirt be y Tanzanian shillings.

Thus,

$$\begin{cases} 83\ 000 - (2x + 2y) = 9\ 000 & (1) \\ 83\ 000 - (x + 3y) = 0 & (2) \end{cases}$$

Rearranging the equations (1) and (2) gives,

$$\begin{cases} 2x + 2y = 74\ 000 & (3) \\ x + 3y = 83\ 000 & (4) \end{cases}$$

Use any method to solve equations (3) and (4). In this case, substitution method is used.

Using equation (4); express x in terms of y as follows:

$$x = 83\ 000 - 3y \qquad (5)$$

Substituting equation (5) into equation (3), we have,

$2(83\ 000 - 3y) + 2y = 74\ 000$

$166\ 000 - 6y + 2y = 74\ 000$

$-4y = 74\ 000 - 166\ 000$

$-4y = -92\ 000$

$$y = \frac{-92\ 000}{-4}.$$

Hence, $y = 23\ 000$.

Substitute $y = 23\ 000$ into (5) to obtain the value of x, that is,

$x = 83\ 000 - (3 \times 23\ 000)$

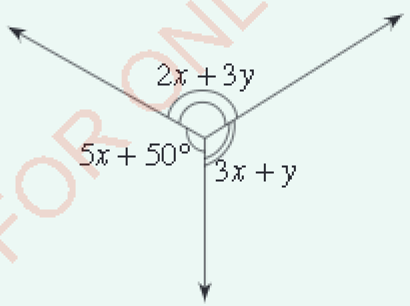
$x = 83\ 000 - 69\ 000 = 14\ 000$.

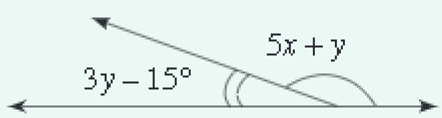
Hence, $x = 14\ 000$.

Therefore, the price of each tie is 14 000 Tanzanian shillings and the price of each shirt is 23 000 Tanzanian shillings.

Exercise 8

1. The sum of two numbers is 109 and their difference is 29. Find the numbers.
2. Two numbers are such that the first number plus three times the second number is 1, and the first number minus three times the second number is $\frac{1}{7}$. Find the two numbers.
3. The sum of the number of boys and girls in a class is 36. If twice the number of girls exceeds the number of boys by 12, find the number of girls and that of boys in the class.
4. Twice the length of a rectangle exceeds three times the width of the rectangle by one centimetre. If one third of the difference of the length and the width is one centimetre, find the dimensions of the rectangle.
5. The cost of 4 pencils and five pens together is 6 000 Tanzanian shillings, while the cost of 6 pencils and 8 pens together is 9 400 Tanzanian shillings. Calculate the cost of one pencil and one pen.
6. Half of Paul's money plus one fifth of Hamisa's money is 14000 Tanzanian shillings. Three quarters of Paul's money plus two thirds of Hamisa's money is 26 500 Tanzanian shillings. How much does each one have?
7. One third of the sum of two numbers is 50 and one fifth of their difference is 2. Find the numbers.
8. One pair of the opposite sides of a parallelogram is $(4x - y)$ units and $(3x - 5)$ units while the other pair is $(2x + y)$ units and $2\frac{1}{2}$ units. Find the values of x and y.
9. The sides of an equilateral triangle are given as $(3\frac{1}{2}x + y)$ centimetres, $(2y - 6)$ centimetres, and $(x + y - 2)$ centimetres. Find the values of x and y.
10. Use the following two figures to calculate the values of x and y:





Inequalities with one unknown

Inequalities in mathematics involve comparing quantities of similar items.

For example; 10 is greater than 3 is represented as $10 > 3$. Similarly, 2 is less than 6 is represented as $2 < 6$.

The signs $=$, $>$, $\geq$, $<$, $\leq$, and $\neq$ are used when comparing the unknowns in the same way as when comparing numbers. Sentences with the symbol '$=$' are called equations. Mathematical sentences which use $\neq$, $>$, $\geq$, $\leq$ or $<$ are called inequalities.

For example;

(a) John is older than Jane.

(b) John is taller than Jane.

(c) He finished the equations in less than thirty minutes.

Note: The sharp end (vertex) of the symbols $>$ and $<$ always points to a smaller number. The symbols $\leq$ and $\geq$ are sometimes referred to as at most and at least, respectively.

Example 1

Solve for x if $x - 3 < \frac{1}{2}$.

Solution

Given $x - 3 < \frac{1}{2}$.

Add 3 to both sides:

$x - 3 + 3 < \frac{1}{2} + 3$

$x < 3\frac{1}{2}$.

Therefore, $x < 3\frac{1}{2}$.

Example 2

Solve for x if $5x + 2 > 1$.

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Solution

Given $5x + 2 > 1$.

Subtract 2 from both sides:

$5x + 2 - 2 > 1 - 2$

$5x > -1$

Divide both sides by 5:

$\frac{5x}{5} > \frac{-1}{5}$

$x > -\frac{1}{5}$.

Therefore, $x > -\frac{1}{5}$.

Multiplication and division by negative numbers in inequalities

When both sides of an inequality are multiplied or divided by a negative number, the inequality sign changes direction. That is, it is reversed.

For instance, we know that, $20 > 4$, but when this inequality is multiplied by a negative number, say -2, the inequality sign changes and we get $-40 < -8$. Similarly, when $20 > 4$ is divided by, say -2, the inequality changes and we get $-10 < -2$.

Example 3

Solve for x if $-4x + 3 \geq \frac{1}{2}$.

Solution

Given $-4x + 3 \geq \frac{1}{2}$.

$-4x + 3 - 3 \geq \frac{1}{2} - 3$ (By subtracting 3 from both sides)

$-4x \geq -2\frac{1}{2}$

$\frac{-4x}{-4} \leq -2\frac{1}{2} \div \frac{-4}{1}$ (Dividing both sides by -4)

$x \leq -\frac{5}{2} \times -\frac{1}{4}$

$x \leq \frac{5}{8}$.

Therefore, $x \leq \frac{5}{8}$.

Note that: From $-4x \geq -2\frac{1}{2}$ to $x \leq \frac{5}{8}$, two operations were involved, which were; (a) Dividing by -4 (b) Changing $\geq$ to $\leq$

The following rules are useful when solving inequalities:

(a) Adding an equal value to each side or subtracting an equal value from each side does not change the inequality sign.

(b) Multiplying or dividing each side by the same positive number maintains the inequality sign.

(c) Multiplying or dividing each side by the same negative number changes the inequality sign.

Exercise 9

1. Which of the following inequalities are true?
   (a) $3 < 30$
   (b) $5 > -2$
   (c) $3 < -30$
   (d) $2 > -2$
   (e) $-3 < -2$
   (f) $2 > -4$
   (g) $1 > -2$
   (h) $-3 < -3$
   (i) $-4 < -5$
   (j) $5 < 10$
   (k) $6 \geq 2$
   (l) $25 \leq 24$
   (m) $-5 \leq -7$
   (n) $-8 \leq -4$
   (o) $4 > 4$
   (p) $8 = -8$
2. Represent the following numbers on a number line:
   (a) $-4, -2, 0, 2, 4$
   (b) $-2 \leq k \leq 1$
   (c) $-5 < y \leq -2$
   (d) $-2 < x < 4$
3. List the numbers which satisfy each of the following conditions:
   (a) $x < 6$ if x is a counting number.
   (b) $x \leq 4$ if x is a whole number.
   (c) $x > 3$ if x is an odd number.
   (d) $x > -3$ and $x \leq 4$ if x is an integer.

4. Solve each of the following inequalities:
   (a) $5x > 12$
   (b) $4 - x < 10$
   (c) $2x - 5 \leq \frac{1}{2}$
   (d) $6x + 4 \geq 2$
   (e) $\frac{x}{3} \leq 10$
   (f) $2 - x > 8$
   (g) $\frac{x}{2} - \frac{1}{2} < \frac{1}{2}$
   (h) $4x - \frac{3}{4} > 2x + \frac{1}{4}$
5. In each of the following, insert < or > in the box provided to make a true statement:
   (a) $-5 < 8$, then $-5 \times (-3)$ ☐ $8 \times (-3)$.
   (b) $13 > 7$, then $13 \times (-2)$ ☐ $7 \times (-2)$.
   (c) $8 > -2$, then $8 \times (-5)$ ☐ $2 \times (-5)$.
   (d) $6 > -3$, then $6 \times (-4)$ ☐ $-3 \times (-4)$.
   (e) $a > b$, then $a \times (-9)$ ☐ $b \times (-9)$.

In question 6 to 10, represent the integers on a number line under the given conditions:

6. $-6 < a < -2$
7. $-4 < b < -1$
8. $c < +5$ and $d > -3$
9. $-3 \leq e \leq +5$
10. $f < -4$ and $g > -9$

Word problems involving inequalities

Just like in equations, some word problems can lead to inequalities.

Example 1

In order to pass an examination, a candidate must obtain a minimum average of 61 marks in two tests. If Vivian obtained 54 marks in the first test, find the lowest possible marks she should obtain in the second test in order to pass the examination.

Solution

Let x be the minimum score in the second test for her to pass the examination. This implies that, the average of the scores in the two tests should be greater or equal to 61.

Thus,

$\frac{1}{2}(54 + x) \geq 61$

$27 + \frac{1}{2}x \geq 61$

$\frac{1}{2}x + 27 - 27 \geq 61 - 27$

$\frac{1}{2}x \geq 34$

$2\left(\frac{1}{2}x \geq 34\right)$

$x \geq 68.$

Therefore, the lowest possible marks that Vivian should obtain in the second test in order for her to pass the examination is 68.

Example 2

If 5 is subtracted from twice a certain number y, the result is less than 11. Find the value of y.

Solution

$2y - 5 < 11$

$2y - 5 + 5 < 11 + 5$ (Add 5 to both sides)

$2y < 16$

$\frac{2y}{2} < \frac{16}{2}$ (Divide by 2 on both sides)

$y < 8.$

Therefore, $y < 8.$

Exercise 10

1. A rectangle has a length of 9 centimetres, and an area of not less than 45 square centimetres. Find the possible values of its width.
2. Mgeni sold 200 coconuts at x Tanzanian shillings each. If he got more than 30 000 Tanzanian shillings, find the value of x.
3. Three times a whole number x is less than 2. Find the value of x.

4. If 9 is added to twice a certain whole number x, the result is greater than 63. Find the value of x.
5. A number is such that, 2 more than thrice the number is not more than 4 less than five times the number. Form an inequality to represent this information and solve it.
6. When 5 is added to an integer n, the result is greater than 7. Find the smallest possible value of n.
7. If two quarters of a certain number is subtracted from 1, the result is always greater than 0. Write the inequality for this statement.

Chapter summary

1. Algebra is related to arithmetic and is useful in many life activities.
2. When a term does not change in value, it is a constant and if it changes to different values, it is a variable.
3. The number which multiplies a variable is called the coefficient of the variable.
4. Any number used in a place of a variable is the value of that variable.
5. When you open brackets, you multiply each term in the brackets by the number outside the brackets.
6. When a number of brackets are to be opened, it is known as expanding the algebraic expression.
7. When an algebraic expression is written as a product of its factors, it is factorised.
8. Algebraic expressions can be simplified by collecting like terms.
9. The value of an expression can be calculated when the values of the variables are substituted in the expression.
10. An equation is a statement which connects two expressions with an equal sign (=).

Revision exercise

1. Write each of the following expressions in the simplest form:
   (a) $14a + 7a - 5a$
   (b) $9m - 4m + 2m$
   (c) $0.5x + 1.7y - 1.2x + 2.4y$
   (d) $\frac{5x}{2} - \frac{3}{2} + \frac{5x}{2} + \frac{9}{4}$

2. Simplify each of the following expressions and state:
   (a) The number of terms.
   (b) The coefficient of each term.
      (i) $16m + 3n - 9m - 9n + 3m$
      (ii) $0.7x + 2.3y - 4.4x + 3.6y$
      (iii) $\frac{1}{2}x + \frac{3}{2}y - \frac{3}{4}z + \frac{3}{2}x - \frac{1}{4}y + z$
      (iv) $4ab + 5ac - 7ad - 7ad + 3ac$

3. Simplify each of the following algebraic expressions:
   (a) $7m - 9n + 6m + 8n - 3m$
   (b) $5x + 9y + 7x - 8y - 3x$
   (c) $5k - 5n + 6k - 17k - 5$
   (d) $(3x - y) - 2(4x - 3y)$

4. Multiply each of the following algebraic expressions:
   (a) $5x - 7y$ by 4
   (b) $4a - 5c + 2$ by -5
   (c) $11p + 7q - 5$ by $2a$
   (d) $\frac{3}{2}m - \frac{5}{2}$ by $-6n$
   (e) $7x - 2y + 3z$ by 0.5

5. Find the answers to the following algebraic expressions:
   (a) $8ab \times 5mn \times 3$
   (b) $3n \times (-2m) \times 7$
   (c) $(-2x)\left(\frac{a}{2} - 3b\right)$

6. Complete each of the following statements:
   (a) $5ax - 2ay + 3az = a$ (.........)
   (b) $9ab - 15ac - 6a = -3a$ (.........)
   (c) $axy - bxy + cxy = xy$ (.........)
   (d) $25mn + 5mp - 5mq = 5m$ (.........)
   (e) $ax + bx + ay - by = x$ (.........) $+ y$ (.........)

7. Find the value of each of the following expressions when $m = 5$ and $n = 2$:
   (a) $6mn - 2mn + 4mn$
   (b) $\frac{m}{5} - \frac{mn}{2}$
   (c) $\frac{2m}{4} + \frac{5mn}{10}$

8. Express each of the following statements in an algebraic form:
   (a) The sum of a number m and three times another number n is 225.
   (b) The difference between 100 and another number n is m.
   (c) When the sum of x and y is multiplied by 3, the result is 49.
   (d) The product of twice a certain number x and three times a certain number y is 249.

9. Solve each of the following equations:
   (a) $5x - 7\frac{1}{2} = -\frac{3}{2}x$
   (b) $3 - 2x = 15$
   (c) $\frac{x}{5} - 3 = 3$
   (d) $\frac{3}{5}x - 3 = 3$
   (e) $\frac{7}{x} = 8$
   (f) $6x - 7 = 2x + 3$
   (g) $\frac{3}{2}x + \frac{x}{5} = \frac{5}{2}x - 5$
   (h) $5x - \frac{x-2}{3} = 5 + \frac{3x+1}{2}$

10. John is 5 centimetres taller than Jane, and the sum of their heights is 307 centimetres. Find their heights.

11. The age of James is one third the age of his father. If the sum of their ages is 66 years, find their ages.

12. If the four angles of a quadrilateral are given in degrees as $(4x - 20°)$, $(3x + 40°)$, $(3x + 50°)$, and $(x + 30°)$, find the value of x.

13. Solve each of the following simultaneous equations by using the substitution method:

(a) $\begin{cases} y - 2x = 0 \\ 2y + 3x = 21 \end{cases}$

(b) $\begin{cases} 2x - y = 0 \\ x + 2y = 11 \end{cases}$

(c) $\begin{cases} x - y = -4 \\ 5x - y = 4 \end{cases}$

(d) $\begin{cases} 2x - 3y = -1 \\ -2x + y = -13 \end{cases}$

14. Solve each of the following simultaneous equations by using the elimination method:

(a) $\begin{cases} 3x - 2y = 17 \\ x + 2y = 11 \end{cases}$

(b) $\begin{cases} 5x - 3y = 19 \\ 4x - 3y = 14 \end{cases}$

(c) $\begin{cases} 0.1x + 0.2y = 1.1 \\ 1.5x - 0.2y = 10.1 \end{cases}$

(d) $\begin{cases} x - 2y = -1 \\ 3x - 4y = 3 \end{cases}$

15. Solve each of the following pairs of simultaneous equations by using the combination method:

(a) $\begin{cases} 2x + y = 31 \\ 3x + 2y = 52 \end{cases}$

(b) $\begin{cases} 9(x - y) - (x + y) = 14 \\ (x + y) + 5(x - y) = 28 \end{cases}$

(c) $\begin{cases} 4y - 2x = 2 \\ 6y - x = 6 \end{cases}$

(d) $\begin{cases} \frac{x}{2} - \frac{y}{3} = -1 \\ \frac{x}{4} + \frac{y}{3} = \frac{5}{2} \end{cases}$

16. If the sum of the ages of a father and his daughter is 60 years, and the age of the father is five times the age of the daughter, find their ages.

17. Two numbers x and y are such that, the sum of $2x$ and y is 62, while the sum of x and $2y$ is 79. Find the numbers.

18. Two numbers are such that, one number is greater than the other. If 25 is subtracted from the greater number, the result is half the smaller number. When the two numbers are added, the result is 91. Find the two numbers.

19. Find the value of x in each of the following:

(a) $x - 2 > \frac{1}{2}$

(b) $3x + 1 \leq 4$

(c) $-5x - 2 > 3$

(d) $\frac{1}{2} - 4x \leq -\frac{1}{2}$

(e) $\frac{x}{3} - \frac{1}{4} \geq \frac{1}{8}$

(f) $10x + 3 < 2\frac{1}{2}x - 12$

20. Twice the sum of two numbers is at most 87. If one of the numbers is 13, what is the largest possible value of the second number?

21. Solve the following simultaneous equations by elimination and substitution methods.

$$\begin{cases} \frac{3}{2}x - 0.2y = 4 \\ -x + 3y = \frac{9}{2} \end{cases}$$

22. Formulate an algebraic equation basing on the following statement: In a class of 42 students, six of them came late, and the rest came early.

23. If 1 is subtracted from both the numerator and the denominator of a fraction, the result is $\frac{2}{5}$. If 2 is added to twice the numerator and 6 is added to thrice the denominator of the same fraction, the result is $\frac{1}{3}$. Find the fraction.

24. A certain school placed two orders with the stationery dealers. The first order was of 50 boxes of marker pens and 20 boxes of correction fluid for the total amount of 40 000 Tanzanian shillings. The second order was 10 boxes of marker pens and 20 boxes of correction fluid for the total amount of 24 000 Tanzanian shillings.
   (a) Formulate a system of simultaneous equations.
   (b) Calculate the cost of a box of marker pens and a box of correction fluid.

25. When Juma went to the National Park, he was interested with two kinds of wild animals only; zebras and elephants. If the total number of zebras and elephants he saw was 102, formulate the equation representing the word problem.

Project 7

Formulate two equations with one unknown and two equations with two unknowns. By using any method, solve the formulated problems. Give the interpretation of the obtained solutions.

Chapter Nine

Numbers II

Introduction

The concept of real numbers came from the generalization of the concept of rational numbers. A real number is a value of a continuous quantity that can represent a distance along a line or as an infinite decimal expansion. Rational numbers and irrational numbers make up the set of real numbers. In this chapter, you will learn about rational numbers, representation of rational numbers on a number line, basic operations on rational numbers, irrational numbers, real numbers, representation of real numbers on a number line, and absolute values of real numbers. The competencies developed will help you in dealing with various real life problems, where numbers are used.

Rational numbers

A rational number is any number which can be represented in the form of $\frac{a}{b}$, where a and b are both integers, such that b is not equal to zero (that is, $b \neq 0$). The condition $b \neq 0$ is essential because division by zero is not defined. The set of rational numbers is denoted by the symbol $\mathbb{Q}$.

Note:

Integers include positive whole numbers, negative whole numbers, and zero. For example; –3, –2, –1, 0, 1, 2, 3 is a list of integers from – 3 to +3.

The negative fractions, integers, zero, and positive fractions together form what is called rational numbers. Examples of rational numbers are:

$$\frac{-34}{3}, \frac{-20}{3}, \frac{-7}{4}, \frac{-6}{5}, \frac{1}{2}, \frac{0}{2}, \frac{1}{3}, \frac{4}{4}, \frac{9}{5}, \frac{100}{5}, \text{ and } \frac{30}{1}.$$

There are infinite (unterminating) rational numbers. From the definition of a rational number, all integers are rational numbers.

For example; $5 = \frac{5}{1}$, $-7 = \frac{-7}{1}$, and $0 = \frac{0}{1}$.

Also, all terminating decimals are rational numbers.

For example; $0.5 = \frac{5}{100}$, $-0.071 = \frac{-71}{1000}$, and $12.65 = \frac{1265}{100}$.

All repeating (recurring) decimals are rational numbers.

For example; $0.\dot{3} = \frac{1}{3}$, $0.\dot{2}\dot{3} = \frac{23}{99}$, and $1.56\dot{7} = \frac{1411}{900}$.

Representation of rational numbers on a number line

Activity 1

Individually or in groups, perform the following tasks:

1. Construct a 10 centimetre long number line.
2. Divide each unit interval into 10 equal units.
3. Prepare at least 10 cards of rational numbers of your choice.
4. Place the cards prepared in task 3 into a number line constructed in task 2.
5. Under the guidance of your teacher, discuss as a class what could be the best way of going through task 1 to task 4 for better results.

The representation of any positive rational number $\frac{a}{b}$ is done by first dividing the unit interval into 'b' equal parts. Then, the 'a' of these parts is taken along the number line to reach the point corresponding to $\frac{a}{b}$ to the right of zero if the number is positive, and to the left of zero if the number is negative.

Example 1

Represent $\frac{13}{5}$ and $-\frac{13}{5}$ on the same number line.

Solution

Change $\frac{13}{5}$ into a mixed number, that is,

$$\frac{13}{5} = 2\frac{3}{5}$$
$$= 2 + \frac{3}{5}.$$

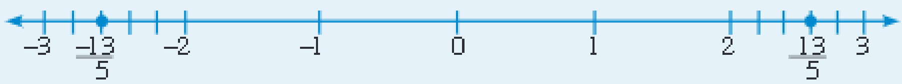
To draw a number line, take 2 units from 0 and then, divide the third unit into 5 equal parts. Take 3 parts out of the 5 parts to complete a part representing $\frac{13}{5}$.

Also, convert $-\frac{13}{5}$ into a mixed number, that is,

$$\frac{-13}{5} = -2\frac{3}{5}$$
$$= -\left(2 + \frac{3}{5}\right).$$

Repeat the same procedure from 0 to the negative side of the number line to show the representation of $-\frac{13}{5}$.

Therefore, the following number line is the representation of $\frac{13}{5}$ and $-\frac{13}{5}$.



Example 2

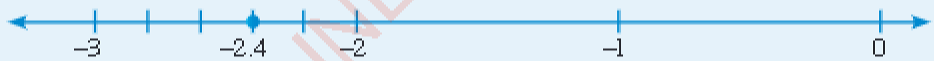
Represent –2.4 on a number line.

Solution

$-2.4 = -2\frac{4}{10} = -2\frac{2}{5}$.

To draw a number line, take 2 units from 0 to the left and then, divide the third unit into 5 equal parts. Take 2 parts out of the 5 parts to complete a part representing –2.4.

Therefore, the following number line is the representation of –2.4.



Exercise 1

1. Draw a number line, and then locate and label on it the points corresponding to the following rational numbers:

(a) 5	(c) $\frac{1}{4}$	(e) $\frac{3}{4}$	(g) 0
(b) $\frac{7}{3}$	(d) 4.5	(f) $\frac{3}{2}$	(h) $3\frac{3}{4}$

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2. Show the position of each of the following rational numbers on a number line:
   (a) $2\frac{2}{3}$ (c) $\frac{16}{8}$ (e) $\frac{25}{6}$ (g) $\frac{283}{283}$
   (b) $\frac{6}{1}$ (d) $\frac{0}{3}$ (f) $\frac{25}{50}$ (h) $\frac{50}{25}$

3. Draw a number line, and locate on it the points corresponding to the following rational numbers:
   (a) 3 (b) $-\frac{5}{2}$ (c) –3

4. (a) List all negative rational numbers and positive rational numbers from the following list of numbers: –5, 0, 12, –3.416520…, 2.85, 7.14, 12.64646464…, $\frac{11}{5}$, $\frac{-3}{5}$, $-2\frac{1}{3}$, 9.4123, $\frac{6}{0}$, $72\frac{1}{4}$, and 534.
   (b) Which numbers from (a) are not rational numbers and why?

5. Write each of the following rational numbers in the form of $\frac{a}{b}$ in its simplest form:
   (a) 0.04 (c) –27 (e) –0.03 (g) $-6\frac{1}{8}$
   (b) 4 (d) 5.6 (f) $4\frac{1}{4}$ (h) $-2\frac{2}{3}$

6. Represent each of the following rational numbers on a number line:
   (a) –4 (e) $-\frac{5}{3}$ (i) $-\frac{3}{2}, -\frac{5}{4}, -\frac{3}{4}, -\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{5}{4}, \frac{3}{4}$
   (b) –2.5 (f) –0.9, –0.8, –0.3, 0.2, and 0.7
   (c) 4 (g) $-\frac{3}{10}$
   (d) 1.8 (h) 1.2 and 1.6

Basic operations on rational numbers

Activity 2: Recoginsing basic operations on rational numbers

Individually or in groups, perform the following tasks:

1. Prepare a card, and write on it 4 rational numbers of your choice.
2. Create number sentences using the four numbers and any combination of addition, subtraction, multiplication, or division to get 18.

3. Share your work with other students, and check the correctness of your answers.
4. Repeat task 1 to 3 using four numbers different from those already used.
5. With the guidance of your teacher, make a concluding remark on the activity.

The basic operations, that is, addition, subtraction, multiplication, and division of rational numbers, are the same as those of fractions and decimals.

Addition and subtraction of rational numbers

Example 1

Find the sum of $-\frac{3}{5}$ and $\frac{1}{4}$.

Solution

Make their denominators equivalent using the LCM, which is 20.

$$-\frac{3}{5}+\frac{1}{4}=\frac{-12+5}{20}$$

$$=-\frac{7}{20}.$$

Therefore, the sum of $-\frac{3}{5}$ and $\frac{1}{4}$ is $-\frac{7}{20}$.

Example 2

Find the value of $2\frac{1}{3}-\frac{5}{9}$.

Solution

Convert $2\frac{1}{3}$ into an improper fraction, that is, $2\frac{1}{3}=\frac{7}{3}$.

Thus, $2\frac{1}{3}-\frac{5}{9}=\frac{7}{3}-\frac{5}{9}$.

Make their denominators equivalent using the LCM, which is 9.

$$\frac{7}{3}-\frac{5}{9}=\frac{21-5}{9}$$

$$=\frac{16}{9}$$

$$=1\frac{7}{9}.$$

Therefore, $2\frac{1}{3}-\frac{5}{9}=1\frac{7}{9}$.

Example 3

Compute 2.14 − 8.753.

Solution

$2.14 - 8.753 = -(-2.14 + 8.753)$

(taking the negative sign out of the brackets)

$= -(8.753 - 2.14)$ (interchanging the numbers)

$= -6.613.$

Therefore, $2.14 - 8.753 = -6.613$.

Example 4

Find the sum of 43.7 and 1.2.

Solution

$43.7 + 1.2 = 44.9$

Therefore, the sum of 43.7 and 1.2 is 44.9.

Note: For the rational number $-\frac{2}{5}$, it is always true that $-\frac{2}{5} = \frac{-2}{5} = \frac{2}{-5}$.

Generally, if a and b are positive integers, then, $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$.

Exercise 2

1. Calculate each of the following:

(a) $-\frac{1}{2} + \frac{1}{3} =$

(b) $-\frac{3}{4} + \left(-\frac{1}{2}\right) =$

(c) $\frac{-9}{16} + \frac{9}{16} =$

(d) $-\frac{1}{4} + \left(-\frac{2}{3}\right) =$

2. Work on each of the following:

(a) $\frac{1}{4} - \left(-\frac{2}{3}\right) =$

(b) $\frac{-9}{11} - \frac{3}{10} =$

(c) $-\frac{7}{8} - \left(-\frac{1}{6}\right) =$

(d) $\frac{56}{17} + \frac{2}{5} + 1\frac{3}{4} =$

(e) $1\frac{1}{5} - 3\frac{2}{5} =$

(f) $-\frac{5}{9} - \left(-\frac{2}{12}\right) =$

(g) $1\frac{1}{7} + 2\frac{2}{9} =$

(h) $\frac{1}{2} - \frac{3}{5} + \frac{1}{3} =$

3. Calculate each of the following:
   (a) $4.7 - 1.9$
   (b) $3.4 - 4.8$
   (c) $-3.5 - 2.1$
   (d) $1.7 - (-0.5)$
   (e) $-2.8 - (-4.2)$
   (f) $3.864 - (-21.33)$
   (g) $1.86 - 0.985$
   (h) $4.87 - (-14.86)$
4. Calculate each of the following:
   (a) $5.004 + 2.136$
   (b) $0.95 + 0.666$

Multiplication of rational numbers

Multiplication of rational numbers is similar to multiplication of fractions. The following are the steps for multiplying rational numbers:

Step 1: Multiply the numerators and the denominators.

Step 2: Simplify the resulting number to its lowest term.

Example 1

Find the product of $\frac{-5}{3}$ and $\frac{4}{7}$.

Solution

The product can be obtained as follows.

$$\frac{-5}{3} \times \frac{4}{7} = \frac{-5 \times 4}{3 \times 7}$$

$$= \frac{-20}{21}.$$

Therefore, the product of $\frac{-5}{3}$ and $\frac{4}{7}$ is $\frac{-20}{21}$.

Example 2

Find the product of $\frac{5}{3}$ and $\frac{4}{7}$.

Solution

$\frac{5}{3} \times \frac{4}{7} = \frac{5 \times 4}{3 \times 7}$

$= \frac{20}{21}.$

Therefore, the product of $\frac{5}{3}$ and $\frac{4}{7}$ is $\frac{20}{21}$.

Example 3

Find the product of $-\frac{5}{3}$ and $-\frac{4}{7}$.

Solution

$\frac{-5}{3} \times \frac{-4}{7} = \frac{-5 \times -4}{3 \times 7}$

$= \frac{20}{21}.$

The product of $-\frac{5}{3}$ and $-\frac{4}{7}$ is $\frac{20}{21}$.

Generally, if a and b are any two rational numbers, then:

(i) $a \times b = ab$.

(ii) $(-a) \times (-b) = ab$.

(iii) $(-a) \times b = a \times (-b) = -(a \times b)$.

Example 4

Find the value of $-\frac{4}{3} \times \frac{5}{8}$.

Solution

$-\frac{4}{3} \times \frac{5}{8} = -\frac{20}{24}$

$= -\frac{5}{6}.$

Therefore, $-\frac{4}{3} \times \frac{5}{8} = -\frac{5}{6}$.

Example 5

Find the value of $-3\frac{1}{2} \times \left(-4\frac{1}{5}\right)$.

Solution

Convert $-3\frac{1}{2}$ and $-4\frac{1}{5}$ into improper fractions, that is,

$$-3\frac{1}{2} \times \left(-4\frac{1}{5}\right) = -\frac{7}{2} \times -\frac{21}{5}$$
$$= \frac{147}{10}$$
$$= 14\frac{7}{10}.$$

Therefore, $-3\frac{1}{2} \times \left(-4\frac{1}{5}\right) = 14\frac{7}{10}$.

Example 6

Find the product of -6.24 and -0.33.

Solution

$-6.24 \times (-0.33) = 2.0592$.

Therefore, the product of -6.24 and -0.33 is 2.0592.

Exercise 3

Find the value of each of the following, and give your answers in the simplest form:

1. $\frac{7}{3} \times \frac{8}{5} \times \frac{1}{3} =$
2. $-0.074 \times (-0.75) =$
3. $\frac{4}{7} \times \left(-\frac{2}{3}\right) =$
4. $\left(-2\frac{1}{4}\right) \times \left(-3\frac{5}{9}\right) =$
5. $2\frac{1}{4} \times 1\frac{3}{5} =$
6. $0.0001 \times (-1) =$
7. $\frac{2}{9} \times \frac{-3}{10} \times \frac{-3}{4} =$
8. $\left(\frac{3}{8} \times \frac{5}{9}\right) \times \frac{-4}{5} =$
9. $\frac{9}{8} \times \frac{-6}{15} \times \frac{7}{81} =$
10. $2.125 \times (-4.25) =$

11. $3.8 \times (-0.5) =$

12. $-\frac{7}{8} \times \left(-2\frac{2}{3}\right) =$

13. $-\frac{6}{7} \times \left(-\frac{14}{15}\right) =$

14. $\frac{1}{4} \times \left(-2\frac{3}{4}\right) \times \frac{8}{3} =$

15. $-\frac{35}{6} \times \left(-\frac{10}{21}\right) =$

16. $-4.56 \times (-1.23) =$

17. $\left(\frac{3}{7} \times \frac{7}{8}\right) \times \frac{-2}{3} =$

18. $\frac{3}{9} \times \left(\frac{4}{9} \times \frac{5}{8}\right) =$

19. $-1.548 \times -3.132 =$

20. Determine whether the following statements are True or False:
    (a) The product of two negative rational numbers is a positive rational number.
    (b) The product of a negative rational number and zero is zero.
    (c) The product of zero and a negative rational number is a negative rational number.
    (d) The product of a negative rational number and a positive rational number is always a negative rational number.
    (e) If the product of two rational numbers is a positive rational number, then one of them is a positive rational number and the other is a negative rational number.
    (f) If the product of two rational numbers is a positive rational number, then both of them are positive rational numbers.

Division of rational numbers

Division of rational numbers follows the same procedures as division of whole numbers and fractions.

For any two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ where $q \neq 0, s \neq 0$, $\frac{p}{q}$ divided by $\frac{r}{s}$ is the same as $\frac{p}{q}$ multiplied by the reciprocal of $\frac{r}{s}$, where a reciprocal of a number is equal to one divided by that number, that is, $\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{1}{\frac{r}{s}} = \frac{p}{q} \times \frac{s}{r}$.

Example 1

Find the value of $\frac{4}{9} \div \left(-\frac{7}{6}\right)$.

Solution

$$\frac{4}{9} \div \left(-\frac{7}{6}\right) = \frac{4}{9} \times \left(-\frac{6}{7}\right)$$
$$= -\frac{24}{63}$$
$$= -\frac{8}{21}.$$

Therefore, $\frac{4}{9} \div \left(-\frac{7}{6}\right) = -\frac{8}{21}.$

Example 2

Find the value of $-20\frac{4}{5} \div \left(-18\frac{1}{3}\right)$.

Solution

Convert the mixed numbers into improper factions.

$$-20\frac{4}{5} \div \left(-18\frac{1}{3}\right) = -\frac{104}{5} \div -\frac{55}{3}$$
$$= -\frac{104}{5} \times -\frac{3}{55}$$
$$= \frac{312}{275} \text{ or } 1\frac{37}{275}.$$

Therefore, $-20\frac{4}{5} \div \left(-18\frac{1}{3}\right) = 1\frac{37}{275}.$

Example 3

Find the value of $-10 \div 0.001$.

Solution

Convert the decimal into a fraction, that is,

$0.001 = \frac{1}{1000}.$

Thus,

$$-10 \div 0.001 = -10 \div \frac{1}{1000}$$
$$= -10 \times 1\ 000$$
$$= -10\ 000.$$

Therefore, $-10 \div 0.001 = -10\ 000.$

Exercise 4

Compute each of the following, and express the answers in their lowest terms:

1. $-1 \div 1\frac{5}{12} =$
2. $0.209 \div 0.019 =$
3. $-\frac{18}{5} \div \frac{3}{10} =$
4. $\left(8\frac{1}{3} \div 4\frac{2}{3}\right) \times 1.5 =$
5. $\frac{7}{4} \div \left(-2\frac{1}{8}\right) =$
6. $(-0.52925 \div 0.145) \times 0.4 =$
7. $2\frac{3}{4} \div 3 =$
8. $\left(\frac{2}{7} \div \frac{18}{7}\right) \div \left(-\frac{21}{27}\right) =$
9. $9.4572 \div 0.0852 =$
10. $\frac{9}{14} \div \frac{6}{7} =$
11. $-\frac{4}{3} \div \left(-\frac{8}{7}\right) =$
12. $-\frac{17}{5} \div -\frac{34}{5} =$
13. $\left(-9\frac{5}{48} \div 9\frac{10}{96}\right) \times 1\frac{1}{2} =$
14. $\frac{7}{6} \div \left(-\frac{14}{9}\right) =$
15. $-15\frac{1}{2} \div \left(-\frac{7}{2}\right) =$
16. $-\frac{21}{32} \div \frac{63}{48} =$
17. $\left(\frac{41}{5} \div 20\right) \div -2\frac{1}{5} =$
18. $36.25 \div 2.5 =$
19. $27 \div \left(-\frac{15}{7}\right) =$
20. $8\ 000 \div (-2.5) =$

Irrational numbers

A number which can be expressed as non-terminating and non-repeating decimal is called an irrational number. Irrational numbers cannot be expressed in the form of $\frac{a}{b}$, where a and b are both integers and $b \neq 0$. It is not possible to locate irrational numbers on the number line. Irrational numbers can be approximated to rational numbers.

Example 1

Write 0.12 as a fraction.

Solution

The given number can be written in a form of $\frac{a}{b}$ as follows.

$$0.12 = \frac{12}{100}$$

$$= \frac{3}{25}.$$

Therefore, $0.12 = \frac{3}{25}$.

Note: Repeating decimals can also be written in the form of $\frac{a}{b}$.

Example 2

Write $0.\dot{1}\dot{5}$ as a fraction.

Solution

$0.\dot{1}\dot{5}$ can be written as a fraction as follows.

Let $x = 0.\dot{1}\dot{5}$ (1)

$100x = 15.\dot{1}\dot{5}$ (2)

Subtracting (1) from (2), we have,

$$99x = 15$$

$$x = \frac{15}{99}.$$

Therefore, $0.\dot{1}\dot{5} = \frac{5}{33}$.

Every terminating or repeating decimal represents a rational number. Similarly, every rational number can be represented as a terminating or repeating decimal. There are numbers which neither terminate nor repeat. For example:

(a) $\sqrt{2} = 1.4142135\ldots$

(b) $\sqrt{5} = 2.2360679\ldots$

(c) $\pi = 3.142\ldots$

(d) $\sqrt{7} = 2.6457513\ldots$

Example 3

Compare $\sqrt{31}$ and $\sqrt{71}$.

Solution

Since the numbers in the square roots are not perfect squares, then these numbers are irrational.

In order to compare them, square the number as follows.

$(\sqrt{31})^2 = \sqrt{31} \times \sqrt{31}$

$= 31.$

$(\sqrt{71})^2 = \sqrt{71} \times \sqrt{71}$

$= 71.$

Now, it is easier to compare 31 and 71. Since $31 < 71$, then $\sqrt{31} < \sqrt{71}$.

Therefore, $\sqrt{31} < \sqrt{71}$.

Example 4

Compare $\sqrt[3]{43}$ and $\sqrt[3]{61}$.

Solution

Since the numbers in the cube roots are not the perfect cubes, then these numbers are irrational.

In order to compare them, raise both numbers to power 3.

$\left(\sqrt[3]{43}\right)^3 = \sqrt{43} \times \sqrt{43} \times \sqrt{43}$

$= 43.$

$\left(\sqrt[3]{61}\right)^3 = \sqrt{61} \times \sqrt{61} \times \sqrt{61}$

$= 61.$

Now, it is easier to compare 43 and 61, that is, $43 < 61$. Since $43 < 61$, then $\sqrt[3]{43} < \sqrt[3]{61}$.

Therefore, $\sqrt[3]{43} < \sqrt[3]{61}$.

Note: π is usually approximated to a rational number $\frac{22}{7}$, but it is an irrational number.

Exercise 5

1. Find the rational number in the form of $\frac{a}{b}$ represented by each of the following decimals:
   (a) 0.875
   (b) 1.475
   (c) $0.4\dot{5}\dot{6}$
   (d) $19.\dot{1}27\dot{6}$
   (e) $0.\dot{7}\dot{8}$

2. Which of the following rational numbers are repeating decimals?
   (a) $\frac{1}{9}$ (b) $\frac{22}{7}$ (c) $\frac{2}{3}$ (d) $\frac{20}{11}$

3. Which of the following numbers are:
   (i) Whole numbers?
   (ii) Integers?
   (iii) Rational numbers?
   (iv) Irrational numbers?
   (a) $\sqrt{8}$
   (b) 1.17
   (c) $\frac{22}{7}$
   (d) 0
   (e) $\frac{-10}{2}$
   (f) $-\sqrt{47}$
   (g) -3
   (h) $\frac{3}{12}$
   (i) 77.62
   (j) $\frac{4}{9}$
   (k) $0.\dot{1}\dot{2}$
   (l) 56
   (m) 3
   (n) 0.0004
   (o) $\sqrt{11}$
   (p) -0.7
   (q) 1
   (r) π

4. Use the appropriate sign (>, <, or =) between each of the following pairs of numbers to make the statements true:
   (a) 4.16 ___ 416
   (b) $\frac{2}{5}$ ___ $\frac{200}{500}$
   (c) $\sqrt{4}$ ___ $\sqrt{2}$
   (d) $0.\dot{3}$ ___ $\frac{1}{3}$
   (e) $\sqrt{21}$ ___ $\sqrt{53}$
   (f) $\sqrt[3]{83}$ ___ $\sqrt[3]{97}$
   (g) $\sqrt{19}$ ___ $\sqrt{5}$
   (h) $\sqrt[3]{-83}$ ___ $\sqrt[3]{-47}$

Activity 3: Recoginising real numbers

Individually or in groups, perform the following tasks:

1. Write any 6 rational numbers.
2. Write any 4 irrational numbers.
3. What is the difference between those rational numbers written in task 1, and those irrational numbers written in task 2?

Real numbers are the numbers which include both rational numbers and irrational numbers. They can be both positive or negative and the set of real numbers is denoted by the symbol $\mathbb{R}$. All the natural numbers, integers, decimals, and fractions are real numbers. This means, any rational or irrational number is a real number. Also, for any two real numbers, a and b, only one of the following is true.

Either $a = b$ or $a < b$ or $a > b$. For example; if:

(i) $a = 3$ and $b = 7$, then, $a < b$ or $b > a$ that is, $3 < 7$ or $7 > 3$.

(ii) $a = \frac{12}{3}$ and $b = 4$, then, $a = b$ that is, $\frac{12}{3} = 4$.

(iii) $a = -3$ and $b = -5$, then, $a > b$ that is, $-3 > -5$ or $b < a$ that is $-5 < -3$.

(iv) $a = -4$ and $b = 2$, then, $a < b$, that is, $-4 < 2$.

Examples of positive real numbers are:
$1,\ 2, 3, \frac{2}{3}, \sqrt{7}$, 1.67 and 1.020020002…

Examples of negative real numbers are:
$-1, -2, -3, \frac{-3}{2}, -\sqrt{7}, -1.067, -1.020020002\ldots, -1.067, -1.020020002\ldots$

Note: 0 is also a real number. Every real number corresponds to a single point on the number line. Also, every point on the number line corresponds to a real number.

Representation of real numbers on a number line

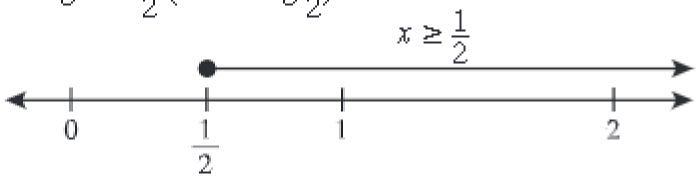
The steps for representing real numbers on a number line are as follows:

Step 1: Draw a horizontal line, and locate the point '0'. This point is known as the origin.

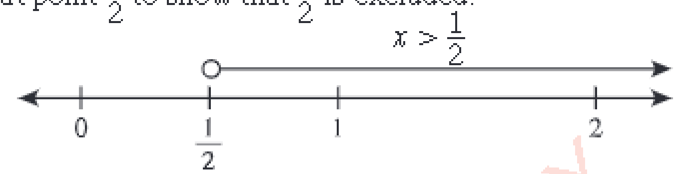
Step 2: If the given number is positive, mark it to the right side of the origin. If it is negative, mark it to the left side of the origin.

Step 3: Divide each unit into the values which are equal to the denominator of the fraction.

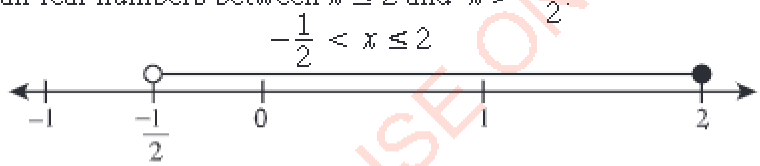
Consider the representation of all real numbers x such that $x \geq \frac{1}{2}$ on a number line. All points to the right of $\frac{1}{2}$ (including $\frac{1}{2}$) are considered and are shown as follows.



The solid dot at point $\frac{1}{2}$ appears because $\frac{1}{2}$ is included in the inequality $x \geq \frac{1}{2}$. When representing all real numbers for which $x > \frac{1}{2}$ on the number line, an open circle is used at point $\frac{1}{2}$ to show that $\frac{1}{2}$ is excluded.



Consider for example; the representation of all real numbers x such that $-\frac{1}{2} < x \leq 2$. These are all real numbers between $x \leq 2$ and $x > -\frac{1}{2}$.



Point $-\frac{1}{2}$ is excluded while point 2 is included.

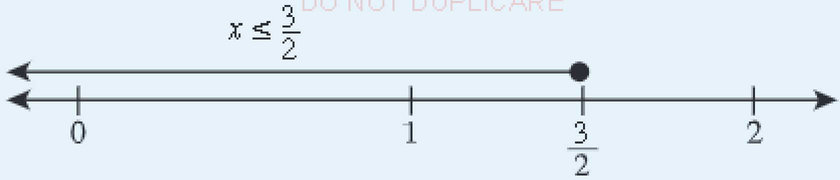
Example 1

Represent all real numbers x such that $x \leq \frac{3}{2}$ on a number line.

Solution

Draw a number line taking 1 unit from 0 and then, divide the second unit into two equal parts. Take 1 part out of the 2 parts to complete $\frac{3}{2}$, which is a required number to be represented.

Therefore, the following is the required number line representing $x \leq \frac{3}{2}$.

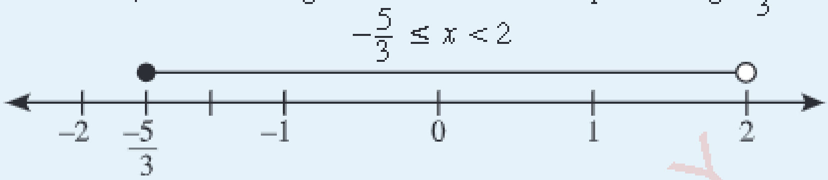


Example 2

Represent all real numbers x such that $-\frac{5}{3} \le x < 2$ on a number line.

Solution

Draw a number line taking 2 units from 0 on both sides and then, divide the second unit on the negative side into 3 equal parts. Take 2 parts out of the 3 parts to complete $-\frac{5}{3}$, which is the required part to be represented on this side. Therefore, the following is the number line representing $-\frac{5}{3} \le x < 2$.



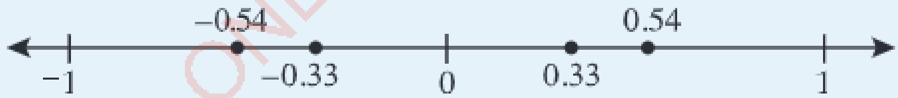
Example 3

Compare the following real numbers using a number line:

(a) 0.54 and 0.33 (b) −0.54 and −0.33

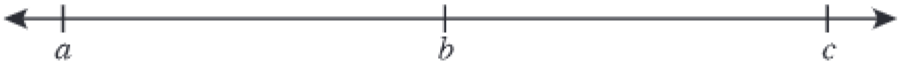
Solution

Locate the required numbers to be compared on the number line as follows.



(a) On the number line, 0.54 is on the right of 0.33. Therefore, $0.54 > 0.33$.
(b) On the number line, −0.54 is on the left of −0.33. Therefore, $-0.54 < -0.33$.

For any real numbers a, b, and c, if $a < b$ and $b < c$, then $a < c$. This can be shown on the following number line.



From the diagram, $a < b$ and $b < c$. Therefore, $a < c$.
For example; since $4.23223 < 5.12112$ and $5.12112 < 8.01001$, it follows that $4.23223 < 8.01001$.

Given any three real numbers, a, b and c and if $a < b$, then $a + c < b + c$.
For example; since $-8.13 < -4.72$, it follows that $-8.13 + 3 < -4.72 + 3$ or $-5.13 < -1.72$.

Also, the square roots of positive real numbers can be compared as shown:

(i) $\sqrt{3} > \sqrt{2}$

(ii) $\sqrt{5} > \sqrt{3}$

(iii) $-\sqrt{3} < -\sqrt{2}$

(iv) $-\sqrt{4} < -\sqrt{3}$

Exercise 6

1. Compare the following numbers using the symbols >, < or =.
   (a) 0.432 and 0.437
   (b) −0.127 and 0.001
   (c) 3.724 and 3.716
   (d) −0.129 and −0.128
   (e) π and 3.14
   (f) $-\sqrt{7}$ and $-\sqrt{5}$

2. Use the symbol >, < or =, to compare each of the following pairs of real numbers:
   (a) $-\frac{3}{8}$ and -0.375
   (b) 0.273 and 0.273
   (c) $\sqrt{2}$ and $-\sqrt{2}$
   (d) $\frac{3}{11}$ and -0.222

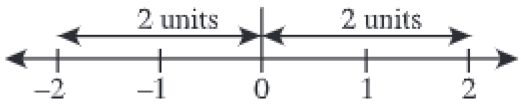
3. Represent each of the following inequalities on a number line:
   (a) $x \geq 0$
   (b) $x > -\frac{2}{5}$
   (c) $x \leq \frac{5}{2}$

4. Represent each of the following real numbers x on a number line, such that:
   (a) $-\frac{5}{2} < x < -\frac{1}{2}$
   (b) $\frac{7}{2} > x \geq \frac{1}{2}$
   (c) $-\frac{2}{3} < x \leq 4$
   (d) $-2\frac{1}{3} < x < \frac{5}{2}$
   (e) $2 < x < 5\frac{1}{4}$
   (f) $-3 \leq x \leq 4$

Absolute value of a real number

The absolute value or modulus of a real number x, denoted by $|x|$, is the non-negative value of x regardless of its sign.

Consider the following number line.



The distance from 0 to –2 and that from 0 to 2 is the same, that is, 2 units. Both 2 and –2 have the absolute value of 2. The absolute value of a number is the magnitude of that number regardless of its sign.

The absolute value of any number x is written as $|x|$ such that, $|x| = x$ if $x \geq 0$ and $|x| = -x$ if $x < 0$, and its distance is represented on a number line from 0. This distance is always positive or zero.

For example;
$|0.75| = 0.75$, $|-0.143| = 0.143$, $\left|-\frac{1}{2}\right| = \frac{1}{2}$, $|0| = 0$, and $|-2| = 2$.
In general, for any two opposite numbers x and $-x$, $|x| = |-x|$.

The following are the steps for solving absolute valued equations:

Step 1: Take the given equation out of the absolute value and write the ± sign to the left of it.

Step 2: Obtain two equations from the given equation by setting the given equation as positive on one side and as negative on the other side.

Step 3: Solve the two equations for the unknown.

Example 1

Find the values of x if $|x| = 4$.

Solution

$|x| = 4$

$\pm x = 4$ (Taking the given equation out of the absolute value sign by indicating the sign $\pm$)

Either, $+x = 4$ or $-x = 4$ (Setting the equation in the absolute sign as a positive equation on one side, and as a negative equation on the other side)

$x = 4$ or $x = -4$ (Solving the two equations for the unknown)

Therefore, $x = 4$ or $x = -4$.

Example 2

Solve for x if $|6 - x| = 1$.

Solution

$|6 - x| = 1$

$\pm(6 - x) = 1$

Either, $+(6 - x) = 1$ or $-(6 - x) = 1$

$6 - x = 1$ or $-6 + x = 1$

$-x = 1 - 6$ or $x = 1 + 6$

$-x = -5$ or $x = 7$.

Therefore, $x = 5$ or $x = 7$.

Example 3

Solve for x if $|x + 2| = 2$, and show the solution on a number line.

Solution

$|x + 2| = 2$

$\pm(x + 2) = 2$

Either $+(x + 2) = 2$ or $-(x + 2) = 2$

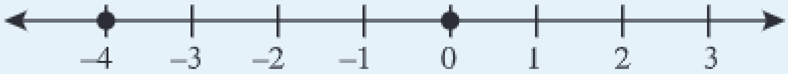
$x + 2 = 2$ or $-x - 2 = 2$

$x = 2 - 2$ or $-x = 2 + 2$

$x = 0$ or $-x = 4$.

Therefore, $x = 0$ or $x = -4$.

$x = 0$ and $x = -4$ are represented as shown on the following number line.



Example 4

Indicate $|x| \leq 2$ on a number line.

Solution

$$|x| \leq 2$$

$$\pm(x) \leq 2$$

Either $+(x) \leq 2$ or $-(x) \leq 2$

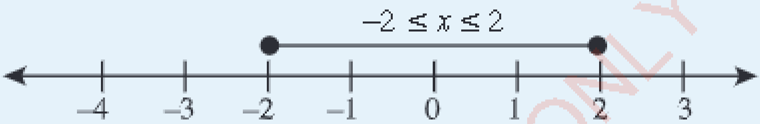
$$x \leq 2 \text{ or } -x \leq 2$$

$$x \leq 2 \text{ or } x \geq -2.$$

(Division by a negative number changes the inequality symbol)

Hence, $-2 \leq x \leq 2$.

Therefore, the representation of this solution is shown on the following number line.



Example 5

Find the solution of $|2x+1| > 3$, and show it on a number line.

Solution

$$|2x+1| > 3$$

$$\pm(2x+1) > 3$$

Either $+(2x+1) > 3$ or $-(2x+1) > 3$

$$2x + 1 > 3 \text{ or } -2x - 1 > 3$$

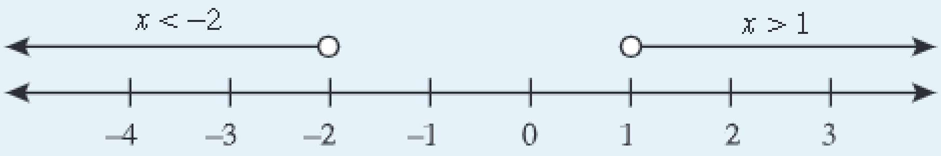
$$2x > 3 - 1 \text{ or } -2x > 3 + 1$$

$$\frac{2x}{2} > \frac{2}{2} \text{ or } \frac{-2x}{-2} < \frac{4}{-2}$$

(Division by a negative number changes the inequality symbol)

Therefore, $x > 1$ or $x < -2$.

The required representation of this solution is as shown on the following number line.



Example 6

What is the absolute value of $-\frac{4}{5} + \frac{13}{10}$?

Solution

Given $-\frac{4}{5} + \frac{13}{10}$, then

$$\left|-\frac{4}{5} + \frac{13}{10}\right| = \left|\frac{-8+13}{10}\right|$$

$$= \left|\frac{5}{10}\right|$$

$$= \frac{1}{2}.$$

Therefore, the absolute value of $-\frac{4}{5} + \frac{13}{10}$ is $\frac{1}{2}$.

Note:

If you divide or multiply both sides of the inequality by a negative number, the inequality symbol changes its direction.

Exercise 7

1. Find the absolute value of each of the following:
   (a) 62
   (b) $\sqrt{2}$
   (c) -8
   (d) $-\frac{4}{7}$
   (e) -12.5
   (f) $-pq$

2. What is the absolute value of each of the following?
   (a) $\frac{4}{3} - \frac{3}{2}$
   (b) $-3.68 + 3.89$
   (c) $\frac{3}{2} - \frac{4}{3}$
   (d) $-\frac{1}{3} \times 11 + \frac{10}{3}$
   (e) $0.111 - 0.889$
   (f) $-0.111 - (-0.01001)$

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3. Find the solution for each of the following equations:
   (a) $|x| = 3$
   (b) $|2-x| = 5$
   (c) $|2x| = 6$
   (d) $|10 - x| = 1$
   (e) $|6 - x| = 3$
   (f) $\left|x - \frac{1}{3}\right| = \frac{5}{3}$

4. Show the solution of each of the following equations on a number line:
   (a) $|x - 5| = 3$
   (b) $|-2 - x| = 4$
   (c) $|3 - x| = \frac{10}{3}$
   (d) $\left|\frac{1}{2}x + 2\right| = 6$

5. Find the solution for each of the following inequalities:
   (a) $|x| > 2$
   (b) $|6 - x| < 1$
   (c) $|x| \leq 3$
   (d) $|-2 - x| \leq 1$
   (e) $|x| < \frac{5}{2}$
   (f) $|x + 2| > 2$

6. Show the solution of each of the following on a number line:
   (a) $|x - 4| \leq 5$
   (b) $|7 - x| < 0$
   (c) $|x| > 2.5$
   (d) $\left|\frac{1}{2} - x\right| > 3$
   (e) $\left|-\frac{2}{3}x - 4\right| < 2$
   (f) $|4x - 2| \geq 8$
   (g) $2|x| = 6$
   (h) $3|x - 3| = 4$

Chapter summary

1. A rational number is any number written in the form of $\frac{a}{b}$, where both a and b are integers, except that, $b \neq 0$.
2. Every terminating or repeating decimal represents a rational number.
3. A number which neither terminates nor repeats is an irrational number.
4. An irrational number cannot be written in the form of $\frac{a}{b}$, where a and b are integers, but $b \neq 0$.
5. Rational and irrational numbers together form the set of real numbers.
6. For any two real numbers a and b, only one of the following is true; Either $a = b$ or $a < b$ or $a > b$.
7. The absolute value of a number is its value, regardless of its sign.
8. For all problems involving inequalities, multiplying or dividing by a negative number changes the inequality symbol.

Revision exercise

1. Write each of the following rational numbers in the form of $\frac{a}{b}$.

(a) $6\frac{1}{2}$
(b) $-9\frac{3}{11}$
(c) $0.\dot{8}\dot{1}$
(d) 193.5
(e) 23
(f) $245\frac{2}{9}$
(g) -4.3
(h) $114.9\dot{7}$
(i) 0.012
(j) $0.\dot{1}2\dot{3}$

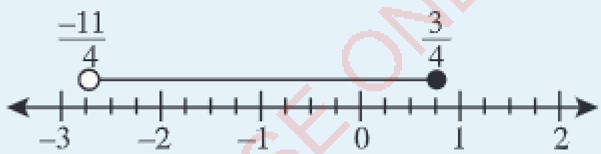
2. List twelve rational numbers which lie between:

(a) -1 and 0
(b) 4 and 5
(c) -3 and -4

3. Represent all real numbers x such that $x < \frac{5}{3}$, on a number line.

4. Represent $-4 \leq x < 4$ on a number line for all real numbers x.

5. Write an inequality for real numbers x represented on the following number line:



6. Find the value of each of the following expressions:

(a) $\frac{3}{8} - \frac{1}{16} + \frac{5}{4}$
(b) $1\frac{2}{3} + 2\frac{3}{4} + 3\frac{4}{5}$
(c) $-\frac{17}{2} + \frac{1}{10} - \frac{3}{10}$
(d) $62.15 - 88.765 + 45.678$
(e) $2\frac{1}{3} + 4\frac{1}{5} + \frac{1}{2}$
(f) $\left(7\frac{5}{8} + 3\frac{1}{4}\right) - \left(2\frac{2}{5} + 4\frac{1}{2}\right)$

7. Find the value of each of the following expressions:

(a) $\frac{2}{3} \times 3\frac{9}{16}$

(b) $2\frac{1}{2} \times \frac{7}{8} \div \left(-3\frac{3}{4}\right)$

(c) $-\frac{4}{5} \div \frac{33}{25}$

(d) $6\frac{1}{4} + \left(4\frac{1}{2} \div 2\right)$

(e) $-\frac{4}{5} \times \left(\frac{3}{7} \times 4\frac{7}{10}\right)$

(f) $5\frac{1}{2} - \left(-7 \times \frac{3}{11}\right)$

(g) $\dfrac{\frac{1}{2} + \frac{1}{3}}{\frac{2}{3} + \frac{4}{5} - 1\frac{1}{4}}$

(h) $\dfrac{6\frac{2}{3} \times 2\frac{4}{5} - 3}{\left(2\frac{2}{3} \times 2\frac{4}{5}\right) - 3}$

(i) $-\frac{16}{5} \times \left(-\frac{3}{8} \times -\frac{2}{9}\right)$

8. Which of the following numbers are:

(a) Rational? (b) Irrational? (c) Real?

$0.1234\ldots, \frac{22}{7}, -\sqrt{5}, 10, \frac{1}{3}, 0, -\frac{1}{4}, \pi, \sqrt{11}, -1984, 8.01001\ldots, 2.16.$

9. Use the symbol >, <, or = to compare each of the following pairs of numbers:

(a) $\frac{22}{7}$ and 3.14

(b) $-\sqrt{5}$ and $-\sqrt{6}$

(c) 1.01001 and 1.01001…

(d) 6.012 and 4(1.503)

(e) 2 and –2.3

(f) 0 and –0.9898

10. Represent the solution of each of the following on a number line:

(a) $|2 - x| = 1$

(b) $|2x - 5| = 3$

(c) $|x - 5| = 5$

(d) $|2 - x| \leq 2$

(e) $2|x| = 6$

(f) $|x + 5| \geq 7$

11. Show the solution of each of the following on a number line:

(a) $|-3 + x| = 0$

(b) $|x - 4| = 5$

(c) $\left|x - \frac{1}{2}\right| > 4$

12. In a certain school, the number of girls is two thirds of the number of boys. How many boys are there if the school has a total of 2 355 students?

13. Akbar deposited $\frac{7}{24}$ of his salary in his savings bank account, $\frac{1}{8}$ of his salary was spent on house rent, $\frac{1}{3}$ of the same salary was spent on food, and he remained with 60 000 Tanzanian shillings. Find his salary.

14. Twenty-five pieces of wire, each $14\frac{1}{5}$ centimetres long, were cut from a reel whose length was 500 centimetres long. What was the length of the remaining wire after cutting the pieces?

15. Show on a number line the values which are less than half the sum of $-\frac{3}{8}$ and $\frac{3}{8}$ by 12.

16. A woman spent a total of 53 750 Tanzanian shillings to buy $3\frac{1}{2}$ kilograms of onions at 2 000 Tanzanian shillings per kilogram, $6\frac{1}{2}$ kilograms of tomatoes at 1 500 Tanzanian shillings per kilogram, 3 litres of cooking oil at 3 000 Tanzanian shillings per litre, and 12 litres of paraffin. Find the price of paraffin per litre.

17. In a certain year, a school harvested 182 bags of maize from its farm. If $\frac{4}{13}$ of the harvest was reserved for the students, calculate the amount of money obtained by the school after selling the remaining amount of maize at 6 750 Tanzanian shillings per bag.

18. If $\frac{9}{24}$ of the candidates who sat for the National Form Four Basic Mathematics Examination at a certain school got at least grade C, how many candidates got below grade C if there were 288 candidates?

Project 8

Construct a table that includes some natural numbers, integers, whole numbers, rational numbers, irrational numbers, and two pairs of numbers each with the same absolute value. Give out your views on all the numbers in the table.

Chapter Ten

Ratios, profit, and loss

Introduction

Ratios are used in calculating proportions, rates, percentages, and dividing the profit or amount between two or more objects. Profit can be defined as a positive gain from an investment or business operation after subtracting all the expenses. It is a surplus remaining after deducting the total cost from the total revenue. Loss is a negative gain from an investment or business operation after subtracting all the expenses. Profit and loss projection can serve the goal of planning for a new business or for an ultimate goal of controlling operations. In this chapter, you will learn about ratios, proportions, profit made, loss made, percentage profit, percentage loss, and simple interest. The competencies developed will help you in daily life situations such as managing business, assets, revenues and expenditures, as well as predicting profits or losses that may arise in business and investment among other applications.

Ratios

Activity 1: The concept of ratio

Individually or in groups, perform the following task:
Shop A sells 5 kilograms of rice for 10 000 Tanzanian shillings and shop B sells 4 kilograms of rice for 7 200 Tanzanian shillings.

(a) Which is a better value for money?
(b) Are the rates the same? Why?
(c) What will 7 kilograms of rice cost in shop A?

Share your results with your fellow students through discussion.

A ratio is the relationship of two or more numbers or quantities expressed in the same units by division. It is a quantitative relationship between two or more amounts showing the number of times one value contains the other or is contained within the other. Ratios are sometimes treated as fractions. For example; Jaha and Siwema shared 40 000 Tanzanian shillings. If Jaha received 15 000 Tanzanian shillings and Siwema got 25 000 Tanzanian shillings, find the ratio of the amounts they received.

The ratio is 15 000 to 25 000 or 15 000 ÷ 25 000 = $\frac{15\ 000}{25\ 000} = \frac{3}{5}$.

Therefore, the ratio of 15 000 to 25 000 is 3 to 5 and is usually written as 3 : 5.

Similary, the ratio of 30 to 135 is the same as $\frac{30}{135}$ or $\frac{2}{9}$, written as 2 : 9, with 2 and 9 being the terms of the ratio.

In general, a ratio between the numbers p and q is written as $p : q$, which is equivalent to $p \div q$ *or* $\frac{p}{q}$. Thus, a ratio is also written as a fraction. It should be noted that multiplying or dividing both terms of a ratio by the same number does not change the ratio. In other words, 2 : 4 = 4 : 8 (multiplying both terms of the ratio by 2). To reduce a ratio to its lowest terms, express the ratio as a fraction and reduce the fraction to its lowest term.

Example 1

Simplify $4\frac{1}{10} : 12\frac{3}{10}$.

Solution

$$4\frac{1}{10} \div 12\frac{3}{10} = \frac{41}{10} \div \frac{123}{10}$$

$$= \frac{41 \times 10}{10 \times 123}$$

$$= \frac{1}{3}.$$

Therefore, $4\frac{1}{10} : 12\frac{3}{10} = 1:3$.

Example 2

A man's monthly income is 1 274 000 Tanzanian shillings. He spends 1 078 000 Tanzanian shillings every month. Find the ratio of his:

(a) Income to expenditure. (b) Savings to income.

Solution

Given;

Income = Tsh 1 274 000

Expenditure = Tsh 1 078 000

Hence,

Savings = Tsh (1 274 000 – 1 078 000)

= Tsh 196 000.

(a) Income : Expenditure $= \frac{1\ 274\ 000}{1\ 078\ 000}$

= 13 : 11.

Therefore, Income : Expenditure = 13 : 11.

(b) Savings : Income $= \frac{196\ 000}{1\ 274\ 000}$

= 2 : 13.

Therefore, Savings : Income = 2 : 13.

Example 3

A mathematics club has 21 members of which 13 are females and the rest are males. What is the ratio of males to all club members?

Solution

Number of females = 13

Total number of club members = 21

Number of males = Total number of club members – Number of females

= 21 – 13

= 8.

Therefore, the ratio of males to all club members = 8 : 21.

Example 4

Express the ratio 200 cm : 1 m in its lowest form.

Solution

Using comparison of metric units

1 m = 100 cm.

Thus, 200 cm : 1 m

= 200 cm : 100 cm

= 2 : 1.

Therefore, the ratio 200 cm : 1 m in its lowest form is 2 : 1.

Example 5

A special cereal mixture contains rice, wheat, and corn in the ratio 2 : 3 : 5, respectively. If a bag of the mixture contains 3 kilograms of rice, how much corn does it contain?

Solution

Let x = amount of corn in kg

The items in the ratio as a fraction is $\frac{\text{rice}}{\text{corn}} = \frac{2}{5} = \frac{3}{x}$

Thus, $\frac{2}{5} = \frac{3}{x}$

$2x = 3 \times 5$

$x = 7.5.$

Therefore, the mixture contains 7.5 kilograms of corn.

Alternatively

Let x be the amount of the mixture in kg.

Then $\frac{2}{10}x = 3$ kg

$2x = 10 \times 3$ kg

$2x = 30$ kg

$\frac{2x}{2} = \frac{30}{2}$ kg

$x = 15$ kg.

Thus, the amount of corn in the mixture is $\frac{5}{10} \times 15$ kg = 7.5 kg.

Therefore, the mixture contains 7.5 kilograms of corn.

Example 6

A clothing store sells T-shirts in only three colours: red, blue, and green. The colours are in the ratio of 3 to 4 to 5, respectively. If the store has 20 blue T-shirts, how many T-shirts are there in the store?

Solution

Let x be the number of red T-shirts and

y be the number of green T-shirts.

Writing the items in ratios as fraction is:

$$\frac{\text{red}}{\text{blue}} = \frac{3}{4} = \frac{x}{20} \quad (1)$$

$$\frac{\text{green}}{\text{blue}} = \frac{5}{4} = \frac{y}{20} \quad (2)$$

Solving equations (1) and (2) for x and y:

From equation (1), we have,

$$\frac{3}{4} = \frac{x}{20}$$

$$4x = 3 \times 20$$

$$\frac{4x}{4} = \frac{60}{4}$$

$$x = 15.$$

From equation (2), we have,

$$\frac{5}{4} = \frac{y}{20}$$

$$4y = 5 \times 20$$

$$\frac{4y}{4} = \frac{100}{4}$$

$$y = 25.$$

Thus, the total number of T-shirts is $15 + 20 + 25 = 60$.

Therefore, the store has 60 T-shirts.

Exercise 1

Express each of the following ratios in its lowest term:

1. 8 : 12
2. 40 : 50
3. 100 : 50
4. 77 : 35
5. 48 : 36
6. 160 : 96
7. 144 : 81
8. $3\frac{1}{2} : 1\frac{1}{4}$
9. $14 : \frac{1}{2}$
10. $1\frac{1}{2} : 2\frac{1}{2}$
11. $\frac{1}{4} : \frac{1}{7}$
12. $\frac{2}{5} : 1\frac{3}{5}$
13. 0.17 : 1
14. 3.5 : 0.07
15. 7 : 0.007

16. 120 : 105
17. Tsh 136 to Tsh 4
18. 50s to 90s
19. 0.75 km to 0.5 km
20. 120 km to 105 km

Complete the following ratios:

21. 25 : ____ = 5 : 6
22. 15 : 20 = 18 : _____
23. 24 : 8 = ____ : 7
24. $\frac{30}{6} = \frac{}{9}$
25. $\frac{4}{7} : \frac{20}{}$
26. 4 : 9 = ___ : 63
27. 12 : ___ = 3 : 7
28. ___ : 8 = 21 : 24
29. A herd of 52 horses consists of 12 white horses and black horses. What is the ratio of white horses to black horses?
30. The ratio of girls to boys in a swimming club was 2 : 4. If 14 members were girls, how many members were there?
31. If 4A = 5B = 6C, find the ratio A : B : C.
32. Find the ratio A : B : C, when:
    (a) A : B = 3 : 5 and A : C = 6 : 7
    (b) B : C = $\frac{1}{2} : \frac{1}{6}$ and A : B = $\frac{1}{3} : \frac{1}{5}$
33. Divide 37 000 Tanzanian shillings into three portions such that the second portion is $\frac{1}{4}$ of the third portion and the ratio between the first and the third portion is 3 : 5. Find each portion.

Proportions

Activity 2: Recognising proportions

Individually or in groups, perform the following task:

Ashura shared a box of chocolates with her brother Amir. She said "2 for you and 3 for me" as she divided them out in the ratio 2 : 3. She continued until all the chocolates had been divided up. When she had finished, she said to Amir, "Ok, you got $\frac{2}{3}$ of the chocolates, because I divided them in the ratio 2 : 3".

(a) Was Ashura correct?

(b) Is there any difference between 2 : 3 and $\frac{2}{3}$?

Share your results with your fellow students through presentation

From Activity 2 above, the ratio 2 : 3 compares part to part, and so Amir got $\frac{2}{3}$ of Ashura's chocolates rather than $\frac{2}{3}$ of the total. In fact, the chocolates were divided into 5 equal portions of which Amir received two and Ashura three. So, Amir's number of chocolates was only $\frac{2}{5}$ of the total.

A proportion is a part, a share, or a number considered in comparison to a whole. It is a statement that two ratios are equal. It can be written in two ways:

(a) As two equal fractions; $\frac{a}{b} = \frac{c}{d}$, or

(b) Using colons; $a : b = c : d$.

When two ratios are equal, then the cross multiplication of their ratios are equal, that is, $a : b = c : d$, which gives $a \times d = b \times c$.

When three numbers a, b, and c are in continued proportion, then c is called the third proportional. The third proportional of two numbers a and b is defined as:

$a : b : : b : c$, that is, $\frac{a}{b} = \frac{b}{c}$.

Example 1

Find the proportional parts of 156 in the ratio 3 : 4 : 5.

Solution

Given the total number of parts = 156, ratio = 3 : 4 : 5.

The sum of the terms of the ratio is $3 + 4 + 5 = 12$.

The required proportional parts are:

$$\frac{3}{12} \text{ of } 156 = \frac{3}{12} \times 156 = 39,$$

$$\frac{4}{12} \text{ of } 156 = \frac{4}{12} \times 156 = 52,$$

and $\frac{5}{12}$ of $156 = \frac{5}{12} \times 156 = 65$.

Therefore, the required proportional parts are 39, 52, and 65.

Example 2

The ratio of the number of girls to that of boys in a school is 7 : 12. If the number of boys in the school is 1 380, find

(a) the number of girls in the school.

(b) the total number of students in the school.

Solution

Given

Number of girls : Number of boys = 7 : 12

Number of boys = 1 380

(a) To find the number of girls in the school, let the number of girls be x.

Then,

$$7 : 12 = x : 1\,380$$

$$\frac{7}{12} = \frac{x}{1\,380}$$

$$12x = 7 \times 1\,380$$

$$x = \frac{7 \times 1\,380}{12}$$

$$= 805.$$

Therefore, the number of girls in the school is 805.

(b) The total number of students = Number of girls + Number of boys

$$= 805 + 1\,380$$

$$= 2\,185.$$

Therefore, the total number of students in the school is 2 185.

Example 3

The first, second, and third terms of a proportion are 42, 36, and 35. Find the fourth term.

Solution

Let the fourth term be x.

Thus, 42, 36, 35, x are in proportion.

Product of extreme terms = $42 \times x$.

Product of mean terms = 36×35.

Since the terms make up a proportion,

then, $42x = 36 \times 35$

$$x = \frac{36 \times 35}{42}$$

$$x = 6 \times 5$$

$$x = 30.$$

Therefore, the fourth term of the proportion is 30.

Exercise 2

1. Divide the following in the given ratios:
   (a) 100 in the ratio 7 : 3
   (b) 75 in the ratio 3 : 2
   (c) 16 in the ratio 3 : 3 : 2
   (d) $\frac{1}{2}$ in the ratio 4 : 1
2. If 800 kilograms of rice are shared between two families in the ratio 3 : 2, how much does each family get?
3. Divide 60 000 Tanzanian shillings among Juma, Ali, and John in the ratio 5 : 3 : 2.
4. Juma, Ali, Mary and Kato have 300, 100, 500, and 600 shares in a cooperative shop, respectively. Divide 150 000 Tanzanian shillings among them in the ratio of their shares.
5. Magnesium combines with oxygen in the ratio 3 : 2 by mass to form a new substance. What mass of magnesium would be needed to combine with 1.4 kg of oxygen?
6. Divide 28.6 kg of meat among four families in the ratio 4 : 5 : 6 : 7.
7. A powdery mixture is made up of powders A and B in the ratio 5 : 4. If 72 kg of this mixture are required, how much of each type should be used?
8. An alloy is made up of metals x and y in the ratio 2.4 : 1 by mass. How much mass of x is required if 8 kg of y is used to make the alloy?
9. If the ratio of lead to tin in a solder is 1.8 : 1, how much mass of tin is needed when 90 kg of lead is used?
10. A line segment AB, which is 24 cm long, is divided at a point P (between A and B) in the ratio 7 : 5. Find the lengths of $\overline{AP}$ and $\overline{PB}$.
11. The volumes of two tanks are in the ratio 2 : 5. If the volume of the first tank is 2 600 litres, find the total volume of the two tanks.

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12. A real horse is 1.8 m high. The statue of the horse is 3 m high. What is the ratio of the height of the horse to the height of the statue?
13. Jason has 500, 200, and 100 Tanzanian shilling coins. The ratio of the total amount of these coins is 2 : 3 : 5, and the total amount is 780 000 Tanzanian shillings. Find the numbers of the coins of each kind.
14. Find the fourth proportional of 6, 9, and 12.
15. Find the third proportional of $2n^2$ and $3n^2$.

Profit and loss

Activity 3 : Recognising profit and loss

Individually or in groups, perform the following task:
Juma and Salome went to an electronic shop and bought the following goods: Juma bought a radio for 24 000/= and Salome bought a television set for 132 000/=. After 6 years, Juma and Salome decided to sell those goods. Juma sold his radio for 34 000/=, and Salome sold the television set for 90 000/=. What is the difference of the buying price and selling price for each item? Did Juma get a loss or a profit? Did Salome get a profit or a loss?

Businesses involve buying and selling commodities. Business people are mostly interested in knowing the difference between the buying price and the selling price of the items. When the selling price is higher than the buying price, then the difference obtained is called a profit. When the selling price is lower than the buying price, then the difference obtained is called a loss.

Profit = Selling Price – Buying Price

Loss = Buying Price – Selling price

The profit made can be expressed in percentage profit as,

Percentage profit = $\frac{\text{Profit made} \times 100\%}{\text{Buying price}}$ and the loss made can also be expressed in a percentage loss as,

Percentage loss = $\frac{\text{Loss made} \times 100\%}{\text{Buying price}}$.

Example 1

A house was sold at a profit of 900 000 Tanzanian shillings. If the percentage profit was $37\frac{1}{2}\%$, what was the buying price of the house?

Solution

Given;

Profit made = Tsh 900 000, percentage profit = $37\frac{1}{2}\%$

$$\text{Percentage profit} = \frac{\text{Profit made} \times 100\%}{\text{Buying price}}$$

$$37\frac{1}{2}\% = \frac{\text{Tsh } 900\,000 \times 100\%}{\text{Buying price}}$$

$$\text{or } 37\frac{1}{2}\% \times \text{Buying price} = \text{Tsh } 900\,000 \times 100\%$$

$$\text{Buying price} = \frac{\text{Tsh } 900\,000 \times 100 \times 2}{75}$$

$$= \text{Tsh } 2\,400\,000.$$

Therefore, the buying price of the house was 2 400 000 Tanzanian shillings.

Example 2

A damaged chair that costs Tsh 11 000 was sold at a loss of 10%.

Find: (a) The loss made (b) The selling price.

Solution

Given;

Cost (buying price) = Tsh 11 000, percentage loss = 10%

(a) $$\text{Percentage loss} = \frac{\text{Loss made} \times 100\%}{\text{Buying price}}$$

$$10\% = \frac{\text{Loss made} \times 100\%}{\text{Tsh } 11\,000}$$

$$10\% \times \text{Tsh } 11\,000 = \text{Loss made} \times 100\%$$

$$\text{or Loss made} \times 100 = \text{Tsh } 11\,000 \times 10$$

$$\text{Loss made} = \frac{\text{Tsh } 11\,000 \times 10}{100}$$

$$= \text{Tsh } 1\,100.$$

Therefore, the loss made was 1 100 Tanzanian shillings.

(b) Buying price – Selling price = Loss made.

Tsh 11 000 – Selling price = Tsh 1 100

Selling price = Tsh (11 000 – 1 100)

= Tsh 9 900.

Therefore, the selling price of the chair was 9 900 Tanzanian shillings.

Alternatively

Getting a loss of 10%, means that, the selling price is (100% –10%) of the buying price.

Buying price = 90% of Tsh 11 000

$= \frac{90}{100} \times \text{Tsh } 11\ 000$

= Tsh 9 900.

Therefore, the selling price of the chair was 9 900 Tanzanian shillings.

Example 3

Jenny bought a calculator for 42 000 Tanzanian shillings, and sold it at a loss of $6\frac{2}{3}\%$. How much did she sell the calculator?

Solution

Given;

Buying price = Tsh 42 000, percentage loss = $6\frac{2}{3}\%$;

$$\text{Percentage loss} = \frac{\text{Loss made} \times 100\%}{\text{Buying price}}$$

$$\frac{20}{3}\% = \frac{\text{Loss made} \times 100\%}{\text{Tsh } 42\ 000}$$

$$\frac{20}{3} = \frac{\text{Loss made} \times 100}{\text{Tsh } 42\ 000}$$

$$\text{Loss made} = \frac{\text{Tsh } 420 \times 20}{3}$$

= Tsh 2 800.

Selling price = Buying price – Loss made

= Tsh 42 000 – Tsh 2 800

= Tsh 39 200.

Therefore, Jenny sold the calculator for 39 200 Tanzanian shillings.

Exercise 3

1. Find the profit (or loss) when a toy is bought for:
   (a) Tsh 13 000 and sold for Tsh 13 600
   (b) Tsh 4 000 and sold for Tsh 3 800
   (c) Tsh 4 000 and sold for Tsh 4 800
   (d) Tsh 1 250 and sold for Tsh 1 500
   (e) Tsh 8 000 and sold for Tsh 7 500
2. Find the buying price of an item which is sold for:
   (a) Tsh 16 000 at a profit of 15%
   (b) Tsh 9 800 at a loss of 40%
   (c) Tsh 7 400 at a profit of 32%
   (d) Tsh 30 000 at a loss of 16%
   (e) Tsh 108 500 at a profit of 7%
3. Find the selling price of an item which was bought for:
   (a) Tsh 25 000 at a profit of 24%
   (b) Tsh 18 000 at a profit of 30%
   (c) Tsh 150 000 at a profit of 10%
   (d) Tsh 680 000 at a loss of 6%
   (e) Tsh 80 000 at a loss of $7\frac{1}{2}\%$
4. John sold a damaged mattress at $37\frac{1}{2}\%$ loss. If the loss was 9 000 Tanzanian shillings, find the selling price of the mattress.
5. A book dealer bought 10 books for 20 000 Tanzanian shillings. If $\frac{2}{5}$ of the books were sold at 3 000 Tanzanian shillings each, and the remaining books were sold at 2 500 Tanzanian shillings each, what was the percentage profit?
6. A man decided to sell his car at a 14% loss, which is equivalent to 300 000 Tanzanian shillings. What was the buying price of the car?

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7. A machine costing 180 000 Tanzanian shillings is sold at a profit of 40%. What is its selling price?
8. At a clearance sale, boots which cost 30 000 Tanzanian shillings each were sold at a loss of 25%. Calculate the loss and the clearance price.
9. Alfred bought a second-hand car for 4 700 000 Tanzanian shillings and spent 600 000 Tanzanian shillings on its repair. If he sold the car for 5 600 000 Tanzanian shillings, what was his profit?

Simple interest

When a person is in need, he or she may decide to borrow a certain amount of money from a friend, a bank that offers loans or from a money lender. The person will be required to pay back the amount borrowed after a specific period of time with an additional amount of money. Thus, we define the following terms:

Interest (I) is the extra amount paid in return after taking a loan or after making investment.

Principal (P) is the money borrowed or deposited. It is the initial deposit or loan amount.

Rate (R) is the percentage charge of the principal on which the interest is paid.

Time (T) is the period over which the principal is deposited or borrowed.

Amount (A) is the sum of the principal and the interest. It is the future value of the investment or loan amount.

Thus, Amount = Principal + Interest.

The simple interest for any given period of time is the product of principal, rate, and time. When the interest is calculated uniformly on the original principal throughout the loan period, then it is called a simple interest.

Formula for calculating simple interest

Let P be the principal, R be the rate, and T be the time.

Then, the simple interest I is given by the formula:

$$I = \frac{PRT}{100}.$$

Example 1

Find the time in which a person investing 30 000 Tanzanian shillings will earn 6 000 Tanzanian shillings at the rate of 5% interest per annum.

Solution

Given: P = Tsh 30 000, I = Tsh 6 000, R = 5%, required to find time T.

$$I = \frac{PRT}{100}$$

$$\text{Tsh } 6\,000 = \frac{\text{Tsh } 30\,000 \times 5 \times T}{100}$$

$$T = \frac{6\,000 \times 100}{30\,000 \times 5}$$

$$= 4 \text{ years.}$$

Therefore, the time is 4 years.

Example 2

The interest in 4 years on a principal of 90 000 Tanzanian shillings is 25 200 Tanzanian shillings. Find the rate.

Solution

Given: P = Tsh 90 000, T = 4 years, I = Tsh 25 200, required to find R

$$I = \frac{PRT}{100}$$

$$\text{Tsh } 25\,200 = \frac{\text{Tsh } 90\,000 \times R \times 4}{100}$$

$$R = \frac{25\,200}{90\,000 \times 4} \times 100$$

$$= 7\%.$$

Therefore, the rate is 7%.

Example 3

Alex borrowed 100 000 Tanzanian shillings for 5 years at 10% simple interest. What amount of money did Alex pay back?

Solution

Given: P = Tsh 100 000, R = 10%, T = 5 years, required to find the amount paid back;

$$I = \frac{PRT}{100}$$

$$I = \frac{\text{Tsh } 100\ 000 \times 10 \times 5}{100}$$

$$= \text{Tsh } 50\ 000.$$

Amount = Principal + Interest

$$= \text{Tsh } 100\ 000 + \text{Tsh } 50\ 000$$

$$= \text{Tsh } 150\ 000.$$

Therefore, Alex paid back 150 000 Tanzanian shillings.

Example 4

Mwanaisha borrowed 2 800 000 Tanzanian shillings from a bank which charges an interest rate of 8%. How much interest did she pay after two years and eight months?

Solution

Given: P = Tsh 2 800 000, R = 8%, $T = 2\frac{8}{12} = \frac{8}{3}$ years, required to find I;

$$I = \frac{PRT}{100}$$

$$= \frac{\text{Tsh } 2\ 800\ 000 \times 8 \times 8}{100 \times 3}$$

$$= \text{Tsh } 28\ 000 \times \frac{64}{3}$$

$$\approx \text{Tsh } 597\ 333.$$

Therefore, Mwanaisha paid the interest of 597 333 Tanzania shillings.

Example 5

Manyilizu deposited 540 000 000 Tanzanian shillings in a bank account. After one year, he received 600 000 000 Tanzanian shillings. Find the interest he received.

Solution

Given: P = Tsh 540 000 000, A = Tsh 600 000 000, required to find interest (I);

$$I = A - P$$
$$= \text{Tsh } 600\,000\,000 - \text{Tsh } 540\,000\,000$$
$$= \text{Tsh } 60\,000\,000.$$

Therefore, Manyilizu received an interest of 60 000 000 Tanzanian shillings.

Exercise 4

1. Find the simple interest on:
   (a) Tsh 80 000 for 1 year at the rate of 6.5% per annum.
   (b) Tsh 140 000 for 6 years at the rate of 11.5% per annum.
   (c) Tsh 20 000 for 3 years at the rate of 8% per annum.
   (d) Tsh 200 000 for 2 years at the rate of 6% per annum.
   (e) Tsh 800 000 for 4 years at the rate of 9.5% per annum.
   (f) Tsh 1600 000 for 2 years and 4 months at the rate of 8% per annum.
   (g) Tsh 4 800 000 for 4 years at the rate of 10% per annum.
   (h) Tsh 250 000 for 3 years at the rate of 7.5% per annum.
   (i) Tsh 300 000 for 4 years and 2 months at the rate of 8.5% per annum.
   (j) Tsh 7 000 000 for 5 years at the rate of 12% per annum.
2. Find the number of years in which the interest on:
   (a) Tsh 2 000 000 at the rate of 7% per year is Tsh 480 000.
   (b) Tsh 6 000 000 at the rate of 5% per year is Tsh 300 000.
   (c) Tsh 40 000 at the rate of 8% per year is Tsh 3 600.
3. Find the percentage rate at which the interest on:
   (a) Tsh 200 000 for 4 years is Tsh 56 000.
   (b) Tsh 1 600 000 for 3 years is Tsh 384 000.
   (c) Tsh 6 000 000 for 7 years is Tsh 5 040 000.
4. Find the simple interest on 54 000 000 Tanzanian shillings invested for 18 months at the rate of 12% per annum.
5. Find the rate if the principal of 500 000 Tanzanian shillings is invested for a period of 2 years with an interest of 140 000 Tanzanian shillings.

6. Find the rate on a principal of 75 000 000 Tanzanian shillings invested for a period of 4 years, if the interest is 9 300 000 Tanzanian shillings.
7. For how long was 84 000 Tanzanian shillings invested at the rate of 7.5% per annum if the interest was 27 800 Tanzanian shillings?
8. For how long should 6 720 000 Tanzanian shillings be invested at the rate of 5% per annum to get an interest of 560 000 Tanzanian shillings?
9. How much money should be invested at a rate of 10.5% per year to give an interest of 84 000 Tanzanian shillings for 4 years?
10. How much money will you have to lend in order to get the interest of 480 000 Tanzanian shillings at the rate of 12% per year, if you lend it for 16 months?
11. Find the principal that would earn:
    (a) 540 000 Tanzanian shillings in 10 years at the rate of 5% per year.
    (b) 4 900 000 Tanzanian shillings in 7 years at the rate of 10% per year.
12. Irene invested a certain amount of money and got back an amount of 840 000 Tanzanian shillings. If the bank paid an interest of 70 000 Tanzanian shillings, find the amount she invested.
13. Find the interest on 850 000 at $\frac{50}{3}$% per year for 9 months.
14. Find the interest on a loan of 2 500 000 Tanzanian shillings that is borrowed at 9% per year for 14 months.
15. Find the time period if the interest on 80 000 Tanzanian shillings at 20% per annum is 68 000 Tanzanian shillings.
16. Find the principal that will earn 450 000 Tanzanian shillings in 5 years at the rate of $12\frac{1}{2}$% per year.

Chapter summary

1. A ratio is a comparison by division between two or more quantities which are in the same unit and can be simplified as a fraction.
2. A proportion is a statement that two ratios are equal.
3. A profit is made by selling an item at a price higher than the buying price.
4. A loss is made by selling an item at a price lower than the buying price.
5. Simple interest is an arrangement in which the money borrowed is returned with extra money called interest during its interest period, and is calculated using the formula: $I = \frac{PRT}{100}$.

Revision exercise

1. Decrease:
   (a) 55 years by 5%
   (b) 2 hours by 25%
   (c) Tsh 1 500 000 by 20%
2. Increase:
   (a) 2 000 by 10%
   (b) 80 kg by 25%
   (c) 8 hours by 4.5%
3. A tradesman has a capital of 1 520 000 Tanzanian shillings. If he increases it by 20%, how much will he have?
4. Three relatives share 180 950 Tanzanian shillings so that the first receives twice as much as the second, and the second receives twice as much as the third. How much does the first receive?
5. Find the number of years in which the interest on 40 000 Tanzanian shillings at the rate of 5% per annum is 5 000 Tanzanian shillings.
6. Divide 270 kg in the ratio 1 : 4 : 5.
7. Divide 360° in the ratio 1 : 2 : 5 : 7.
8. Calcium and chlorine combine at the ratio 9 : 16. Calculate the mass of chlorine that will combine with 5 grams of calcium.
9. Find the principal that will earn an interest of 50 000 Tanzanian shillings in 4 years at the rate of 10% per annum.
10. Find the principal that will earn 72 900 Tanzanian shillings at the rate of $2\frac{1}{2}\%$ per annum in 8 years.

Project 9

1. Visit a nearby shop and ask a shopkeeper about the buying prices and the selling prices of some items in the shop. For each item, calculate the percentage gain or the percentage loss.
2. (a) Visit any two nearby banks and request to know the interest rate per annum charged to a loan. Using these rates, calculate the total amount that a business person will pay after 2 years if she or he borrows 30 000 000 Tanzanian shillings.
   (b) Based on the results in (a), how can you advise any person who wants to invest money in either of the banks?

Chapter Eleven

Coordinate geometry

Introduction

Coordinate geometry was introduced by a French mathematician known as Rene Descartes between the years 1596 and 1650. This came after observing a fly for a long time. He realized that, he could describe the position of that fly by its distance from the walls of the room. From that case, he invented the coordinate plane which is sometimes called the Cartesian coordinate plane. Coordinates of a point are ordered pairs of numbers that specify the position of a point on a plane. Coordinate geometry is a system of geometry which describes the positions of points on the coordinate plane using ordered pairs of numbers. In this chapter, you will learn about coordinates of a point, gradient of a straight line, equation of a straight line, graphing straight lines as well as solving linear simultaneous equations graphically. The competencies developed will enable you to locate places correctly. For example; you will be able to locate and plot boundaries, interpret properly traffic information on roads, flights, and marine transport. The knowledge in coordinate geometry will also help you in business for representing some data, in astronomy for locating objects of interest in the sky, and many other applications.

Coordinates of a point

Activity 1: Identifying the coordinates of a point

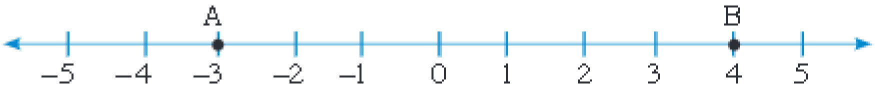
Individually or in groups, perform the following tasks:

1. Stand up and look at the heads of other group members.
2. Observe and locate any students of your choice by taking the following measurements:
   (a) Mesure the perpendicular distances from the student to both left and front walls.
   (b) Record the measurements you have obtained in (a) in the form of (u, v), where u represents the distance from the left wall and v represents the distance from the front wall.

3. Repeat task 1 and 2 several times with the students at different positions.
4. Share your findings with your fellow students through presentation, and conclude your observations with your teacher.

Each point on a plane is represented by means of an ordered pair of real numbers, called the coordinates of that point.

The position of a point on a line is found by using a number line. For example; a position of point A on the following number line is –3, and the position of point B is 4.



The positions of points on a plane surface are found by using two number lines, that is, horizontal number line and vertical number line. Numbers on the number lines on the plane are called coordinates.
A coordinate plane is made up of two number lines intersecting at right angles as shown in Figure 11.1.

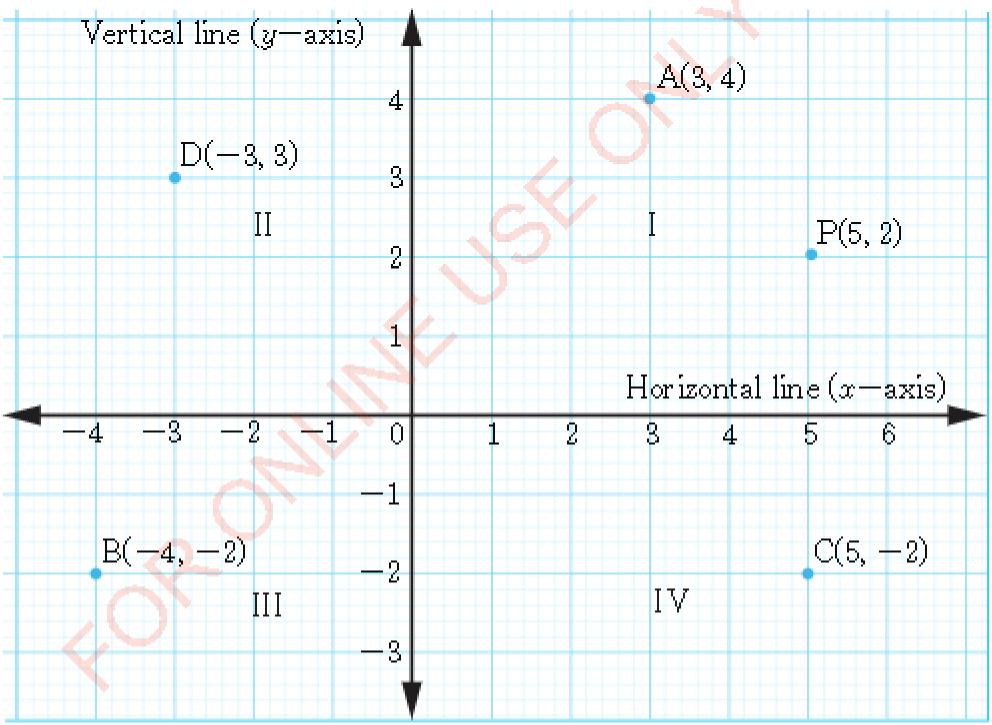


Figure 11.1: *The xy-plane showing the four quadrants*

The horizontal number line is called the x – axis, and the vertical number line is called the y – axis. The point at which the two axes intersect is called the origin. The two axes divide the plane into four parts labelled I, II, III, and IV, called quadrants, as shown in Figure 11.1. The pair of numbers that corresponds to point A is (3,4). The first number represents a distance from the origin along the x–axis, and the second number represents a distance from the origin along the y–axis. This pair is called an ordered pair or coordinates. The order in which the numbers are written is very important.

Note that: The first number is for the x–axis and the second for the y-axis. The first number in the ordered pair is called the first coordinate or abscissa and the second number is called the second coordinate or ordinate. For example; the coordinates of P in Figure 11.1 are (5, 2). It has 5 as the x-coordinate and 2 as the y-coordinate. The coordinates of the origin are (0, 0). The origin is normally denoted by O.

When locating points on a graph paper, we often say that we are plotting points.

Activity 2: Recognisng how to read coordinates on the xy–plane

Individually or in groups, perform the following tasks:

1. Write the ordered pair for each point shown in Figure 11.2:
   (i) A (iii) C (v) E (vii) G (ix) I (xi) K
   (ii) B (iv) D (vi) F (viii) H (x) J (xii) L

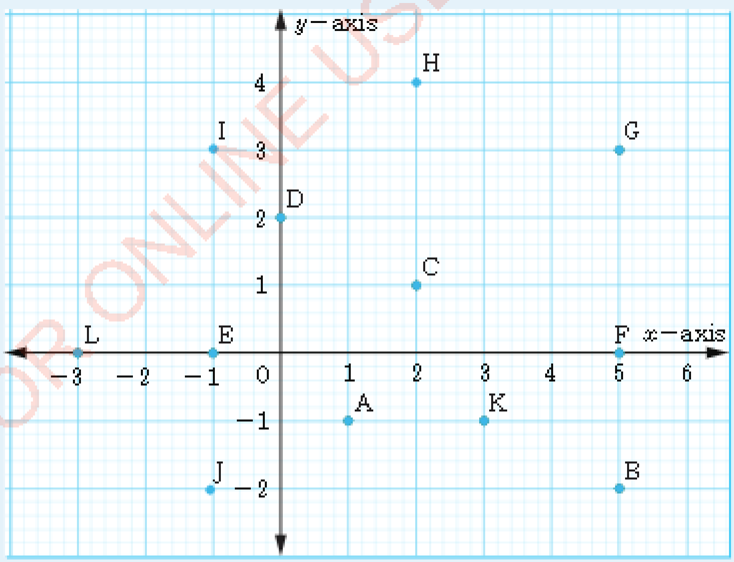


Figure 11.2: *Locating the ordered pair of points*

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2. (a) Draw the coordinate plain on a graph paper and locate each of the following points:

   (i) M(-1, 3)   (iii) R(-3,-2)   (v) Q(5,-1)

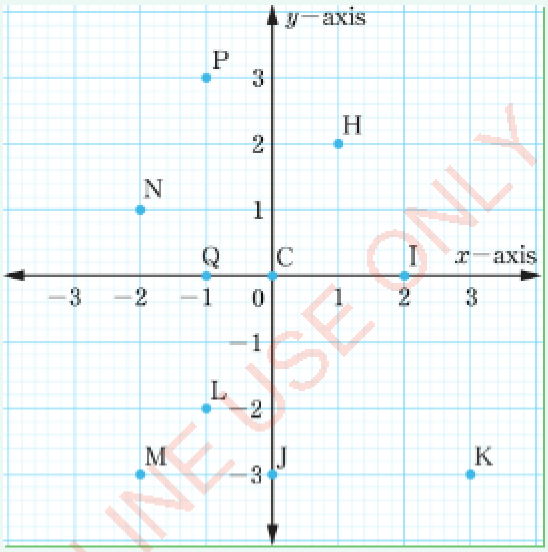
   (ii) S(2,2)   (iv) P(1,-4)   (vi) N(3,4)

   (b) State the quadrant in which each point in (a) is located.

3. Share your answers with your fellow students through presentation, and conclude with your teacher.

Exercise 1

1. Given the following figure:
   (a) Write down the coordinates of each of the labelled points.
   (b) State the quadrant to which the points H, K, L, and N belong.



2. Draw the axes on a graph paper and plot the following points. Join the points with line segments to form polygons. What shape have you drawn in each case?
   (a) (1, 1), (3,1), (3, 3), (1, 3)
   (b) (–2, 1), (2, 5), (2, –2)
   (c) (3, 1), (5.4, 1), (4.3, 2), (3.3, 2)
   (d) (5.5, 3.4), (6.5, 3.4), (6.8, 4.3), (6.0, 4.9), (5.2, 4.3)
   (e) (1.5, –3), (6, 3), (–1.5, 3), (–6, –3)

3. In which quadrant does each of the following points lie?
   (a) P(–4, –10) (c) R(16, –12)
   (b) T(–16, 8) (d) S(4, 2).
4. Locate the following points on the xy–plane: A(1, 2), B(–2, 4), C(0, 0), D(3, –4), E(–3, –2), F(–5, 0), G(4, 0), H(0, 5), I(0, –4)
   (a) Join the points A and B, B and E, A and E by line segments.
   (b) Use a protractor to measure angle BAE.

Gradient of a straight line

Activity 3: Recognising the gradient of a straight line

Individually or in groups, use a graph paper to perform the following tasks:

1. Plot the points A(2, 1), B(5, 6), C(2, 7), and D(6, 1) on the xy–plane.
2. Find the vertical increase and horizontal increase of $\overline{AB}$, $\overline{CD}$, $\overline{BA}$, and $\overline{DC}$.
3. Find the ratios between the vertical increases and the horizontal increases of $\overline{AB}$ and $\overline{BA}$.
4. Give the relationship between the ratios of increases for $\overline{AB}$ and $\overline{BA}$.
5. What happens as you move from A to B along the interval?

A gradient or slope is a measure of steepness of a straight line and is denoted by m. In coordinate geometry, the standard way to define the gradient of a straight line is by finding the ratio between the change in y (vertical increase) to the change in x (horizontal increase), that is,

$$\text{Gradient, } m = \frac{\text{change in } y}{\text{change in } x}.$$

Consider any right-angled triangle formed by the points A(x_1, y_1) and B(x_2, y_2) with the hypotenuse $\overline{AB}$ as shown in Figure 11.3. The gradient is determined by the ratio of the length of the vertical side of the triangle to the length of the horizontal side of the triangle. The length of the vertical side of the triangle is the difference in y-values of the points A and B. The length of the horizontal side of the triangle is the difference in x-values of the points A and B.

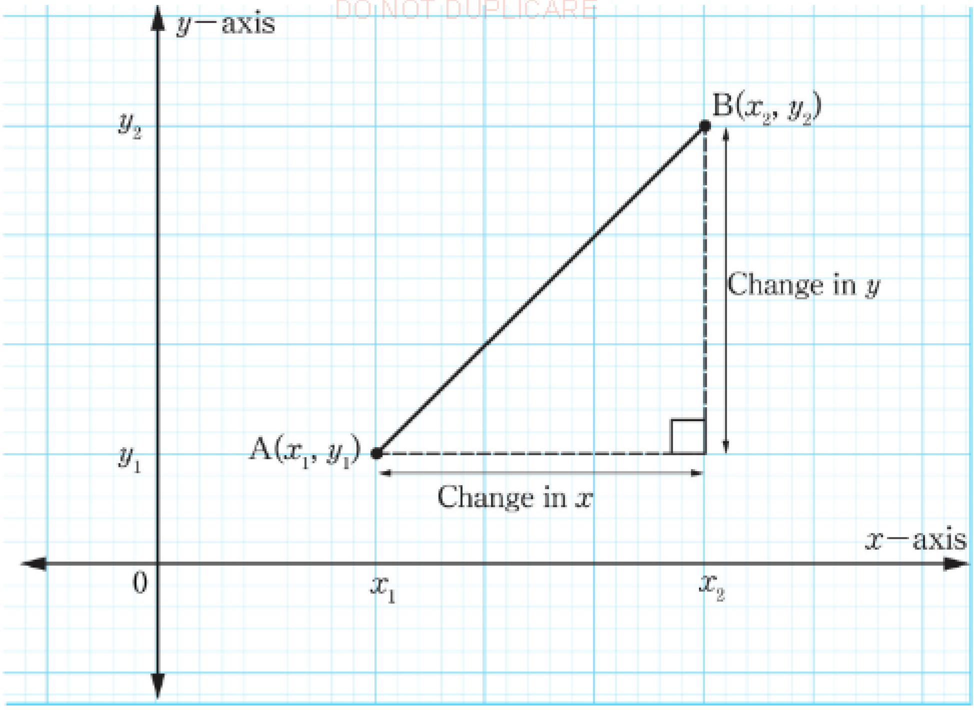


Figure 11.3: *Gradient of a straight line*

The gradient m of the straight line A B is determined by using the following formula:

$$m = \frac{\text{change in } y}{\text{change in } x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ or}$$

$$m = \frac{y_1 - y_2}{x_1 - x_2}.$$

> **Note that:** When calculating the gradient, the order of writing the coordinate values in the equation does not matter. One may choose to start writing the second point and the other may choose to start writing the first point, provided that the coordinates are substracted correspondingly.

The following are the procedures for calculating the gradient of a straight line joining two points A(x_1, y_1) and B(x_2, y_2):

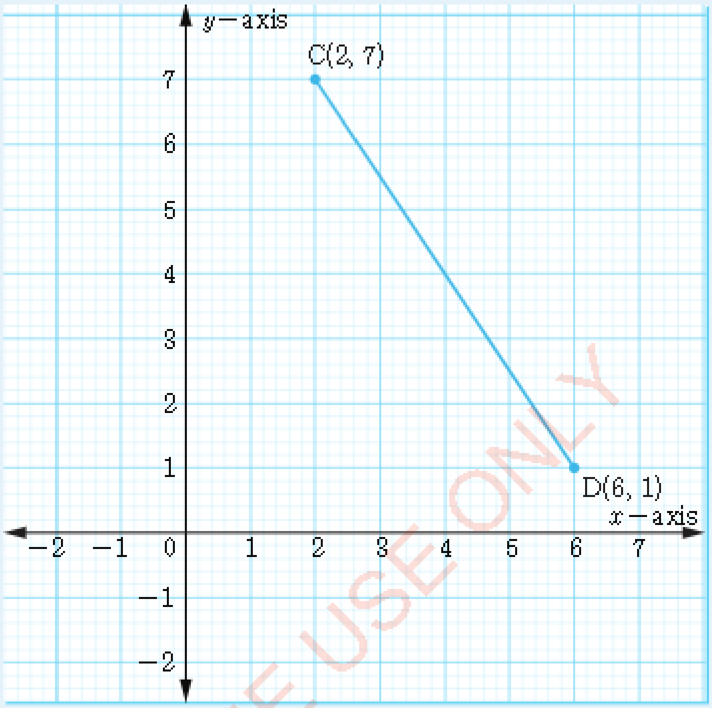
1. Write down the formula for calculating the gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
2. Assign values to (x_1, y_1) and (x_2, y_2).

3. Substitute the values into the formula to obtain the gradient m.
4. Write the final answer which is the gradient m.

Example 1

Use the following figure to find:

(a) The gradient of the line segment CD
(b) The gradient of the line segment DC
(c) Compare the gradients obtained in (a) and (b)



Solution

(a) Gradient of the line segment CD $= \frac{y_2 - y_1}{x_2 - x_1}$,

where $(x_1, y_1) = (6, 1)$ and $(x_2, y_2) = (2, 7)$.

$$m = \frac{7-1}{2-6}$$

$$= \frac{6}{-4}$$

$$= -\frac{3}{2}.$$

Therefore, the gradient of the line segment CD is $-\frac{3}{2}$.

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(b) Gradient of the line segment DC $= \frac{\text{change in } y}{\text{change in } x}$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Where $(x_1, y_1) = (2, 7)$ and $(x_2, y_2) = (6, 1)$

$$= \frac{1 - 7}{6 - 2}$$

$$= \frac{-6}{4}$$

$$= -\frac{3}{2}.$$

Therefore, the gradient of the line segment CD is $-\frac{3}{2}$.

(c) As we move from point C to point D, the y-values decrease as the x-values increase. Therefore, the gradient of the line segment CD is negative. Also, the gradient of the line segment DC is the same as the gradient of the line segment CD.

If a line is vertical, the change in x-values is zero, and the gradient of the line is not defined. This is shown by the line segment PN in Figure 11.4.

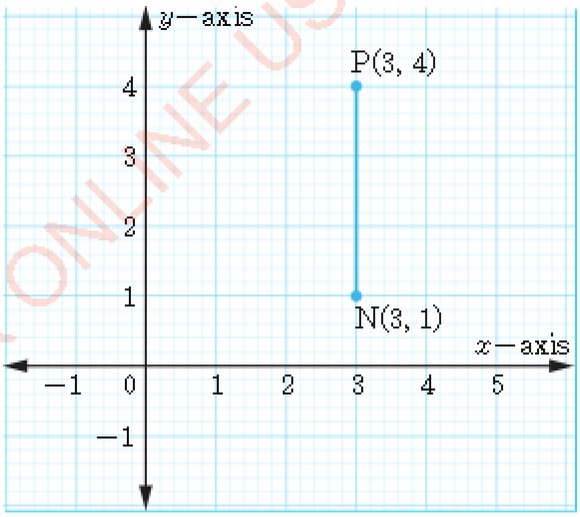


Figure 11.4: *The gradient of PN is not defined*

The gradient of the vertical line segment joining the two points P(3, 4) and N(3, 1) can be obtained as follows:

$m = \frac{y_2 - y_1}{x_2 - x_1}$ where $(x_1, y_1) = (3, 1)$ and $(x_2, y_2) = (3, 4)$

$m = \frac{4 - 1}{3 - 3}$

$= \frac{3}{0}$.

Therefore, $m = \frac{3}{0}$ (undefined).

Since the gradient of $\overline{PN}$ is undefined, we conclude that, $\overline{PN}$ is vertical.

If the line is horizontal, the change in y-values is zero and the gradient of the line is zero, as shown by the line segment DE in Figure 11.5:

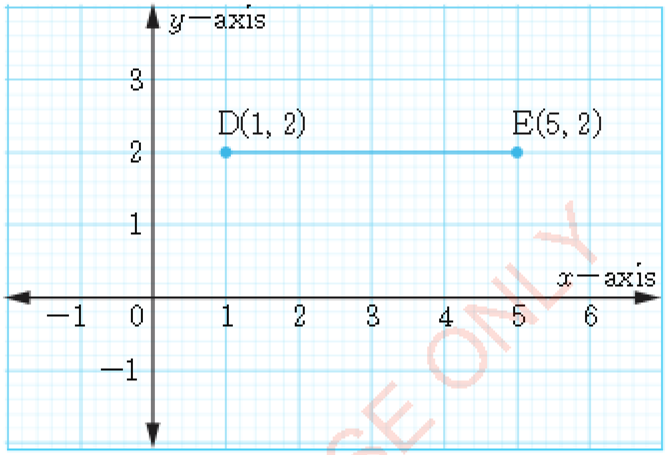


Figure 11.5: *The gradient of line segment DE is zero*

The gradient of the horizontal line segment joining two points D(1, 2) and E(5, 2) can be obtained as follows:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Given $x_1 = 1$, $x_2 = 5$, $y_1 = 2$, and $y_2 = 2$

$m = \frac{2 - 2}{5 - 1}$

$= \frac{0}{4}$

$= 0$.

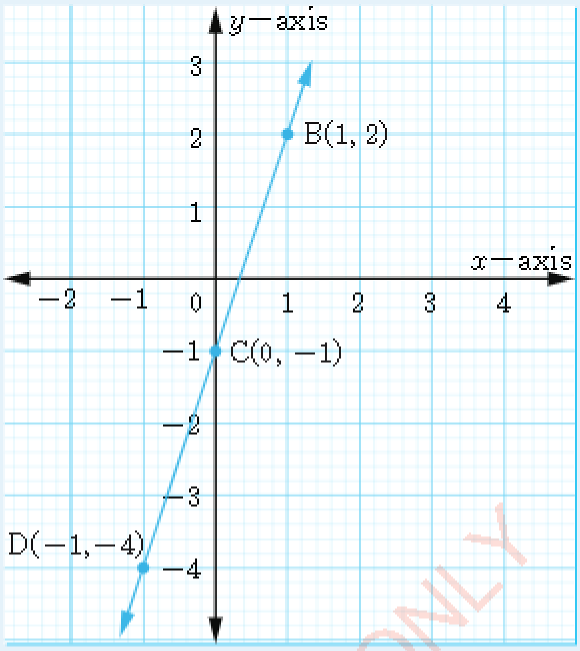
Therefore, $m = 0$.

Since the gradient of the line segment DE is zero, we conclude that $\overline{DE}$ is horizontal.

Example 2

Use the following figure to find the gradient of the straight line joining the points:

(a) B and C (b) B and D.



Solution

(a) Gradient of the line joining B(1, 2) and C(0, – 1) is given by:

$m = \frac{y_2 - y_1}{x_2 - x_1}$,

where $(x_1, y_1) = (0, -1)$

and $(x_2, y_2) = (1, 2)$

$m = \frac{2 - (-1)}{1 - 0}$

$m = \frac{3}{1}$

$= 3.$

Therefore, the gradient of $\overleftrightarrow{BC}$ is 3.

(b) Gradient of the line joining B(1, 2) and D(–1, – 4) is given by:

$m = \frac{y_2 - y_1}{x_2 - x_1}$,

where $(x_1, y_1) = (1, 2)$

and $(x_2, y_2) = (-1, -4)$

$m = \frac{-4 - 2}{-1 - 1}$

$= \frac{-6}{-2}$

$= 3.$

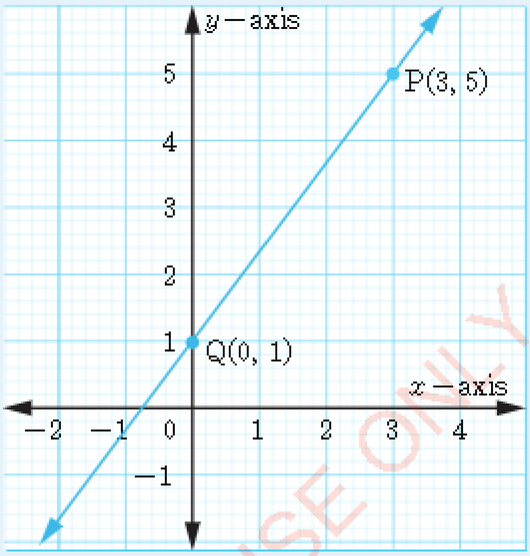
Therefore, the gradient of $\overleftrightarrow{BD}$ is 3.

From Example 2, it can be observed that the gradient of a straight line does not change no matter what pair of points are used, provided that all the points lie on the same straight line.

To calculate the gradient of a straight line, any two points which lie on the line can be used.

Example 3

Find the gradient of the given straight line in the following figure.



Solution

Choose any two points which lie on the straight line. For example; points P(3, 5) and Q(0, 1).

$$m = \frac{y_2 - y_1}{x_2 - x_1},$$

Given, $x_1 = 3$, $x_2 = 0$

$y_1 = 5$, $y_2 = 1$

$$m = \frac{1 - 5}{0 - 3}$$

$$= \frac{-4}{-3}.$$

Therefore, the gradient of the given straight line is $\frac{4}{3}$.

Generally, to calculate the gradient of a straight line, choose any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ both of which lie on the line, then calculate its gradient by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{y_1 - y_2}{x_1 - x_2}$.

Exercise 2

1. Plot each of the following pairs of points on a graph paper and join them by a straight line. For each pair, calculate the gradient of the straight line, and state whether it is positive, negative, zero, or undefined:

   (a) (0, 3), (2, 5)
   (b) (5, 8), (4, 1)
   (c) (1, 5), (4, 7)
   (d) (2, 6), (5, 3)
   (e) (1, 6), (3, –1)
   (f) (3, 6), (–2, –1)
   (g) (0, 2), (6, 2)
   (h) (2, 3), (–1, –3)
   (i) (2, 10), (2, 0)
   (j) (0, 2), (0, 5)
   (k) (1, 0), (4, 0)

2. Calculate the gradient of the straight line which pass through each of the following pairs of points:

   (a) (2, 1), (7, 6)
   (b) (5, 12), (3, 2)
   (c) (3, 7), (1, –1)
   (d) (–1, 5), (–4, 1)
   (e) (–1, 6), (4, 1)
   (f) (–4, 4), (1, –4)
   (g) (–2, 1), (4, 3)
   (h) (–4, 4), (–3, –3)
   (i) (2, 6), (0, 3)
   (j) $\left(3\frac{1}{2}, \frac{-1}{2}\right), \left(-7\frac{1}{2}, 2\right)$
   (k) (99, 6), (119, 1)
   (l) (0.64, –0.62), (–1.36, –0.62)

3. Study the following graph, and answer the questions that follow.

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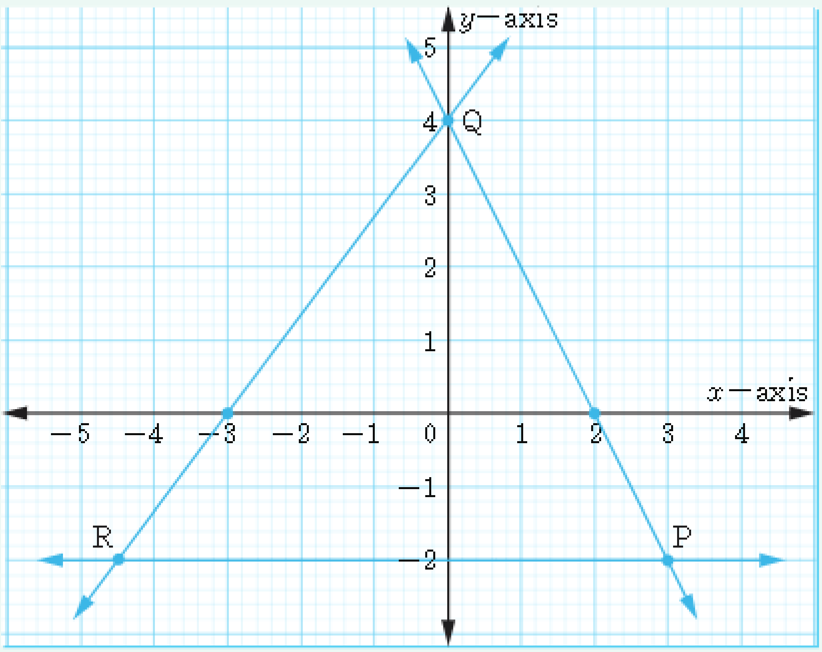


(a) What are the coordinates of the points P, Q, and R?

(b) Find the gradients of $\overline{PQ}$, $\overline{PR}$, and $\overline{QR}$.

(c) Write the name of the polygon formed by joining the points P, Q, and R.

Equation of a straight line

Activity 4 : Determining the equation of a straight line

Individually or in groups, perform the following tasks:

1. Plot the following points on the xy–plane: A(0, 8), B(1, 6), C(2, 4), D(3, 2), E(4,0) and F(5, –2).
2. Join the points in task 1 by straight lines.
3. Find the gradients of the line segments BC, DE, and AF.
4. What can you say about the gradients of the line segments BC, DE, and AF?
5. Find the equation of the straight line joining the points B and C.
6. Share your results with your fellow students through presentation.

Consider a straight line through the points A(3, 7) and B(–2, –3) as shown in the following figure.

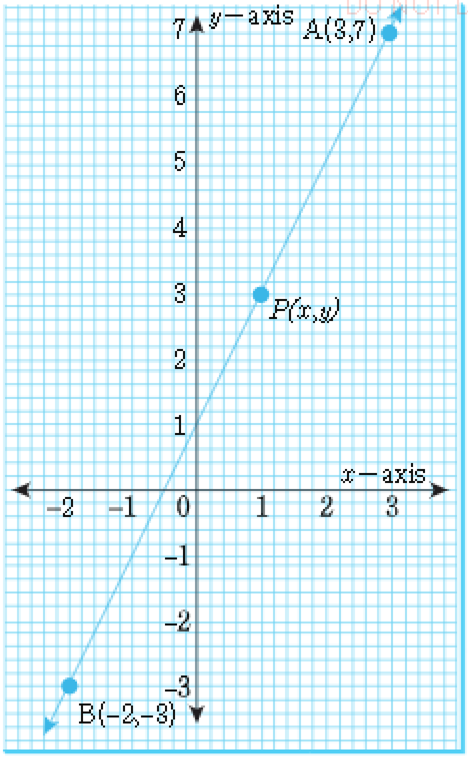


Figure 11.6: *A straight line AB*

Suppose there is another point P(x, y) which lies on the line AB. The gradient of this line can be found using either the pair of points (3, 7) and (–2, –3) or (x, y) and (3, 7), or (–2, –3) and (x, y).

The gradient of the line AB using the points (3, 7) and (–2, –3) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Given; $x_1 = 3$, $x_2 = -2$,

$y_1 = 7$, $y_2 = -3$.

$$m = \frac{-3 - 7}{-2 - 3}$$

$$= \frac{-10}{-5}$$

$$= 2.$$

Therefore, the gradient of the line AB is 2.

Using the points (x, y) and (3, 7):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Given; $x_1 = 3$, $x_2 = x$, $y_1 = 7$, and $y_2 = y$.

$$m = \frac{y - 7}{x - 3}$$

But, $m = 2$, thus,

$$2 = \frac{y - 7}{x - 3}$$

$$\frac{2}{1} = \frac{y - 7}{x - 3}$$

Cross multiplication gives

$$1(y - 7) = 2(x - 3)$$

$$y - 7 = 2x - 6$$

$$y = 2x - 6 + 7$$

$$y = 2x + 1.$$

Therefore, the equation of $\overline{AB}$ is $y = 2x + 1$.

The equation $y = 2x + 1$ is called the equation of the straight line AB.

Generally, the equation of a straight line can be written in the form $y = mx + c$, where m is the gradient and c is the y–intercept. The y–intercept is the point at which the line intersects the y–axis. It is always the point (0, c). For example; $x + y = 0$, $y - x = 1$, and $2x + 3y = 2$ are linear equations whose y–intercepts are 0, 1 and $\frac{2}{3}$ respectively.

The x–coordinate of the point at which a graph cuts the x–axis is called x–intercept. In Figure 11.6, the y–intercept is 1 because the graph cuts the y–axis at point (0, 1). The x–intercept is –0.5 because the graph cuts the x–axis at point (–0.5, 0).

In any equation, the y–intercept is at $x = 0$ and the x–intercept is at $y = 0$.

Thus, the y–intercept for the line $y = 2x + 1$ is $y = 2 \times 0 + 1 = 1$.

Also, the x–intercept for the line $y = 2x + 1$ is obtained as follows:

$$0 = 2x + 1$$
$$0 - 1 = 2x + 1 - 1$$
$$-1 = 2x$$
$$x = -\frac{1}{2}$$
$$= -0.5.$$

Therefore, the x-intercept is -0.5.

Note:

The gradient (slope) and the y–intercept of a straight line can be determined if the equation of the line is written in the form $y = mx + c$, where m is the gradient and c is the y-intercept.

Equation of a line, given two points

Given two points, the equation of a line can be obtained by using the following steps:

1. Find the gradient and substitute it into the general equation $y = mx + c$.
2. Take any of the given points and substitute its coordinates into the new equation $y = mx + c$ to obtain the value of c.
3. Substitute the obtained value of c into the equation obtained in step 1, to obtain the equation of the straight line.

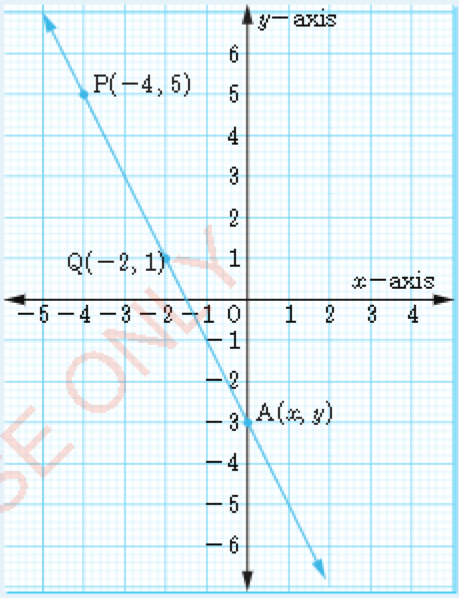
Example 1

Draw a straight line on a graph paper through the points P(–4, 5) and Q(–2, 1), and find:

(a) The gradient of the straight line.
(b) The equation of the straight line.
(c) The y–intercept.

Solution

Plot the ordered pairs P(– 4, 5) and Q(–2, 1) on the graph paper. Join the two points to make a straight line.



(a) The gradient of the line is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1},$$

Given; $x_1 = -4$, $x_2 = -2$, $y_1 = 5$, and $y_2 = 1$.

$$m = \frac{1 - 5}{-2 - (-4)}$$
$$= \frac{-4}{2}$$
$$= -2.$$

Therefore, the gradient of a straight line is -2.

(b) Let A(x, y) be any point on the line, and use the point P(–4, 5) and A(x, y) to obtain the equation of the line, that is,

$m = \frac{y-5}{x-(-4)}$. But, $m = -2$.

$-2 = \frac{y-5}{x+4}$

$\frac{-2}{1} = \frac{y-5}{x+4}$.

By cross multiplying, we get;

$1(y-5) = -2(x+4)$

$y - 5 = -2x - 8$

$y = -2x - 8 + 5$

$y = -2x - 3$.

Therefore, the equation of the straight line is $y = -2x - 3$.

In this case, the point A(x, y) and Q(–2, 1) can also be used to obtain the equation of the line.

(c) From either the graph or the equation $y = -2x - 3$, the y-intercept is –3. The y-intercept can also be found by setting $x = 0$ in the equation $y = -2x - 3$,

$y = -2(0) - 3$

$= 0 - 3$

$= -3$.

Therefore, the y – intercept is –3.

Example 2

Find the equation of a straight line passing through the points (–4, 3) and (1, –7).

Solution

The equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) is obtained by using the following relation:

$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$,

Given; $x_1 = -4$, $x_2 = 1$, $y_1 = 3$, and $y_2 = -7$, and (x, y) is any point on the line other than (x_1, y_1) and (x_2, y_2)

$\frac{y-3}{x-(-4)} = \frac{-7-3}{1-(-4)}$

$\frac{y-3}{x+4} = \frac{-10}{5}$

$\frac{y-3}{x+4} = -2$.

By cross multiplying, we have;

$y - 3 = -2(x + 4)$

$y - 3 = -2x - 8$

$y = -2x - 8 + 3$

$y = -2x - 5$.

Therefore, the equation of the straight line is $y = -2x - 5$.

Equation of a straight line when its gradient and a point are given

In order to find the equation of a straight line with the gradient m passing through the point A(x_1, y_1), we let P(x, y) to be any point on the line.

Using the formula of the gradient, $m = \frac{y-y_1}{x-x_1}$, we obtain the equation $y - y_1 = m(x - x_1)$. This is the equation of a straight line with the gradient m passing through the point A(x_1, y_1).

Example 1

A straight line has a gradient of $\frac{3}{4}$, and it passes through the point (–1, 2). Find:

(a) its equation (b) the y–intercept

Solution

(a) The equation for this line is $y - y_1 = m(x - x_1)$,

$m = \frac{3}{4}$, $x_1 = -1$, and $y_1 = 2$.

Substituting the values in the equation gives:

$$y - 2 = \frac{3}{4}(x - (-1))$$

$$y - 2 = \frac{3}{4}(x + 1)$$

$$4(y - 2) = 3(x + 1)$$

$$4y - 8 = 3x + 3$$

$$4y = 3x + 3 + 8$$

$$4y = 3x + 11$$

$$y = \frac{3}{4}x + \frac{11}{4}.$$

Therefore, the equation of the straight line is $y = \frac{3}{4}x + \frac{11}{4}$.

(b) The y–intercept is $\frac{11}{4}$.

Example 2

Show whether or not the point $P\left(\frac{5}{3}, 4\right)$ lies on the line obtained in Example 1.

Solution

Given a line

$$y = \frac{3}{4}x + \frac{11}{4}.$$

The point P $(\frac{5}{3}, 4)$ lies on the line if it satisfies the equation of the given line. Substitute the values of x and y in the equation to obtain

$$4 = \frac{3}{4}\left(\frac{5}{3}\right) + \frac{11}{4}$$

$$4 = \frac{5}{4} + \frac{11}{4}$$

$$4 = 4.$$

Since the LHS is equal to the RHS, therefore, the point $P\left(\frac{5}{3}, 4\right)$ satisfies the equation, hence it lies on the line $y = \frac{3}{4}x + \frac{11}{4}$.

Finding the equation of a straight line when its gradient and y–intercept are given

Given the gradient and the y–intercept, the equation of a line can also be obtained directly by substituting these values in the general equation of a straight line, $y = mx + c$.

Example 1

Find the equation of a straight line whose gradient is $\frac{3}{2}$ and the y–intercept is 3.

Solution

Substitute the values of $m = \frac{3}{2}$ and $c = 3$ in the general equation of a straight line $y = mx + c$ to get

$$y = \frac{3}{2}x + 3.$$

Therefore, the equation of the straight line is $y = \frac{3}{2}x + 3$.

Example 2

Find the gradient and y–intercept of each of the following straight lines:

(a) $12x + 4y - 15 = 0$

(b) $24 - 8x - 3y = 0$

Solution

(a) Given; $12x + 4y - 15 = 0$.

Make y the subject of the formula:

$4y = -12x + 15$.

Dividing both sides by 4, we get

$y = -3x + \frac{15}{4}$.

Therefore, the gradient is -3 and the y–intercept is $\frac{15}{4}$.

(b) Given; $24 - 8x - 3y = 0$.

Make y the subject of the formula:

$-3y = 8x - 24$.

Dividing both sides by -3:

$y = -\frac{8}{3}x + 8$.

Therefore, the gradient is $-\frac{8}{3}$ and the y–intercept is 8.

Exercise 3

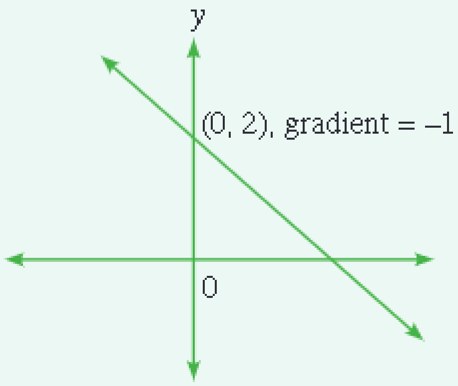
1. Find the equation of each of the lines passing through the following pairs of points:
   (a) (7, 9), (2, 5)
   (b) (5, 3), (0, 1)
   (c) (2, 5), (5, 7)
   (d) (6, 1), (1, 6)
   (e) (4, 0), (3, 0)
   (f) (–2, –2), (3, –4)
   (g) (–4, 3), (8, –6)
   (h) (–7, 2), (–1, 6)
   (i) (–4, 0), (3, –3)
   (j) (0, 0), (–2, 2)

2. Find the y–intercept of each of the lines joining the following pairs of points:
   (a) (0, 0), (–3, –4)
   (b) (–4, –4), (4, 4)
   (c) (2 , 3), (–3 , 1)
   (d) (0, 4), (4, 0)
   (e) (–5, 2), (3, 2)
   (f) (–1, –2), (–5, –5)

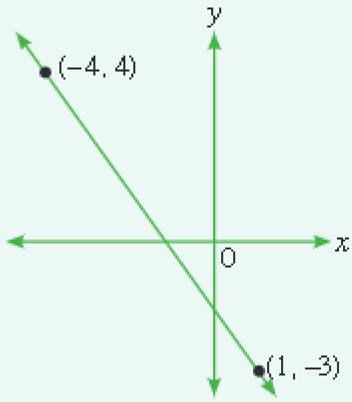
3. Find the equation of the line using the given point and gradient:
   (a) (2, 1), gradient 2
   (b) (0, 5), gradient –2
   (c) (1, –3), gradient 3
   (d) (–2, –4), gradient $\frac{3}{2}$
   (e) (0, 0), gradient –3

4. Find the equation of the line using the given y–intercept and gradient:
   (a) y–intercept = 2, gradient = 1
   (b) y–intercept = 7, gradient = $\frac{3}{4}$
   (c) y–intercept = –16, gradient = 4

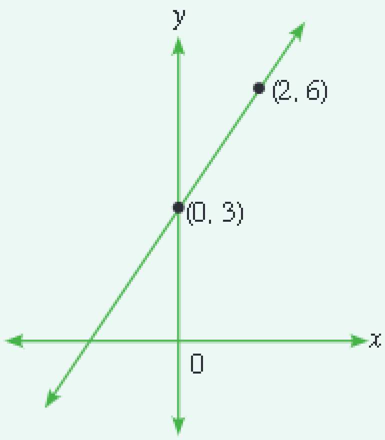
(d)



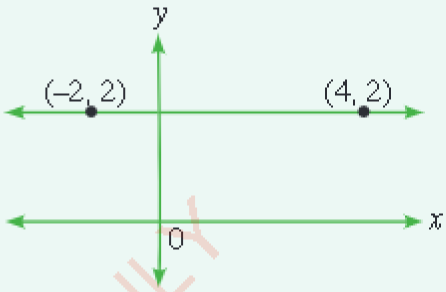
(f)



(e)



(g)



5. Rewrite the following equations in the form $y = mx + c$, and then determine the gradient and the y–intercept of each equation without drawing their graphs:

(a) $7x + 4y = 11$

(b) $14x + 3y = 12$

(c) $2x = 5 + y$

(d) $4x + 5y = 40$

(e) $8x - y = 0$

(f) $39x - 13y + 70 = 0$

Graphing straight lines

When the equation of a straight line is given, its graph can be drawn. In order to draw the graph of the equation of a straight line, find two or more points whose coordinates satisfy the given equation. Plot the points on the xy–plane, and join the points with a straight line.

There are three common methods used to find two or more points of the equation of a straight line. These are:

(a) Using the y–intercept and a point on the line.

(b) Using the x–intercept and y–intercept.

(c) Constructing a table of values.

(a) Graphing straight lines using the y–intercept and a point on the line

Using the y–intercept and a point lying on the straight line, the graph of the line can be drawn or sketched.

It should be noted that, this method does not work if the line is parallel to the y–axis.

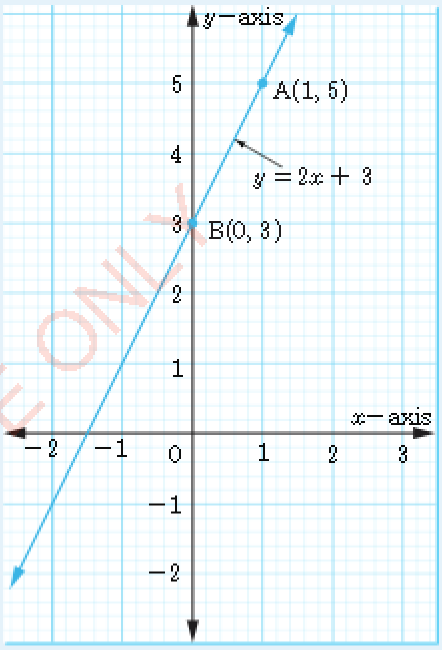
Example

Draw the graph of $y = 2x + 3$.

Solution

The y–intercept is 3. The point at the y–intercept is (0, 3). Choose any value of x, say $x = 1$. Substitute $x = 1$ into the given equation to obtain $y = 5$, giving the point (1, 5).

Thus, we have two points, (0, 3) and (1, 5) which lie on the line $y = 2x + 3$. Locate the two points on the xy–plane and draw a straight line passing through these points as shown in the following figure.



(b) Graphing straight lines using the x–intercept and the y–intercept

In this method, both intercepts are calculated. The x–intercept is calculated by substituting $y = 0$, and the y–intercept is calculated by substituting $x = 0$ in the given equation.

It should be noted that, this method does not work if the line is parallel to any axis or passes through the origin.

Example

Use the x–intercept and y–intercept method to sketch the graph of $2x + 3y + 6 = 0$.

Solution

Calculating the y–intercept:

When $x = 0$, $3y + 6 = 0$

$3y = -6$

$y = -2$.

Therefore, the y–intercept is -2 and its corresponding point is $(0, -2)$.

Calculating the x–intercept:

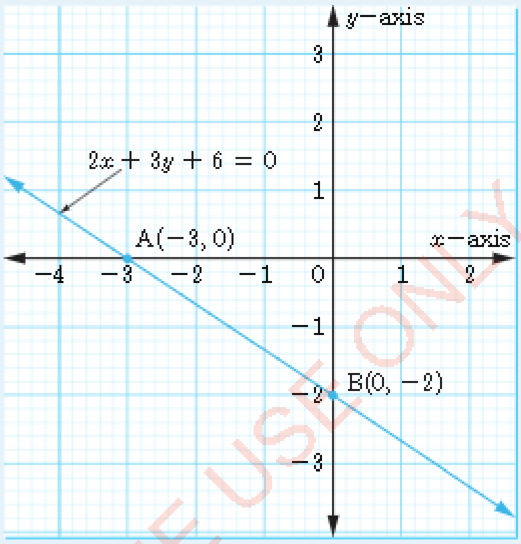
When $y = 0$, $2x + 6 = 0$

$2x = -6$

$x = -3$.

Therefore, the x–intercept is -3 and its corresponding point is $(-3, 0)$

Draw a straight line passing through the x–intercept and y–intercept. Label your graph as shown in the following figure.



(c) Graphing straight lines using a table of values

We can use a table of values to draw straight lines when an equation is given.

Example 1

Given the equation $x + y = 1$, complete the following table of values, and draw the corresponding graph.

Table of values for $x + y = 1$

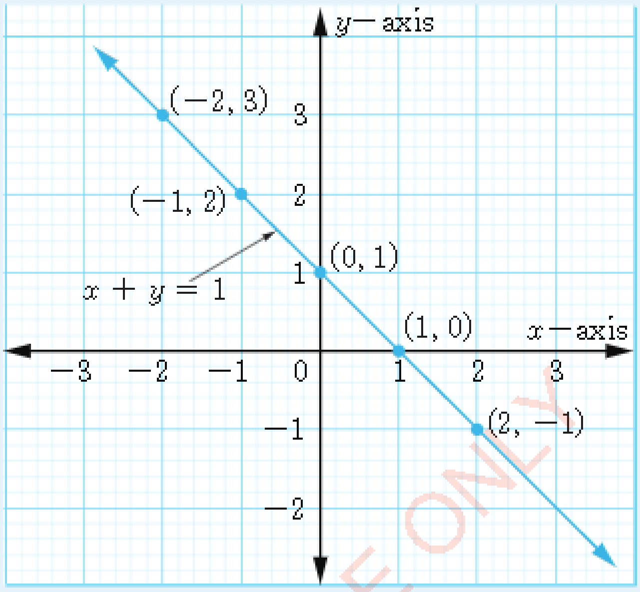
x	–2		0	1	2
y		2		0	

Solution

The following is the completed table of values for $x + y = 1$.

x	–2	–1	0	1	2
y	3	2	1	0	–1

Locate in the xy–plane the ordered pairs obtained in the table of values. Join the points to form a straight line as shown in the following figure.



Example 2

Draw a graph of the equation $y - 2x = 2$.

Solution

A table of values for this equation can be obtained as follows:

Find the corresponding values of y, when some values of x are chosen or vice versa.

When $x = -2$, $y - (2 \times (-2)) = 2$, that is, $y = -2$.

When $x = -1$, $y - (2 \times (-1)) = 2$, that is, $y = 0$.

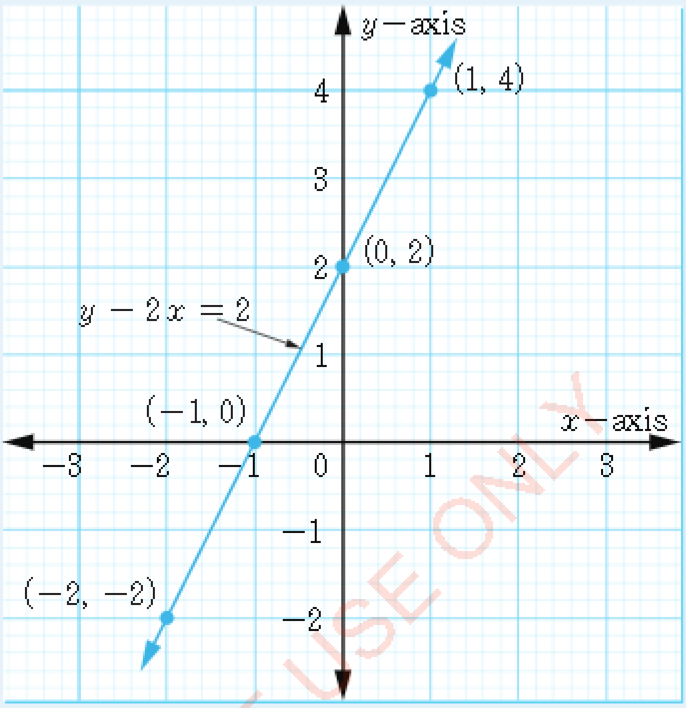
When $x = 0$, $y - (2 \times 0) = 2$, that is, $y = 2$.

When $x = 1$, $y - 2 \times 1 = 2$, that is, $y = 4$.
When $x = 2$, $y - 2 \times 2 = 2$, that is, $y = 6$.

The table of values for $y - 2x = 2$ is as follows:

x	–2	–1	0	1	2
y	–2	0	2	4	6

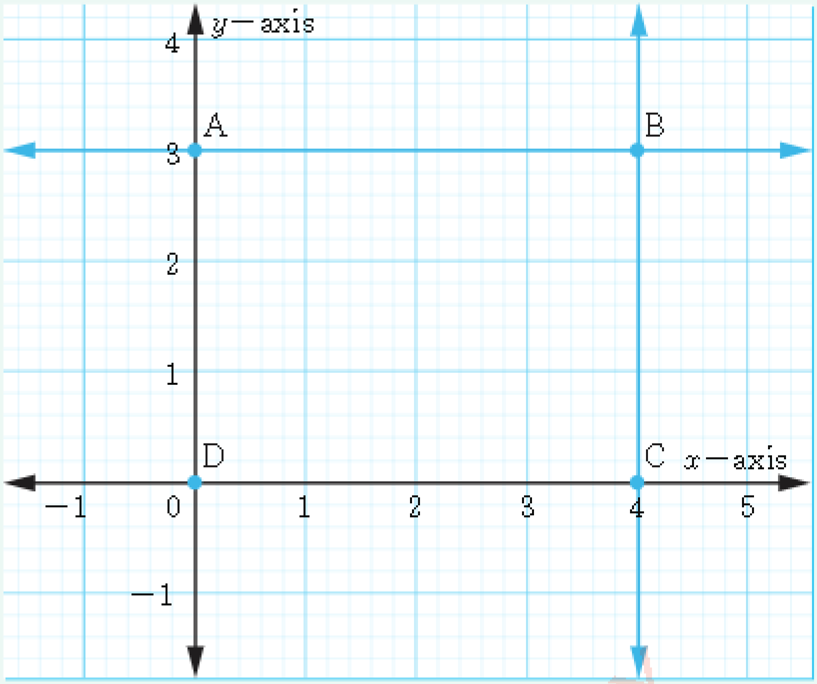
Thus, the graph of this equation is illustrated in the following figure:



Exercise 4

1. Construct a table of values for $2x + y - 1 = 0$, and then use it to draw the graph of this equation. Find the coordinates of the points where the graph meets the x–axis and the y–axis.
2. Draw a graph for each of the following equations:
   (a) $x + 3y = 2$
   (b) $3x - 5y = 1$
   (c) $7x - 2y = 28$
   (d) $3x - y = 2$
   (e) $2x - 9 = 3y$
   (f) $y + 3x = 4$
   (g) $2x = y$

3. Use the following graph to answer the questions that follow.



(a) Find the gradient of each of the lines through the points C and D, and through the points B and C.
(b) Find the equation of each of the lines through the points A and B, and through the points B and C.
(c) Join the points A and C by a straight line, and then find the gradient and equation of the line AC, the y–intercept and the x–intercept.

4. By finding the x–intercept and the y–intercept of each of the following equations, draw their corresponding graphs.

(a) $3x - 2y = 6$
(b) $x - 2y = 4$
(c) $2x + y = 9$
(d) $x + y = 2$
(e) $2x = y + 6$
(f) $2y + 4 = x$
(g) $x - y = 1$
(h) $x + 2y = 2$

Solving linear simultaneous equations graphically

Activity 5: Recognising the point of intersection of two lines graphically

Individually or in groups, perform the following tasks:

1. Some values of the following tables are missing. Use the given equations to complete the tables.

   Table of values of $y = 2x + 1$

x	–3		–1	0		2		4
y		–3		1	3		7	9

   Table of values of $y = -x + 1$

x		–2		0		2		4
y	4	3	2		0		–2	–3

2. From task 1, use a graph paper to draw the two equations on the same xy–plane, and then read and record the point where the two equations intersect.
3. Check if the point you obtained in task 2 satisfies both equations, and give your conclusion.

When solving linear simultaneous equations graphically, the graphs of the equations are drawn on the same xy–plane, and the coordinates of the point of intersection give the solution. Sometimes, the lines of the graphs drawn do not meet or intersect (parallel lines), which means that the simultaneous equations have no solution.

Example 1

Draw the graphs of the following pair of simultaneous equations on the same xy–plane and then, find the point where the two lines intersect: $y = 2x - 1$ and $y = -x + 2$.

Solution

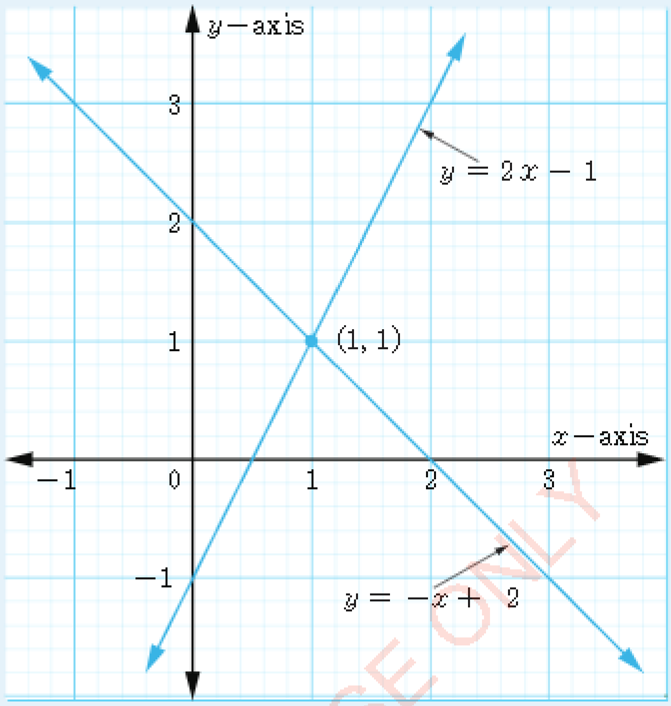
Table of values of $y = 2x - 1$

x	–1	0	1	2
y	–3	–1	1	3

Table of values of $y = -x + 2$

x	–1	0	1	2
y	3	2	1	0

Locate the ordered pairs of values for $y = 2x - 1$ on a graph paper, and join the points to make a line which is the graph of this equation. On the same *xy*–plane, locate the ordered pairs of values for $y = -x + 2$, and join the points to form a straight line which is its graph. Label each graph by its corresponding equation.



Therefore, the two lines intersect at the point (1, 1).

Note that: The point of intersection for both equations is the solution of the simultaneous equations.

Example 2

Find the solution of the following pair of linear simultaneous equations graphically:
$3x - y = 2$ and $2x - y = 3$ (use the values of x from –2 to 2).

Solution

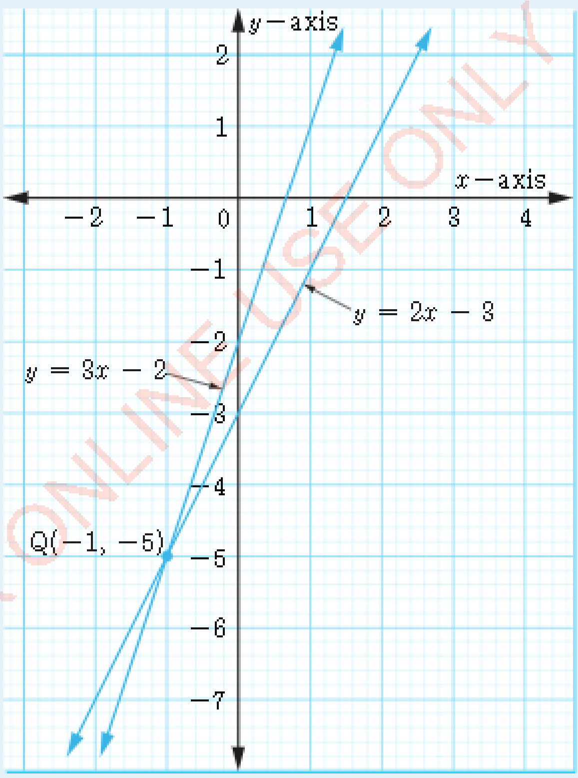
Table of values of $y = 3x - 2$

x	–2	–1	0	1	2
y	–8	–5	–2	1	4

Table of values of $y = 2x - 3$

x	–2	–1	0	1	2
y	–7	–5	–3	–1	1

Locate the ordered pairs of values for $y = 3x - 2$ on a graph paper, and join the obtained points to make a line which is the graph of this equation. On the same xy–plane, locate the ordered pairs of values for $y = 2x - 3$, and join the obtained points to make a straight line which is the graph of this equation. Label each graph by its corresponding equation.



Therefore, the two lines intersect at point Q(–1, –5).

To verify that the point Q(–1, –5) satisfies both equations:

$y = 3x - 2$	$y = 2x - 3$
Substitute $x = -1$ and $y = -5$:	Substitute $x = -1$ and $y = -5$:
$-5 = (3 \times (-1)) - 2$	$-5 = (2 \times -1) - 3$
$-5 = -3 - 2$	$-5 = -2 - 3$
$-5 = -5$.	$-5 = -5$.

Thus, the point satisfies both equations.

Therefore, the solution is (–1, –5).

Exercise 5

Find the solution of each of the following pairs of linear simultaneous equations graphically, and counter check your answers:

1. $\begin{cases} 2x + y = 4 \\ x + y = 3 \end{cases}$
2. $\begin{cases} x + y - 3 = 0 \\ x - y - 1 = 0 \end{cases}$
3. $\begin{cases} x + y = 1 \\ 2x + 3y = 5 \end{cases}$
4. $\begin{cases} x - 2y = -5 \\ x + 3y = 15 \end{cases}$
5. $\begin{cases} x - 2y = 1 \\ x + 2y = 5 \end{cases}$
6. $\begin{cases} 2x + y = 6 \\ 6x - y = 6 \end{cases}$
7. $\begin{cases} y = -2x + 5 \\ y = x - 4 \end{cases}$
8. $\begin{cases} 2y + 12 = 3x \\ 4y + 8 = 2x \end{cases}$
9. $\begin{cases} y + 1 = 3x \\ y - 3 = x \end{cases}$

Chapter summary

1. A coordinate plane is made up of two number lines intersecting at right angles.
2. The horizontal line is called the x–axis.
3. The vertical line is called the y–axis.
4. The point at which the two axes intersect is called the origin, labelled O.
5. The two axes divide the plane into four parts called quadrants.
6. Each point in the coordinate plane has two numbers called coordinates written as an ordered pair (x, y).
7. All points in quadrant I have positive coordinates.
8. All points in quadrant II have negative x values and positive y values.
9. All points in quadrant III have negative values for x and y.

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10. All points in quadrant IV have positive values for x but negative values for y.
11. A gradient of a straight line is denoted by m.
12. A gradient or slope m of a straight line is found by taking $m = \frac{\text{change in } y}{\text{change in } x}$.
13. A vertical line has an undefined gradient.
14. A horizontal line has a gradient whose value is 0.
15. The general form of equation of a line is $y = mx + c$, where m is the gradient and c is a constant known as the y–intercept.
16. Linear simultaneous equations have the uniform solution of an ordered pair (x, y).
17. Graphs of linear simultaneous equations intersect at a common point whose coordinates give the solution of an ordered pair (x, y).

Revision exercise

1. Locate the following pairs of points on a graph paper; join them by a straight line, and find the gradient of each line. State whether the gradient is negative, positive, undefined, or zero:
   (a) (0, 0), (1, 3)
   (b) (–1, –1), (8, 8)
   (c) (1, 0), (0, 1)
   (d) (7, 5), (2, 3)
   (e) (2.5, –3), (6.5, –4)
   (f) (3, 6), (6, 3)

2. Draw a graph of each of the following equations by first rewriting them in the form $y = mx + c$:
   (a) $x + 6y = 5$
   (b) $5x = 3y + 8$
   (c) $4x + 7y = 28$
   (d) $x - y = 1$
   (e) $-4x + 3y = 10$
   (f) $5x - y = 4$

3. Rewrite each of the following equations in the form $y = mx + c$ and determine their gradients and y–intercepts:
   (a) $4x + y = 8$
   (b) $6y = x + 2$
   (c) $2x - y = 2$
   (d) $3x + 4y = 23$
   (e) $3x + 4y + 4 = 0$
   (f) $3y + 4x + 9 = 0$

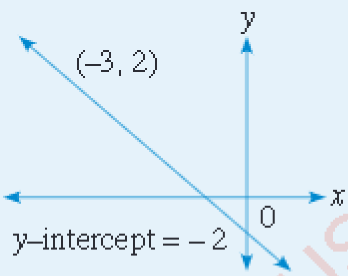
4. Draw the graph of each of the following equations:
   (a) $x - 2y = 4$ (c) $2x + 3y = 9$
   (b) $4x + y = 8$ (d) $6y = x + 2$

5. Use the following points and gradients to draw straight lines:
   (a) $(2, -1)$, gradient $\frac{1}{2}$. (c) $(1, 4)$ with the gradient -4
   (b) $(3, 1)$ and the y–intercept 3 (d) $(-2, 0)$ and the y–intercept $\frac{1}{3}$

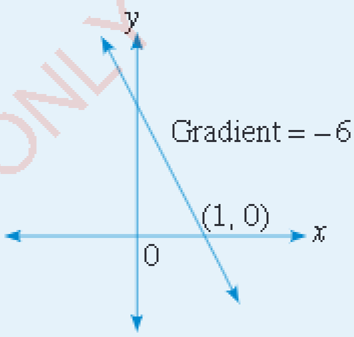
6. Find the solutions to the following linear simultaneous equations graphically, and then verify your answers by substituting the solutions in the equations:
   (a) $\begin{cases} x - y = -3 \\ x + 2y = 3 \end{cases}$ (c) $\begin{cases} 2x + y = 7 \\ 2x - 5y = 4 \end{cases}$ (e) $\begin{cases} y = \frac{1}{2}x + 4 \\ y = 2x - 2 \end{cases}$
   (b) $\begin{cases} 3 + 2y - 3x = 0 \\ 5x = 5 + y \end{cases}$ (d) $\begin{cases} y = \frac{1}{2}x + \frac{1}{2} \\ y = x - 1 \end{cases}$ (f) $\begin{cases} y = 3x - 2 \\ y = x + 2 \end{cases}$

7. Find the equations of the lines in each of the following graphs:

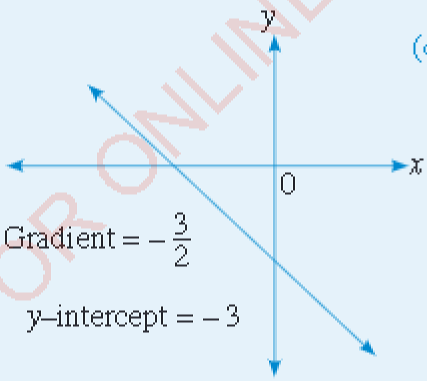
(a)



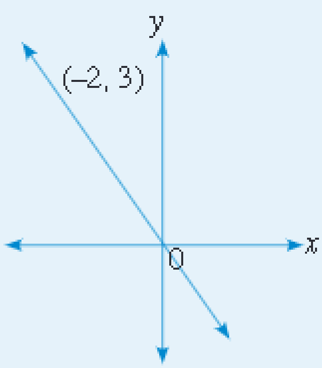
(b)



(c)



(d)



8. (a) Draw the graphs of the following equations on the same coordinate plane.
$y = -\frac{2}{3}x + 4$, $y = 4x + 4$ and $y = \frac{1}{2}x - 3$.
(b) What is the area of the region bounded by the three lines in (a) above?
(c) Find the coordinates of the vertices of the figure formed in (a) above.

9. For each of the following pairs of points, state whether the line passing through them is vertical or horizontal:
(a) (1, 0), (0, 0)
(b) (1, 2), (2, 1)
(c) (3, –2), (3, 5)
(d) (–4, 2), (–4, –1)
(e) (2, 8), (–2, 8)
(f) $\left(\frac{3}{4}, 1\right), \left(\frac{7}{3}, 4\right)$
(g) (0.5, –1), (0.5, –2)
(h) (0, 2), (0, –3)

10. Draw the graphs of the following linear equations on the same xy – plane:
$5y = 6x + 30$, $y = \frac{1}{5}x + 2$, and $3y + 4x = 12$.
(a) What shape is enclosed by the three graphs?
(b) Find the coordinates of the vertices of the figure formed.

11. Find the equation and y–intercept of each line passing through the following pairs of points:
(a) (4, 4), (–3, 2)
(b) (–7, 2), (2, 7)
(c) $\left(\frac{5}{6}, 2\right), \left(6, \frac{1}{6}\right)$
(d) (–2, –1), (4, 1)
(e) $\left(\frac{4}{7}, 6\right), \left(\frac{3}{2}, 6\right)$
(f) (–155, –541), (541, 155)

Project

Individually or in groups, perform the following tasks:
1. Locate a centre of a football pitch.
2. Locate all end points of the diagonals of the football pitch.
3. Measure the distances from the centre to each side of the football pitch.
4. Locate the coordinate points of the end points of diagonals. Use the centre of the pitch as your reference point.
5. Calculate the gradients (slopes) of the diagonals.
6. Find the equations of both diagonals.
7. What relationship can you deduce from the two equations in task 6?

Chapter Twelve

Perimeters and areas

Introduction

The word perimeter comes from the Greek word 'perimetros', which is derived from two words 'peri' which means 'around' and 'metron' which means 'measure', meaning to measure around. Calculating perimeter has several practical applications in daily life situations such as, estimating the length of a fence required to enclose a yard or a sports field, the circumference of a wheel to determine how far it will roll in one revolution, among many other applications. An area is the quantity that expresses the extent of a two dimensional region or shape in a plane. Mathematically, the area of a shape can be measured by comparing the shape to the square of a unit size. Therefore, the area is the number of squares. The first recorded use of areas and perimeters in the East was in ancient Babylon, where they used it to measure the amount of land that was owned by people for taxation purposes. In this chapter, you will learn about the perimeters of polygons, and the circumference of a circle. You will also learn about areas of triangles, quadrilaterals, and circles. The competencies developed will help you in several areas, for example; the perimeter of a building multiplied by the height of the walls gives an estimate of how much paint, siding, sheathing, and plywood will be used. For circular shapes, you can tell the distance along its circumference divided by pi to get the diameter which is used to estimate how much fertilizer or other materials are to be used per acre or any other specified unit. The knowledge will also help you to understand how the estimates of construction of buildings, roads, railways and the like are done.

Perimeters of polygons

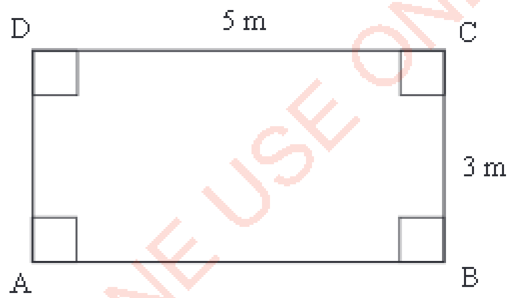
Activity 1: Identifying the formula for the perimeter and the area of a rectangle

Individually or in groups, perform the following tasks:

1. Measure the length and width of a chalkboard.
2. Explain how you will find the perimeter.
3. Divide the chalkboard into squares by using a ruler and a chalk.
4. Find the area and the perimeter of the chalkboard.
5. Examine the similarities and the differences of the perimeter and the area of the chalkboard.
6. Make a conclusion about the formula for the perimeter and the area of a rectangle.

Suppose that, the measurements around a top of a rectangular table are taken, starting from one corner to the next, then to the third, the fourth, and back to the first corner. The total length obtained is called the perimeter of the table.

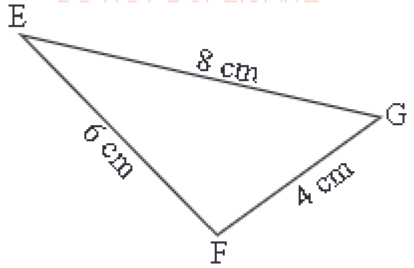
The following figure represents the floor of a small rectangular room.



Moving from A through B, C, D, and back to A gives a total length of 16 metres. This total length is the perimeter of the rectangular room. Therefore, for any rectangle, the perimeter is given by the sum of the lengths of all the four sides, that is,

Perimeter = length + width + length + width
= 2(length) + 2(width)
= 2(length + width).

Also, consider the following triangle EFG:



Moving from E through F, G and back to E, gives a total length of 18 centimetres. This total length is the perimeter of the triangle. Therefore, for any triangle, the perimeter is given by the sum of the lengths of all three sides, that is,

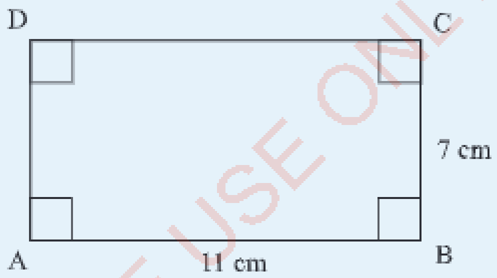
Perimeter = length 1 + length 2 + length 3.

The total length of all the sides of any closed polygon gives a perimeter.

Example 1

Find the perimeter of the rectangle ABCD with $\overline{AB} = 11$ cm and $\overline{BC} = 7$ cm.

Solution



The perimeter of a rectangle = 2(length + width)

$= 2(11 + 7)$ cm

$= 2(18)$ cm

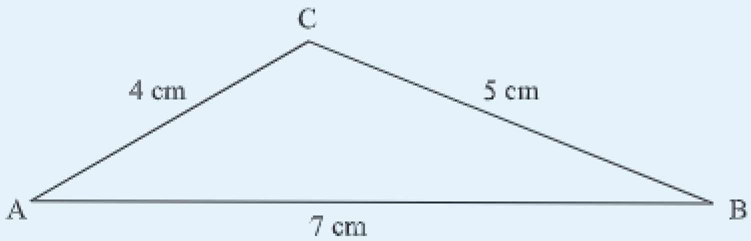
$= 36$ cm.

Therefore, the perimeter of the rectangle ABCD is 36 cm.

Example 2

Find the perimeter of the triangle ABC in which $\overline{AB} = 7$ cm, $\overline{BC} = 5$ cm, and $\overline{AC} = 4$ cm.

Solution



The perimeter of the triangle ABC $= \overline{AB} + \overline{BC} + \overline{AC}$

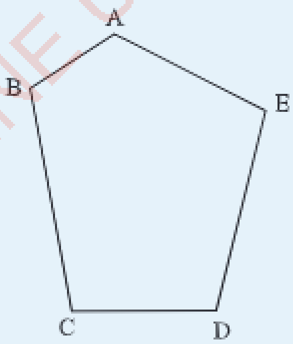
$= (7 + 5 + 4)$ cm

$= 16$ cm.

Therefore, the perimeter of the triangle ABC is 16 cm.

Example 3

Find the perimeter of the pentagon ABCDE, where $\overline{AB} = 2$ cm, $\overline{BC} = 13$ cm, $\overline{CD} = 5$ cm, $\overline{DE} = 9$ cm, and $\overline{EA} = 7$ cm.



Solution

The perimeter of the pentagon

$$ABCDE = \overline{AB} + \overline{BC} + \overline{CD} + \overline{DE} + \overline{EA}$$
$$= (2 + 13 + 5 + 9 + 7)\text{ cm}$$
$$= 36\text{ cm.}$$

Therefore, the perimeter of the pentagon ABCDE is 36 cm.

Example 4

The width of a rectangular garden is 4 metres less than its length. If the perimeter of the garden is 216 metres, find:

(a) the length of the garden.

(b) the width of the garden.

Solution

(a) Let the length of the garden be l.

The width of the garden is $l - 4\text{m}$.

Perimeter of the rectangular garden $= 2(l + w)$

$$216\text{ m} = 2(l + l - 4\text{ m})$$
$$216\text{ m} = 2(2l - 4\text{ m})$$
$$216\text{ m} = 4l - 8\text{ m}$$
$$4l = (216 + 8)\text{ m}$$
$$4l = 224\text{ m}$$
$$l = \frac{224\text{ m}}{4}$$
$$l = 56\text{ m.}$$

Therefore, the length of the garden is 56 metres.

(b) The width of the garden is given by;

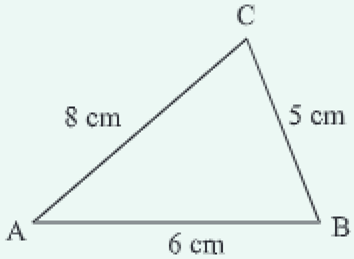
$$w = l - 4\text{m}$$
$$= (56 - 4)\text{m}$$
$$= 52\text{m.}$$

Therefore, the width of the garden is 52 metres.

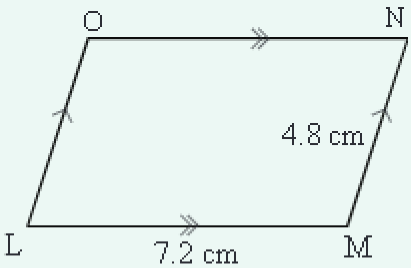
Exercise 1

Find the perimeter of each of the following polygons:

1.



2.



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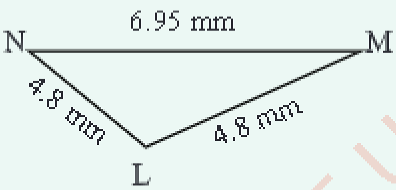
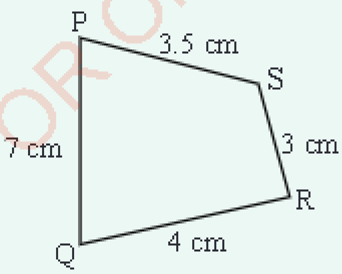


4.



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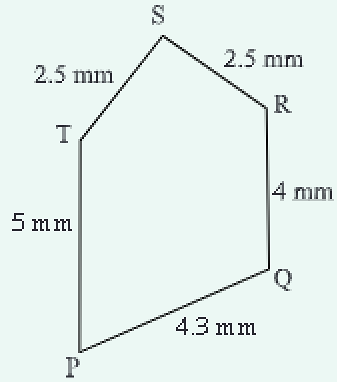

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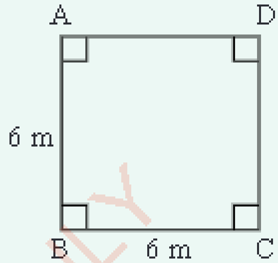
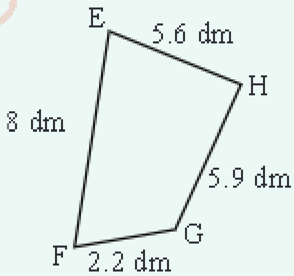
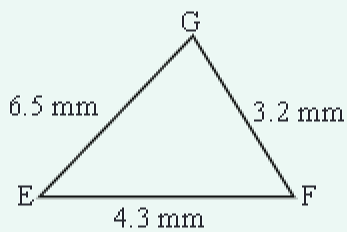


7.



8.



9. Find the perimeter of the floor of the rectangular room which is 10 metres long and 8 metres wide.
10. A football field is 105 metres long and 70 metres wide. Find its perimeter.
11. A rectangular shape of a table measures 20 decimetres by 15 decimetres. Find its perimeter.
12. The perimeter of a square is 96 centimetres. Find the length of one side of the square.
13. The perimeter of a rectangular field is 300 metres. If the length of the field is 90 metres, find its width.
14. The perimeter of the striking face of a matchbox is 12 millimetres. If the width of the face is 1.2 millimetres, find its length.
15. The sum of the length and one-half of the width of a rectangle is 42 metres. The rectangle's perimeter is 100 metres. Find the width of the rectangle.
16. An equilateral triangle has a perimeter of 18 centimetres. Find the length of one side.
17. The perimeter of an isosceles triangle is 15 centimetres. If the length of the base is 7 centimetres, find the length of each of the other sides.
18. Two sides of a triangle are 12 metres and 15 metres long. If the perimeter of the triangle is 34 metres, find the length of the third side.
19. The width of a rectangle is 23 metres less than its length. The perimeter of the rectangle is 94 metres. Find the dimensions of the rectangle.
20. The difference between the length and three times the width of a rectangle is 5 centimetres. Find the length and the width of the rectangle if its perimeter is 82 centimetres.

Circumference of a circle

Activity 2: Recognizing the relationship between the circumference of a circle and its diameter

Individually or in groups, performe the following tasks:

1. Collect ten round objects.

2. Measure the circumference and the diameter of each object using a millimeter measuring tape.

3. Record the measurements in the given table (*c* represents the circumference and *d* represents the diameter).

4. Compute the value of $\frac{c}{d}$ to the nearest hundredths for each object. Record the results in the fourth column of the table.

5. Study the answers you have obtained in task 4.

6. Make a conclusion about the relationship between the circumference and the diameter of a circle. Share your results with other groups. Discuss and make a conclusion with your teacher.

Object	c	d	$\frac{c}{d}$
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

The perimeter of a circle is called a circumference. When the circumference is divided by the diameter (*d*), the result is a constant number called pi, denoted by the symbol π. The value of pi is approximately equal to $\frac{22}{7}$ or 3.14.

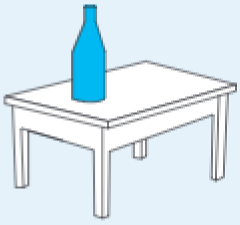
Activity 3: Estimation of the value of pi (π)

Individually or in groups, perform the following tasks:

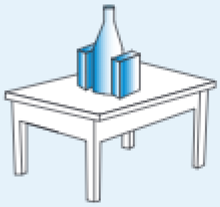
1. (a) Take a round bottle, a tin, or a bucket. Put it on a table.
   (b) Place 2 pieces of wood on the opposite sides of the bottle as shown in the following figures:

(c) Measure the length between the pieces of wood. This gives the diameter of the bottle.

(i)



(ii)



2. (a) Wrap a string round the bottle and mark the ends of the string.
   (b) Measure the length of the string with a ruler to obtain the circumference of the bottle.
3. Estimate the value of pi (π) by using the measurements obtained in 1(c) and 2(b).

$$\pi = \frac{\text{Circumference}}{\text{Diameter}}.$$

4. Estimate the value of π by using the following objects:
   (a) The base of a round plate.
   (b) The base of a bottle.
   (c) A bicycle tyre.

Record the results as shown in the following table.

Object	Circumference	Diameter	$\frac{\text{Circumference}}{\text{Diameter}}$
Base of a bottle			
Base of a round plate			
Bicycle tyre			

What is the estimated value of π?

When given the radius of a circle r or its diameter d, the circumference of the circle can be found as follows:

$$\pi = \frac{\text{Circumference}}{d}$$

$$\text{Circumference} = \pi d$$
$$= 2\pi r \text{ (since } d = 2r\text{)}.$$

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5. Comment on the estimated value of pi.
6. Share your results with other groups, discuss, and make a conclusion with your teacher.

Example 1

Find the circumference of a circle of radius 7 cm (use $\pi = 3.14$).

Solution

Circumference of a circle = $2\pi r$

$= 2 \times 3.14 \times 7$ cm

$= 43.96$ cm.

Therefore, the circumference of the circle is 43.96 cm.

Example 2

Find the circumference of a circle of diameter 21 cm (use $\pi = 3.14$).

Solution

Circumference of the circle = πd

$= 3.14 \times 21$ cm

$= 65.94$ cm.

Therefore, the circumference of the circle is 65.94 cm.

Example 3

The circumference of a circle is 88 cm. Find its radius $\left(\text{use } \pi = \frac{22}{7}\right)$.

Solution

The formula for circumference of a circle is $2\pi r$

Given; circumference of a circle= 88 cm

$$C = 2\pi r$$

$$88 \text{ cm} = 2 \times \frac{22}{7} \times r$$

$$r = \frac{88 \text{ cm} \times 7}{2 \times 22}$$

$$= 2 \text{ cm} \times 7$$

$$= 14 \text{ cm}.$$

Therefore, the radius of the circle is 14 cm.

Example 4

A piece of wire is bent to form a circle of radius 42 cm. The wire is then bent into a rectangular shape with the length of 80 cm. Find the width of the rectangle formed $\left(\text{use } \pi = \frac{22}{7}\right)$.

Solution

Circumference of a circular piece of wire = perimeter of the rectangular shaped wire.

Circumference of a circular piece of wire $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 42 \text{ cm}$$

$$= 2 \times 22 \times 6 \text{ cm}$$

$$= 264 \text{ cm}.$$

Thus, the circumference of the circular piece of wire = 264 cm. But, the circumference of the wire = length of the wire.

Hence, the length of the piece of wire = 264 cm.

The perimeter of the rectangular shaped wire $= 2(l + w)$

$$264 \text{ cm} = 2(80 \text{ cm} + w)$$

$$\frac{264 \text{ cm}}{2} = 80 \text{ cm} + w$$

$$132 \text{ cm} = 80 \text{ cm} + w$$

$$w = 132 \text{ cm} - 80 \text{ cm}$$

$$w = 52 \text{ cm}.$$

Therefore, the width of the formed rectangle is 52 cm.

Example 5

The diameter of a certain semi-circular slice of a watermelon is 14 cm. What will be the perimeter of the slice of the water melon? $\left(\text{use } \pi = \frac{22}{7}\right)$.

Solution

Given;

Diameter (d) = 14 cm

Radius (r) $= \frac{d}{2} = \frac{14 \text{ cm}}{2} = 7$ cm.

Perimeter = circumference

Circumference (c) = $2\pi r$

$= 2 \times \frac{22}{7} \times 7$ cm

$= 44$ cm.

Thus, the perimeter of the semi-circular slice $= \frac{c}{2} + 2r$

$= \frac{44 \text{ cm}}{2} + 2 \times 7$ cm

$= 22$ cm $+ 14$ cm

$= 36$ cm.

Therefore, the perimeter of the slice of the water melon is 36 cm.

Exercise 2

In question 1 to 10, find the circumference of a circle with the given dimension (use $\pi = 3.14$):

1. Radius 10.5 mm
2. Radius 7.77 mm
3. Radius 17.5 mm
4. Radius 12.6 mm
5. Radius 15.26 mm
6. Diameter 10 cm
7. Radius 15 cm
8. Diameter 20 cm
9. Radius 24 cm
10. Diameter 20.4 cm

11. The circumference of a circular floor of a room is 25.14 m. Find the radius of the floor (use $\pi = 3.14$).

12. The circumference of a circular fence is 6 600 metres. Find the radius of the fence $\left(\text{use } \pi = \frac{22}{7}\right)$.

13. The circumference of a bicycle wheel is 188.40 cm. Find the diameter of the wheel (use $\pi = 3.14$).

14. The circumference of a circular field is 4 840 metres. Find its radius $\left(\text{use } \pi = \frac{22}{7}\right)$.

15. Find the radius of a ring whose circumference is 24.2 dm (use $\pi = 3.14$).

16. Find the circumference of a circle whose diameter is 44 dm (use $\pi = 3.14$).

17. Find the diameter of a circle whose circumference is 66 dm $\left(\text{use } \pi = \frac{22}{7}\right)$.

18. The radius of a circular park is 63 metres. Find the cost of fencing it at 3 000 Tanzanian shillings per metre.

19. Find the circumference of a circle whose diameter is 13 cm (use $\pi = 3.14$).

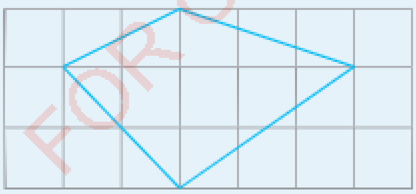
20. A circular pond has 75 cm wide footpath along its edge. A person walks 56 cm per step around the outer edge of the footpath. In 354 steps, this person makes a complete round. Find the diameter of the pond (use $\pi = 3.14$).

Area

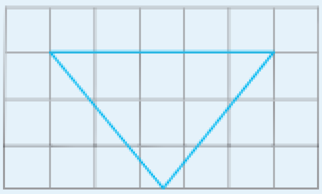
Activity 4: Approximating the area of a polygon figure

Observe the following figures and then perform the tasks that follow:

(a)



(b)



1. Count the number of complete squares in each figure.
2. Count the number of incomplete squares in each figure.
3. Approximate the area of the given figures.
4. Share your findings with your fellow students through discussion.

The area can be defined as the space occupied by a flat shape or the surface of an object. The area of a figure is the number of unit squares that cover the surface of the closed figure. Area is measured in square units such as square centimetres, square metres, and so on. For example; the rectangle in Figure 12.1 is divided into 12 square units. Thus, the area of the rectangle is 12 square units.

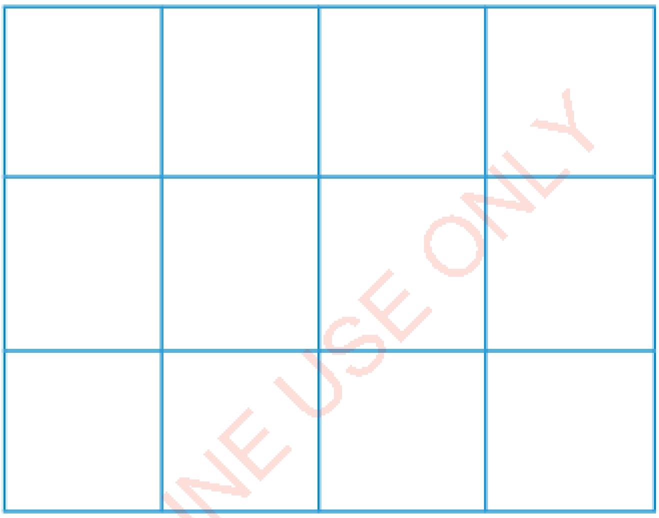
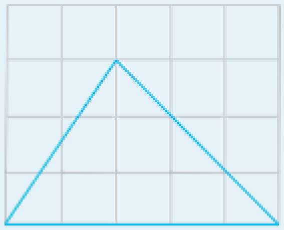


Figure 12.1: *Square units in a rectangle*

Example 1

Find the approximate area of the triangle in the following figure.



Solution

To find the approximate area of this triangle, count the complete squares and incomplete squares and then, find the sum of the number of complete squares and $\frac{1}{2}$ the number of incomplete squares.

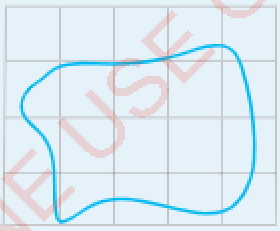
That is, area of the triangle = Number of complete squares + $\frac{1}{2}$ (Number of incomplete squares)

$$\begin{aligned}\text{Area of the triangle} &= 4 + \frac{1}{2}(7)\\ &= 4 + 3.5\\ &= 7.5.\end{aligned}$$

Therefore, the area of the triangle is approximately 7.5 square units.

Example 2

Find the approximate area of the irregular shape given in the following figure.



Solution

Area = Number of complete squares + $\frac{1}{2}$ (Number of incomplete squares)

$$\begin{aligned}&= 4 + \frac{1}{2}(14)\\ &= 4 + 7\\ &= 11.\end{aligned}$$

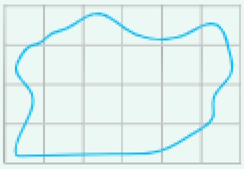
The total number of square units contained in the given shape is approximately 11.

Therefore, the area of the shape is 11 square units.

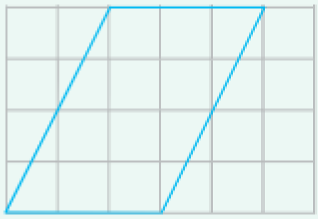
Exercise 3

In question 1 to 6, estimate the area of each figure in square units.

1.



2.



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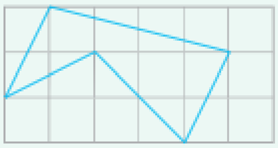

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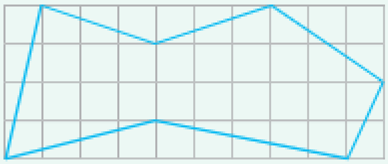

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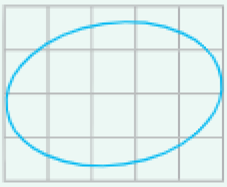

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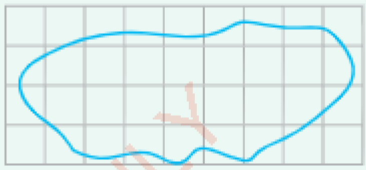
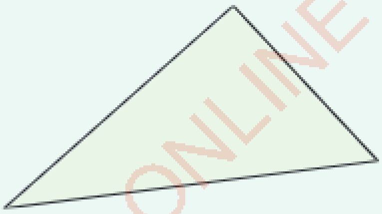


In question 7 to 12, estimate the area of each of the given figures in square units measured in centimetres square units. Use tracing papers to copy the figures.

7.



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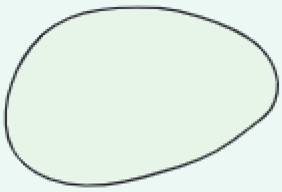

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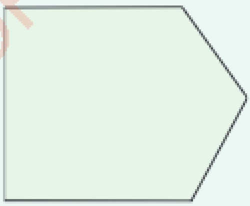
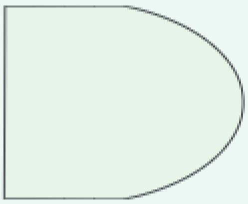
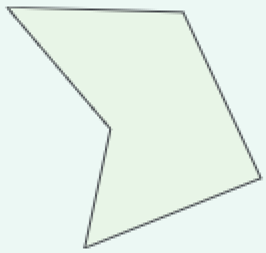


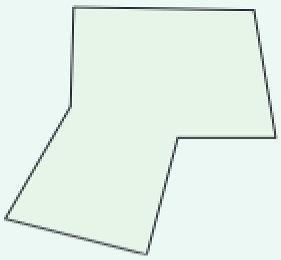
10.



11.

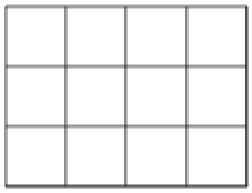


12.



Areas of rectangles and squares

Consider a rectangle 4 cm long and 3 cm wide. The area of the rectangle is determined by drawing square units as shown in the following figure.



There are 12 centimetre square units in the rectangle.

Therefore, the area of the rectangle is 12 cm^2.

Altenatively, this result can be obtained by taking 4 cm $\times$ 3 cm = 12cm^2, which is length $\times$ width.

Thus, the area of a rectangle = length $\times$ width.

Example 1

Find the area of a rectangle whose length is 20 centimetres, and its width is 10 centimetres.

Solution

Area of a rectangle = length $\times$ width

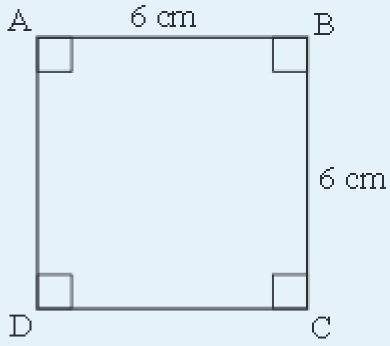
$= 20 \text{ cm} \times 10 \text{ cm}$

$= 200 \text{ cm}^2$.

Therefore, the area of the rectangle is 200 cm^2.

Example 2

Find the area of the following square ABCD.



Solution

If A represents the area of the square ABCD then,

A = length × width

Since the sides of a square are equal, then, length = width. Let the length of ABCD be l, then

$$A = l \times l = l^2$$
$$= 6 \text{ cm} \times 6 \text{ cm},$$
$$= 36 \text{ cm}^2.$$

Therefore, the area of a square ABCD is 36 cm^2.

Example 3

The area of a rectangle is 48 cm^2. Find its length if its width is 6 cm.

Solution

Let the length be l and the width be w, then the area A of a rectangle is

$$A = l \times w$$

$$48 \text{ cm}^2 = l \times 6 \text{ cm}$$

$$l = \frac{48 \text{ cm}^2}{6 \text{ cm}}$$

$$= 8 \text{ cm}.$$

Therefore, the length of the rectangle is 8 cm.

Area of a triangle

Activity 5: Determining the area of a triangle

Individually or in your group, perform the following tasks:

1. On a graph paper, construct ΔPQR with base $\overline{QR}$ and an altitude from the vertex P through Q to the base $\overline{QR}$ and name it *h*.
2. Through P and R, construct the lines parallel to $\overline{QR}$ and $\overline{PQ}$, respectively, which intersect at T.
3. Comment on the relationship between $\overline{PQ}$ and $\overline{TR}$, $\overline{PT}$ and $\overline{QR}$.
4. Cut out the triangle PTR and place it over the triangle PQR such that P falls on R, R falls on P, and T falls on Q.
5. Comment on the relationship between the area of the triangle PQR and the area of the triangle PRT.
6. Compare the total areas of the two triangles, and the area of the rectangle QPTR formed.
7. Identify the formula of the area of a triangle.
8. Post your results on the classroom wall, and share them with other groups through gallery walk under your teacher's supervision.

In Figure 12.2, ABC is a triangle. $\overline{AB}$ is the height of the triangle ABC and $\overline{BC}$ is the base of the triangle ABC. Thus, the area of ΔABC is half the area of the rectangle ABCD, that is,

Area of $\Delta ABC = \frac{1}{2} \times \overline{AB} \times \overline{BC}$

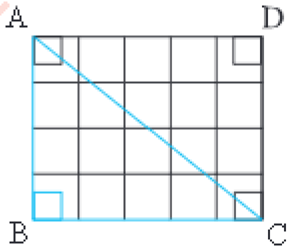
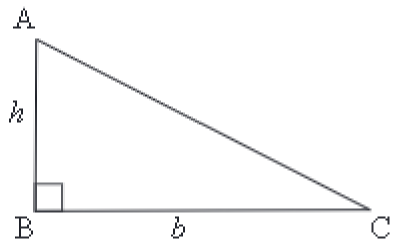


Figure 12.2: *The triangle ABC*

Similary, consider the following right-angled triangle:

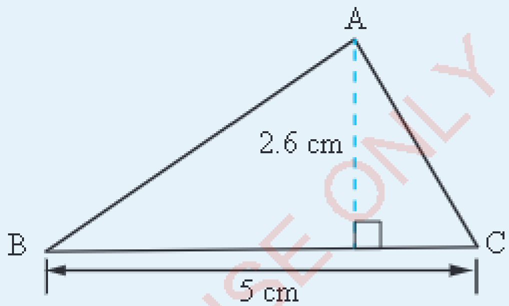


$\overline{AB}$ is the height denoted by h, and $\overline{BC}$ is the base denoted by b. The area A of a triangle is given as, $A = \frac{1}{2} \times \text{base} \times \text{height}$.

Thus, $A = \frac{1}{2} bh$.

Example 1

Find the area of the triangle ABC shown in the following figure.



Solution

From the triangle ABC, base (b) = 5 cm, height (h) = 2.6 cm.

$$\text{Area of a triangle} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 5 \text{ cm} \times 2.6 \text{ cm}$$

$$= 6.5 \text{ cm}^2.$$

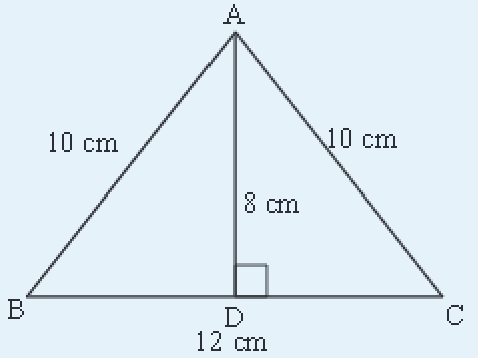
Therefore, the area of the triangle ABC is 6.5 cm².

Example 2

Find the area of an isosceles triangle ABC with sides $\overline{AB}$ = 10 cm, $\overline{AC}$ = 10 cm, $\overline{BC}$ = 12 cm, and the height $\overline{AD}$ = 8 cm.

Solution

From the given information, the following triangle can be obtained:



From the triangle ABC, base = 12 cm and height = 8 cm.

$$\text{Area} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 12 \text{ cm} \times 8 \text{ cm}$$

$$= 48 \text{ cm}^2.$$

Therefore, the area of the triangle ABC is 48 cm².

Example 3

The area of a triangle is 45 cm². If its base is 6 cm, find its height.

Solution

Given, Area = 45 cm², base b = 6 cm, required to find the height h:

$$\text{Area of a triangle} = \frac{1}{2} \times b \times h$$

$$\frac{1}{2} \times 6 \text{ cm} \times h = 45 \text{ cm}^2$$

$$3h \text{ cm} = 45 \text{ cm}^2$$

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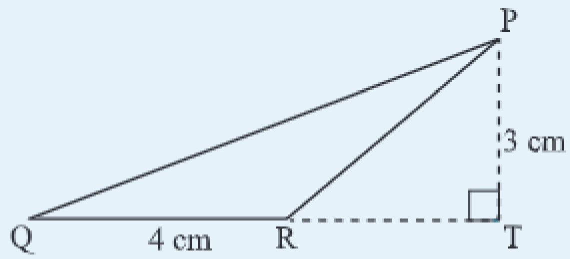
$$h = \frac{45 \text{ cm}^2}{3 \text{ cm}}$$

$$= 15 \text{ cm}.$$

Therefore, the height is 15 cm.

Example 4

Find the area of the triangle PQR as shown in the following figure.



Solution

From the given triangle PQR, base (b) = 4 cm and height (h) = 3 cm.

$$\text{Area} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 4 \text{ cm} \times 3 \text{ cm}$$

$$= 6 \text{ cm}^2.$$

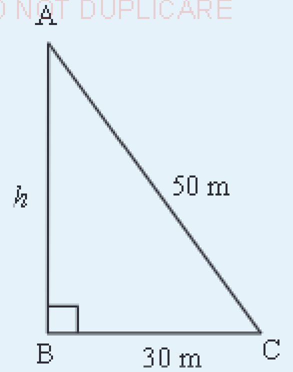
Therefore, the area of the triangle PQR is 6 cm².

Example 5

The cost of printing a pattern in a cloth is 250 Tanzanian shillings per square metres. Find the cost of printing a right triangular piece of cloth whose base is 30 m and its hypotenuse is 50 m.

Solution

From the given information, the following triangle can be obtained:



Before finding the cost, first we calculate the area of the triangular piece of cloth. In order to calculate the area, the height of the triangle must be obtained. The Pythagoras theorem is used to calculate the height:

From the triangle ABC,

$\overline{AB}^2 + \overline{BC}^2 = \overline{AC}^2$

$h^2 + (30\text{ m})^2 = (50\text{ m})^2$

$h^2 + 900\text{ m}^2 = 2\,500\text{ m}^2$

$h^2 = (2\,500 - 900)\text{ m}^2$

$h = \sqrt{1\,600\text{ m}^2}$

$h = 40\text{ m}$.

Thus, the height of the triangle is 40 m.

The area of the triangle is given by:

$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 30\text{ m} \times 40\text{ m}$

$A = 600\text{ m}^2$.

Now, calculate the cost of printing 600 m², if the cost of printing 1 m² is 250 Tanzanian shillings:

Let x be the cost of printing 600m²,

Cost of printing 1 m² = 250 Tanzanian shillings

Cost of printing 600 m² = x

$$x = \frac{\text{Tsh } 250 \times 600\text{ m}^2}{1\text{ m}^2}$$

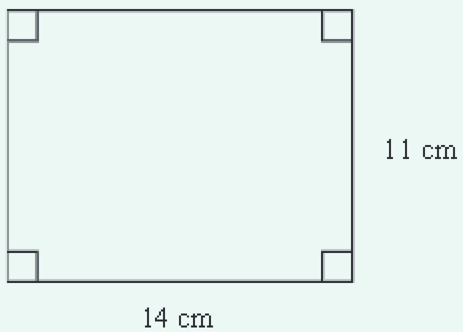
= 150 000 Tanzanian shillings.

Therefore, the cost of printing the triangular piece of cloth is 150 000 Tanzanian shillings.

Exercise 4

In question 1 to 9, find the area of each of the figures:

1.



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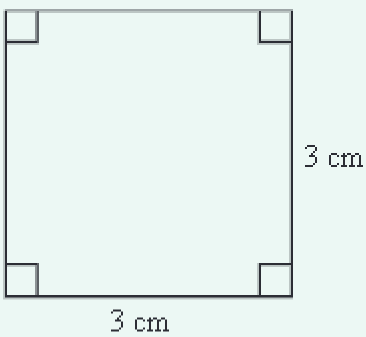

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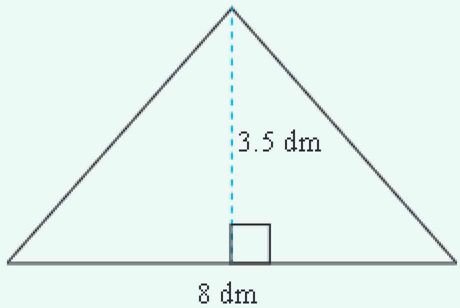

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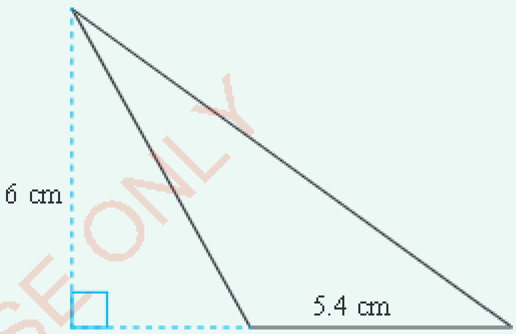

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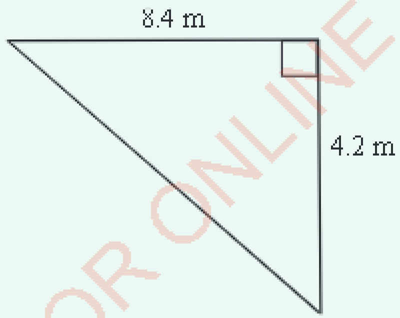

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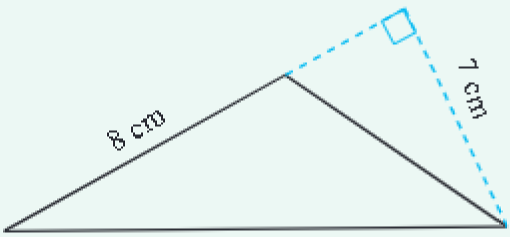
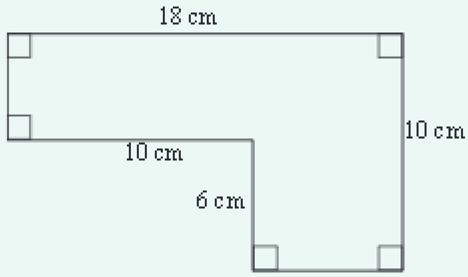
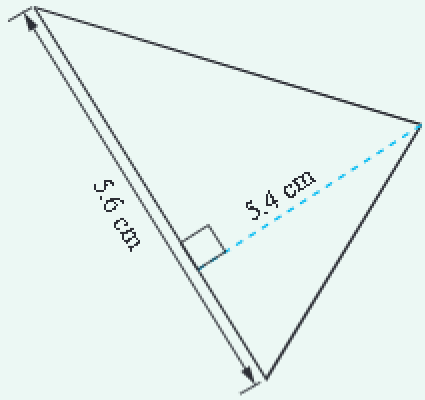
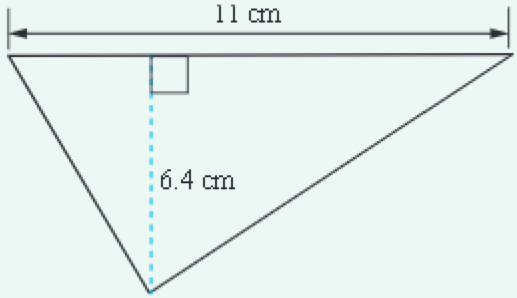


7. 

8. 

9. 

10. The floor of a room has a square shape of length 8 m. Find the area of the floor.
11. A rectangular field is 95 m long and 75 m wide. Find the area of the field.
12. Find the area of a triangle whose base is 20 cm and, height is 15 cm.
13. The height of a triangle ABC from point A to a horizontal line BC is 12 cm. If $\overline{BC}$ is 24 cm, find the area of the triangle.
14. Find the area of an isosceles triangle whose base is 10 cm and one of the equal sides is 13 cm.
15. The area of a rectangle is 880 cm². If the length of the rectangle is 44 cm, find its width.
16. A rectangular top of a table is 2 m long. If its area is 3.96 m², find its width.
17. The area of a triangle is 36 m². If its base is 9.6 m, find its height.
18. Find the area of a triangle MNO with $\overline{MN}$ = 7.5 cm, $\overline{NO}$ = 13 cm, and $M\hat{N}O = 90°$.
19. The perimeter of a rectangle is 144 mm. If the width of the rectangle is 30 mm, find the area of the rectangle.

20. A rectangle is 72 mm long and 40 mm wide. If a triangle with a base of 60 mm is equal to the area of the rectangle, find the height of the triangle.
21. Find the area of the right-angled triangle ABC if $\overline{AB} = 4$ cm, $\overline{BC} = 5$ cm, and $\overline{AC} = 3$ cm.
22. A rectangular box has the length of 10 centimetres, width 6 centimetres, and height 8 centimetres. What is the total area of the sides of the box?

Areas of parallelograms and trapezia

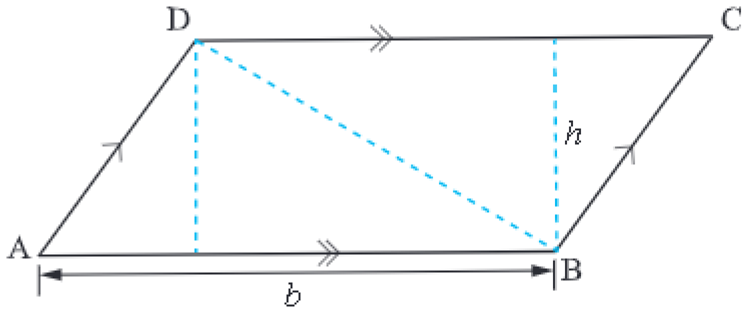
Activity 6: Recognising the formula for calculating the area of a parallelogram

Individually or in your groups, perform the following tasks:

1. On a graph paper, draw a parallelogram with vertices A, B, C and D, with $\overline{AB}$ as its base.
2. Consider line segment AB as the base, and construct the height BE.
3. By using a pair of scissors or razor blade, cut out the triangular piece BCE at an angle of 90° by following the height of the parallelogram.
4. Place the triangular piece BCE to the opposite side of the parallelogram. Make sure the triangular piece fits exaclty without any overlaps or gaps.
5. What did you observe in task 4? What is the name of the resulting figure?
6. Identify the formula which you can use to find the area of the resulting figure in task 5.
7. Use the formula you have identified in task 6 to recognise the area of the parallelogram.
8. Share you findings with your fellow students through presentation, and make a conclusion with your teacher.

Area of a parallelogram

Consider the following figure ABCD:



Area of $\Delta ABD = \frac{1}{2}bh$

Area of $\Delta DBC = \frac{1}{2}bh$

Area of parallelogram ABCD = Area of ΔABD + Area of ΔDBC

$$= \frac{1}{2}bh + \frac{1}{2}bh$$

$$= bh.$$

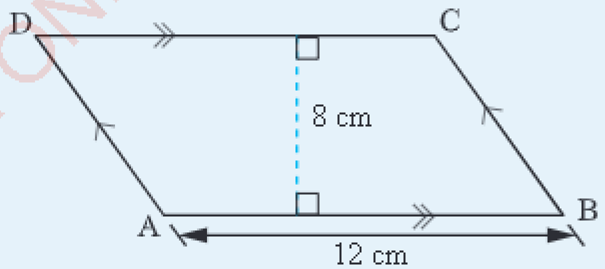
Therefore, the area of any parallelogram = base × height.

Example 1

If ABCD is a parallelogram in which $\overline{AB} = 12$ cm, and the distance between $\overline{AB}$ and $\overline{CD}$ is 8 cm, find its area.

Solution

The area of the parallelogram can be obtained from the given information:



Area of a parallelogram $= b \times h$

$= 12 \text{ cm} \times 8 \text{ cm}$

$= 96 \text{ cm}^2$.

Therefore, the area of the parallelogram ABCD is 96 cm².

Example 2

The altitude of a parallelogram is four times its base. If its area is 1 296 cm², find its base and its altitude.

Solution

Let b = base of the parallelogram

h = altitude of the parallelogram

A = area of the parallelogram

Given; $h = 4b$

$A = bh$

$= b \times 4b$

$1\,296 \text{ cm}^2 = 4b^2$

$$\frac{1\,296 \text{ cm}^2}{4} = \frac{4b^2}{4}$$

$$\sqrt{324 \text{ cm}^2} = \sqrt{b^2}$$

$18 \text{ cm} = b$.

$h = 4b$

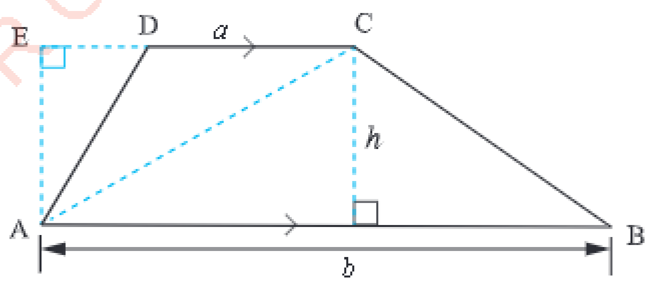
$h = 4 \times 18 \text{ cm}$

$h = 72 \text{ cm}$.

Therefore, the base of the parallelogram is 18 cm and its altitude is 72 cm.

Area of a trapezium

Consider the following figure ABCDE:



Area of $\Delta ABC = \frac{1}{2}bh$

Area of $\Delta ACD = \frac{1}{2}ah$

Area of trapezium ABCD = Area of ΔACD + Area of ΔABC

$$= \frac{1}{2}ah + \frac{1}{2}bh$$

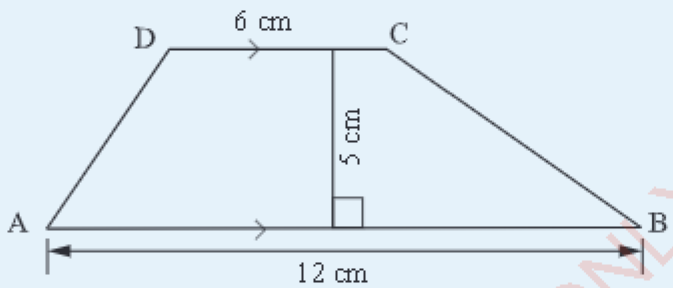
$$= \frac{1}{2}(a + b)h.$$

Therefore,

the area of any trapezium $= \frac{1}{2} \times$ sum of the lengths of the parallel sides $\times$ height.

Example 1

Find the area of the trapezium given in the following figure:



Solution

$$\text{Area} = \frac{1}{2} \times (a + b)h$$

$$= \frac{1}{2} \times (6 \text{ cm} + 12 \text{ cm}) \times 5 \text{ cm}$$

$$= \frac{1}{2} \times 18 \text{ cm} \times 5 \text{ cm}$$

$$= 9 \text{ cm} \times 5 \text{ cm}$$

$$= 45 \text{ cm}^2.$$

Therefore, the area of the trapezium is 45 cm^2.

Example 2

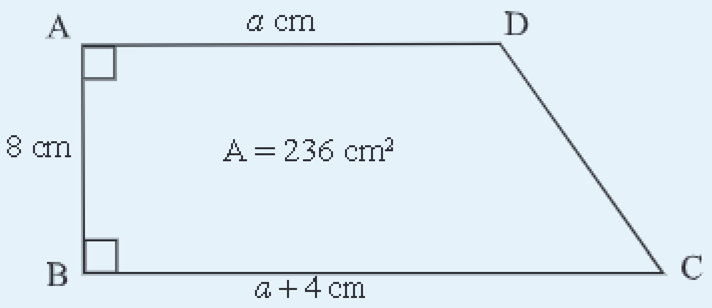
The area of a trapezium whose height is 8 cm, is 236 cm^2. If one of its parallel sides is longer than the other by 4 cm, find the lengths of the two parallel sides.

Solution

Let a = length of the shorter side,

$b = a + 4$ cm = length of the longer side

From the given information, the following trapezium can be constructed:



$$\text{Area} = \frac{1}{2} \times (a + b)h$$

$$236 \text{ cm}^2 = \frac{1}{2} \times (a + a + 4 \text{ cm}) \times 8 \text{ cm}$$

$$236 \text{ cm}^2 = \frac{1}{2} \times (2a + 4 \text{ cm}) \times 8\text{cm}$$

$$236 \text{ cm}^2 = (2a + 4 \text{ cm}) \times 4 \text{ cm}$$

$$2a + 4 \text{ cm} = \frac{236 \text{ cm}^2}{4 \text{ cm}}$$

$$2a + 4 \text{ cm} = 59 \text{ cm}$$

$$2a = (59 - 4) \text{ cm}$$

$$2a = 55 \text{ cm}$$

$$a = \frac{55 \text{ cm}}{2}$$

$$a = 27.5 \text{ cm}$$

$$b = (27.5 + 4) \text{ cm}$$

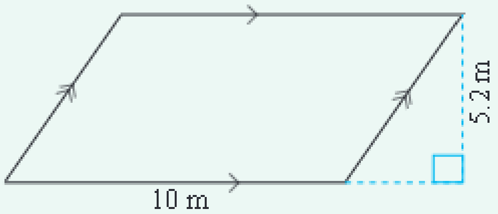
$$b = 31.5 \text{ cm}.$$

Therefore, the lengths of the two parallel sides are 27.5 cm and 31.5 cm.

Exercise 5

In question 1 to 10, find the area of each figure:

1.



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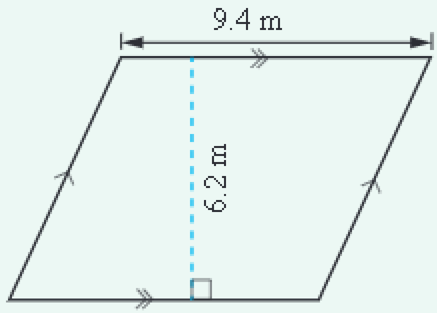

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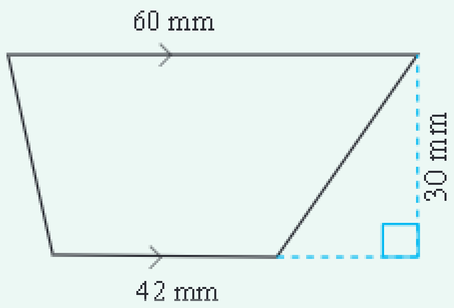

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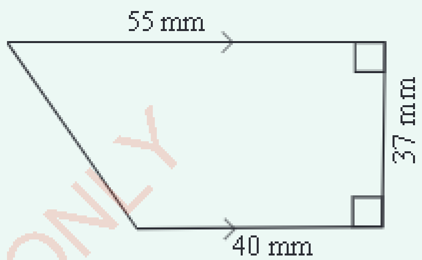

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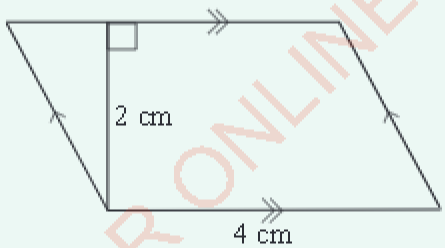

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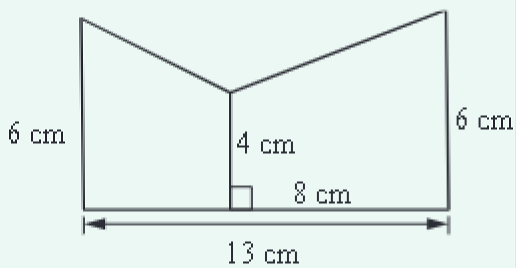

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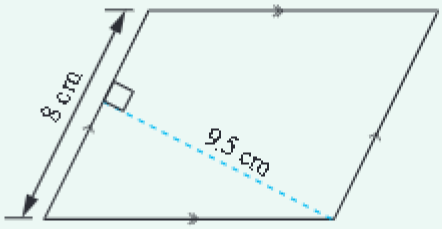
7.



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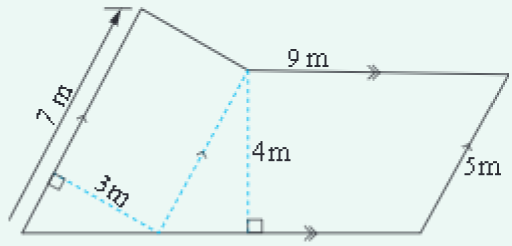

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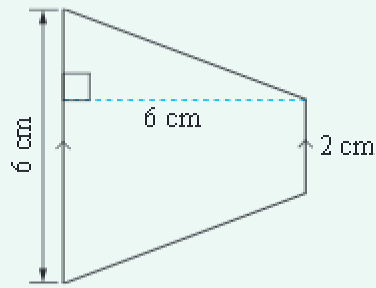
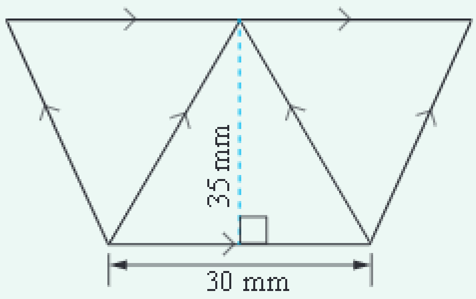


10.



11. The area of a parallelogram is 154 square centimetres. If its height is 22 centimetres, find its base.
12. Find the area of a parallelogram whose base is 21 centimetres, and its height is 13 centimetres.
13. Find the area of a trapezium whose parallel sides are 14 centimetres and 6 centimetres long and its height is 20 centimetres.
14. The height of a trapezium is 13 centimetres. If one of the parallel sides is 20 centimetres, and the area of the trapezium is 260 square centimetres, find the length of the other parallel side.
15. The height of a parallelogram is 160 millimetres. If the area of the parallelogram is 25 600 square millimetres, find its base.
16. The area of a trapezium is 3 000 square millimetres. If the parallel sides are 300 millimetres and 150 millimetres long, find the height of the trapezium.
17. A grazing field in a shape of a parallelogram has an area of 14 400 square metres. If the perpendicular distance from the base of the field is 72 metres, find the length of the base of the field.

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18. A trapezium with the height of 60 millimetres and the base of 90 millimetres has an area of 6 000 square millimetres. Find the length of the other side.
19. The area of a rhombus is 64 square centimetres. If the height of the rhombus is 6.4 centimetres, find its perimeter.
20. Find the area of a rhombus whose base is 100 millimetres, and its height is 84.75 millimetres.

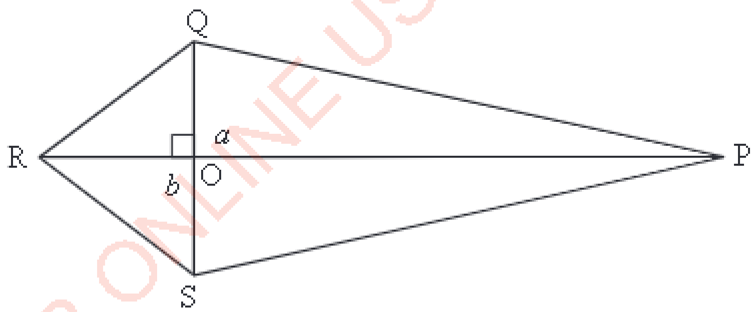
Area of a kite

Activity 7: Determining the area of a kite by using familiar shapes

Individually or in groups, perform the following tasks:

1. Obtain four kites of different sizes and measure their dimensions.
2. Divide the kites into other shapes such as triangles, rectangles, and the like.
3. Using your ruler, measure and estimate the areas of the resulting shapes in task 2.
4. Share your answers with other members of your class and discuss.

A kite is a special quadrilateral in which two pairs of adjacent sides are equal. Consider the following kite PQRS, with diagonals PR (a) and QS (b):



$\overline{OQ} = \overline{OS} = \frac{\overline{QS}}{2} = \frac{b}{2}$, $\overline{PR} = a$.

Area of a kite = Area of ΔPQR + Area of ΔPSR

Area of $\Delta PQR = \frac{1}{2} \times a \times \frac{b}{2}$

Area of $\Delta PSR = \frac{1}{2} \times a \times \frac{b}{2}$

Thus, area of the kite = Area of ΔPQR + Area of ΔPSR

$$\frac{1}{2}\times a\times\frac{b}{2}+\frac{1}{2}\times a\times\frac{b}{2}=\frac{ab}{4}+\frac{ab}{4}$$
$$=\frac{2ab}{4}$$
$$=\frac{ab}{2}$$

Therefore, the area of a kite = $\frac{1}{2}ab$,

where a = length of a long diagonal of a kite
b = length of a short diagonal of a kite.

Example 1

Find the area of a kite whose long and short diagonals are 22 centimetres, and 12 centimetres, respectively.

Solution

Given; $a = 22$ cm and $b = 12$ cm.

Area of the kite $=\frac{1}{2}ab$

$$=\frac{1}{2}\times 22\text{ cm}\times 12\text{ cm}$$
$$=132\text{ cm}^2.$$

Therefore, the area of the kite = 132 cm^2.

Example 2

The area of a kite is 126 square centimetres, and one of its diagonals is 21centimetres long. Find the length of its other diagonal.

Solution

Given; area of the kite = 126 cm^2

Length of one of its diagonals = 21 cm

Area of the kite $=\frac{1}{2}ab$

$$126\text{ cm}^2=\frac{1}{2}\times a\times 21\text{ cm}$$

$$126\text{ cm}^2\times 2=21a\text{ cm}$$

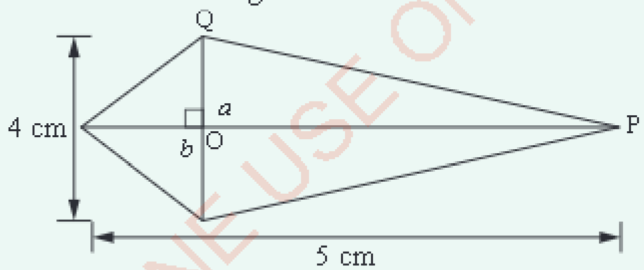
$$a = \frac{252 \text{ cm}^2}{21 \text{ cm}}$$

$a = 12$ cm.

Therefore, the length of the other diagonal is 12 centimetres.

Exercise 6

1. Find the area of a kite whose lengths of diagonals are 3 centimetres and 10 centimetres.
2. The diagonals of a kite are $\sqrt{10}$ centimetres and $\sqrt{5}$ centimetres. Find the area of the kite.
3. The diagonals of a kite are $(2x + 1)$ centimetres and $\frac{1}{8}$ centimetres. Express the area of the kite in a simplified form.
4. Find the area of a kite, if the dimensions of its diagonals are $(x + 3)$ units and $(x - 6)$ units.
5. Two equilateral triangles with sides of length l units are connected so that they share a side. If each triangle has height h units, express the area of the formed shape in terms of h.
6. Draw a kite of diagonals 4 centimetres and 12 centimetres long, and find its area.
7. Find the area of the following kite.



Area of an enclosed circle

Activity 8: Recognising the area of an enclosed circle

Resources: Manila paper or other piece of paper, pencil, a razor blade or a pair of scissors, a pair of compasses, a ruler, and a pen.

Individually or in your groups, perform the following tasks:

1. Draw a circle of a convenient radius on a manila paper.
2. Measure the diameter and use it to calculate the circumference of the circle.

3. Cut the circular plane shape from the manila paper. Divide it into twelve equal parts as shown in Figure 12.3.
4. Carefully, arrange all the parts to form a rectangular shape as shown in Figure 12.4.
5. Measure the length and width of the rectangular shape and record your results.
6. Divide the circumference of the circle you obtained in task 2 into equal parts, and then compare your answer with the length of the rectangle. What have you observed?
7. Divide the diameter you measured in task 2 by 2 to get the radius.
8. Compare the radius in task 7 with the width of the rectangle. What have you observed?
9. Find the area of the rectangular shape by using the length and width you have obtained.
10. Use any alternative method you know to find the area of an enclosed circle.
11. Share your findings with the rest of the class through presentation, and make a conclusion with your teacher.

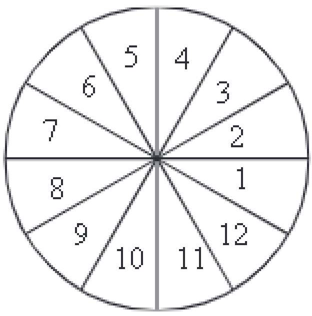


Figure 12.3: *A circle divided into twelve equal parts*

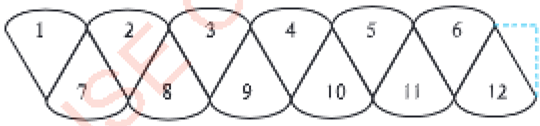


Figure 12.4: *A rectangular shape*

From task 8, you might have observed that the length of the rectangle constructed is half the circumference of the circle. Also, the width of the rectangular shape is the same as the radius of the circle. Thus, the area of an enclosed circle can be obtained by calculating the area of the rectangle constructed.

The area of the parallelogram in Figure 12.4 can be determined by multiplying the base by its height.

In Figure 12.4, it can be noted that,

Height = radius (r),

$$\text{Base} = \frac{1}{2}\,(\text{Circumference of a circle})$$

$$= \frac{1}{2} \times 2\pi r$$

$$= \pi r.$$

Thus, the area enclosed by a circle in Figure 12.3 = base × height.

$$\text{Area} = \pi r \times r$$

$$= \pi r^2 \text{ square units.}$$

Therefore, the area (A) enclosed by a circle of radius r is given by $A = \pi r^2$.

Example 1

Find the area enclosed by a circle whose radius is 7 cm (use $\pi = 3.14$).

Solution

Given; the radius $r = 7$ cm, $\pi = 3.14$, required to find an enclosed area:

$$\text{Area} = \pi r^2$$

$$= 3.14 \times 7 \text{ cm} \times 7 \text{ cm}$$

$$= 153.86 \text{ cm}^2.$$

Therefore, the area enclosed by the circle is 153.86 cm².

Example 2

The area enclosed by a circle is 616 mm². Find its circumference $\left(\text{use } \pi = \frac{22}{7}\right)$.

Solution

Given the area A = 616 mm²

$$\text{Area} = \pi r^2$$

$$616 \text{ mm}^2 = \frac{22}{7} \times r^2$$

$$r^2 = \frac{616 \times 7}{22} \text{ mm}^2$$

$$r^2 = 196 \text{ mm}^2$$

$$\sqrt{r^2} = \sqrt{196 \text{ mm}^2}$$

$$r = 14 \text{ mm}$$

Circumference of a circle (C) = $2\pi r$

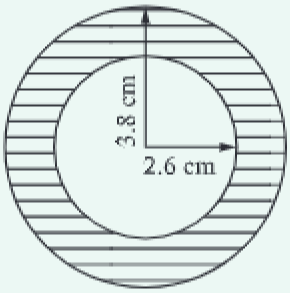
$$C = 2 \times \frac{22}{7} \times 14 \text{ mm}$$

$$C = 88 \text{ mm.}$$

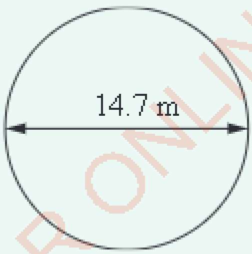
Therefore, the circumference of the circle is 88 mm.

Exercise 7

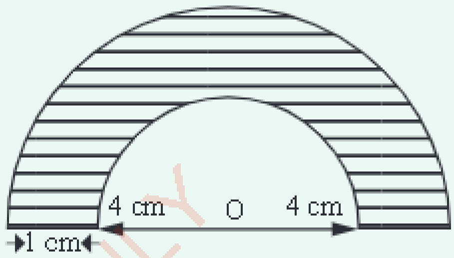
1. Find the area of the shaded region in the following figure (use $\pi = 3.14$).



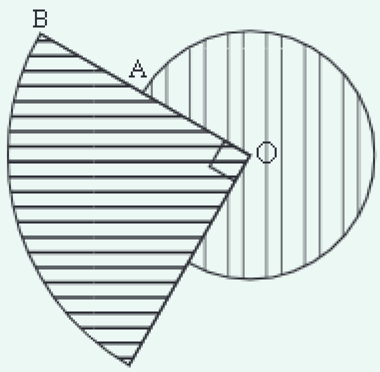
2. The following figure shows a circular fishing pond with a diameter of 14.7 metres. Find its enclosed area (use $\pi = 3.14$).



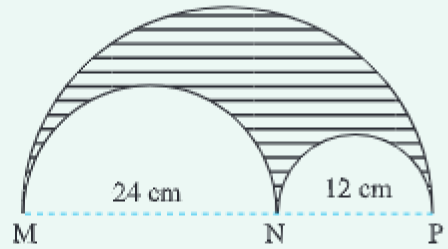
3. Find the enclosed area of the following shaded semi-circular strip with centre O (use $\pi = 3.14$).



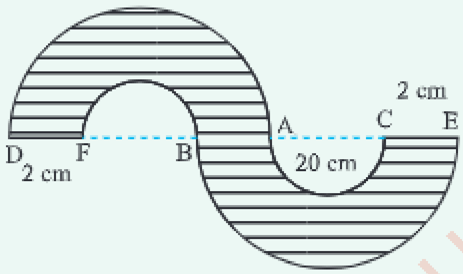
4. In the following figure, O is the common centre. If $\overline{AO} = \overline{AB} = 7$ centimetres, find the enclosed area of the shaded region (use $\pi = 3.14$).



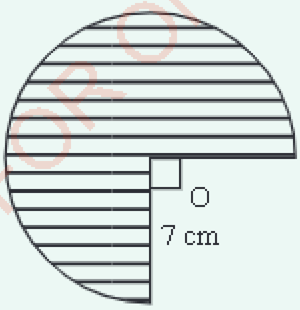
5. The following figure is formed by three semi-circular figures. Find the enclosed area of the shaded region (use $\pi = 3.14$).



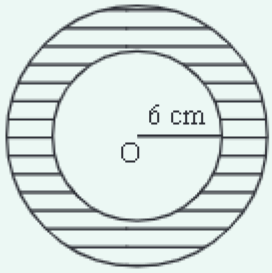
6. Arcs AD, BE, FB, and AC are semi-circles. Find the enclosed area of the shaded region (use $\pi = 3.14$).



7. If O is the centre of the circle, find the enclosed area of the shaded region (use $\pi = 3.14$).



8. Find the enclosed area of the shaded region if the width of the shaded part is 3 centimetres (use $\pi = 3.14$).



9. The enclosed area of a circle is 616 cm². Find its radius $\left(\text{use } \pi = \frac{22}{7}\right)$.

10. Find the circumference of a circle whose enclosed area is 2 464 cm² (use $\pi = 3.14$).

11. The circumference of a circle is 66 decimetres. Find its enclosed area $\left(\text{use } \pi = \frac{22}{7}\right)$.

12. Find the enclosed area between two concentric (same centre) circles of radii 10.5 centimetres and 14 centimetres (use $\pi = 3.14$).

13. Find the enclosed area between two concentric circles of radii 21 centimetres and 14 centimetres (use $\pi = 3.14$).

14. Find the enclosed area of a circle of diameter 154 millimetres (use $\pi = 3.14$).

15. Find the enclosed area of a circle of diameter 280 millimetres $\left(\text{use } \pi = \frac{22}{7}\right)$.
16. Find the enclosed area of a circle whose diameter is 10.5 decimetres (use $\pi = 3.14$).
17. The floor of a circular water tank has a radius of 3.5 metres. Find the enclosed area of the floor (use $\pi = 3.14$).
18. Find the enclosed area of a circle of radius 35 centimetres $\left(\text{use } \pi = \frac{22}{7}\right)$.
19. Find the enclosed area of a circle whose diameter is 700 metres (use $\pi = 3.14$).
20. The enclosed area of a circle is 9 856 mm^2. Find its radius $\left(\text{use } \pi = \frac{22}{7}\right)$.
21. A chord of 48 centimetres is 7 centimetres from the centre of a circle. Calculate the enclosed area of the cirle (use $\pi = 3.14$).

Chapter summary

1. A perimeter is a total length of the sides of any polygon.
2. An area is the number of unit squares that cover the surface of a closed figure.
3. Some units for measurement of area are:
   (a) Square millimetre (mm^2)
   (b) Square centimetre (cm^2)
   (c) Square decimetre (dm^2)
   (d) Square metre (m^2)
   (e) Square decametre (dam^2)
   (f) Square hectometre (hm^2)
   (g) Square kilometre (km^2)

4. The following are the formulae for calculating areas of some shapes:
   (a) Area of a rectangle = length × width = lw.
   (b) Area of a square = length of a side squared = l^2.
   (c) Area of a triangle = $\frac{1}{2}$ × base × height = $\frac{1}{2}bh$.
   (d) Area of a parallelogram = base × height = bh.
   (e) Area of a rhombus = base × height = bh.

(f) Area of a kite $= \frac{1}{2}ab$, where a and b are diagonals of the kite.

(g) Area of a trapezium $= \frac{1}{2}(a + b)h$.

(h) Area of an enclosed circle $= \pi r^2$ or $\frac{\pi d^2}{4}$.

5. The perimeter of a circle is called a circumference.
6. The value of pi (π) is obtained by $\pi = \frac{\text{Circumference}}{\text{Diameter}}$.

Revision exercise

1. Find the perimeter of a triangle of sides 6 centimetres, 9 centimetres, and 13 centimetres.
2. Find the perimeter of a rectangle of sides 21 decimetres and width 19 decimetres.
3. Find the circumference of a circle of radius 15.4 centimetres (use $\pi = 3.14$).
4. Find the circumference of a circle of diameter 10.5 decimetres $\left(\text{use } \pi = \frac{22}{7}\right)$.
5. The circumference of a circle is 176 centimetres. Find the radius of the circle (use $\pi = 3.14$).
6. Find the area of a triangle whose base is 13 centimetres, and its corresponding height is 4 centimetres.
7. Find the area of a right-angled triangle with the height of 10 decimetres, and the base of 24 decimetres.
8. Find the area of a rectangle which is 24 metres long, and 20 metres wide.
9. The width of a rectangle is 40 centimetres. If the area of the rectangle is 1 700 cm^2,
   (a) find the length of the rectangle.
   (b) find the perimeter of the rectangle.
10. The length of a rectangle is twice its width. If the area of the rectangle is 800 dm^2, find its length and width.
11. Find the area of a parallelogram whose base is 12 centimetres, and its height is 8 centimetres.
12. The area of a parallelogram is 105 dm^2. Find the height of the parallelogram if its base is 15 decimetres.
13. Find the area of a trapezium whose parallel sides are 9 centimetres and 13 centimetres, and its height is 11 centimetres.

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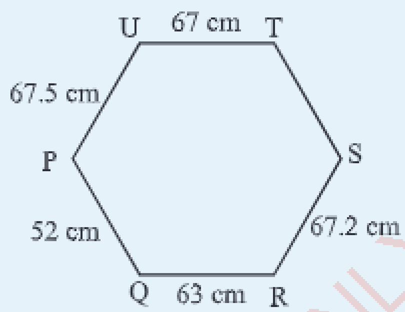
14. The area of a trapezium is 660 m^2. If its height is 40 metres, and the length of one of the parallel sides is 11 metres, find the length of the other side.
15. Find the circumference and area of an enclosed circle of radius 20 decimetres $\left(\text{use } \pi = \frac{22}{7}\right)$.
16. The area of an enclosed circle is 5 400 cm^2. Find its radius (use $\pi = 3.14$).
17. Find the enclosed area between two concentric circles of radii 100 millimetres and 200 millimetres $\left(\text{use } \pi = \frac{22}{7}\right)$.
18. The area of a kite is 96 cm^2. If one of the diagonals is 16 centimetres, find the length of the other diagonal.
19. Find the length of side ST, if the perimeter of PQRSTU is 380 centimetres.



20. January purchased a plot of width 120 metres, and length 360 metres. If he wants to build a fence around the entire plot, how many metres of the materials are needed?

Project

Work in your groups:

1. Collect a rectangular board, nails, a hammer, a pencil, a millimeter tape measure, a meter, a ruler, and coloured threads.
2. Using coloured pencils, draw squares on the board.
3. Hammer nails to every corner of the square.
4. Using the coloured threads, form as many polygons as you can.
5. How could you find the area of each figure on the geoboard?

Answers to Odd-Numbered Questions

Chapter Two

Exercise 1

1. (a) $2 \times 100\,000\,000 + 8 \times 10\,000\,000 + 5 \times 1\,000\,000 + 1 \times 100\,000 + 7 \times 10\,000 + 6 \times 1\,000 + 9 \times 100 + 3 \times 10 + 2 \times 1$

   (b) $8 \times 100\,000\,000 + 6 \times 10\,000\,000 + 2 \times 1\,000\,000 + 5 \times 100\,000 + 5 \times 10\,000 + 4 \times 1\,000 + 9 \times 100 + 1 \times 10 + 7 \times 1$

   (c) $3 \times 100\,000\,000 + 0 \times 10\,000\,000 + 6 \times 1\,000\,000 + 9 \times 100\,000 + 4 \times 10\,000 + 0 \times 1\,000 + 6 \times 100 + 8 \times 10 + 1 \times 1$

3. (a) 978 241 765 (b) 549 847 549 (c) 205 080 495

5. (a) 748 479 768 (b) 907 098 054 (c) 480 999 909

Exercise 2

1. (a) Six hundred seventy-two million one hundred ninety thousand eight hundred and fifty-four.

   (b) Two hundred thirty million four hundred and three thousand two hundred and four.

   (c) Sixty-eight million seven hundred seventy-five thousand one hundred and forty-six.

   (d) Eighty million six hundred ninety thousand and four hundred.

   (e) Four hundred fifteen million nine hundred eighty-two thousand seven hundred and four.

   (f) Nine hundred million.

3. 9 999 5. 111

7. (a) 100 000; one hundred thousand

   (b) 999 999; nine hundred ninety-nine thousand nine hundred and ninety-nine

   (c) 100 000 000; one hundred million

   (d) 999 999 999; nine hundred ninety-nine million nine hundred ninety-nine thousand nine hundred and ninety-nine

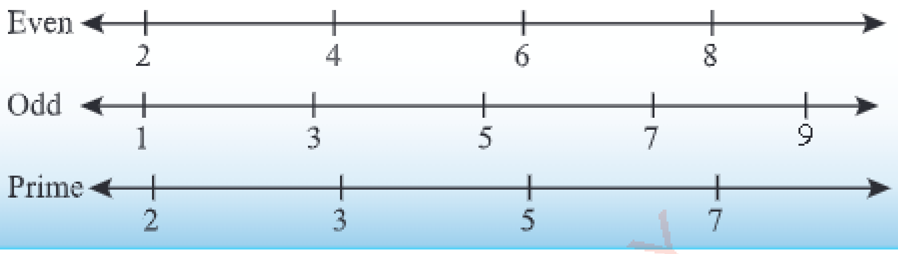
9. (a) 348 740 830 (b) 905 899 572
(c) 346 850 847 (d) 49 206 051

Exercise 3

1. (a) Even: 2, 34, 36, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70.
   (b) Prime: 2, 17, 37, 41, 47, 53, 59, 61, 67.
   (c) Odd: 9, 15, 17, 25, 37, 39, 41, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69.

3. 2

5.



Exercise 4

1. 858 988 898 3. 263 517 478 5. 579 903 054
7. 689 875 598 9. 615 112 101

Exercise 5

1. 935 536 620 3. 235 612 5. 6 854 7. 781 593 760 9. 6 523

Exercise 6

1. True 3. False 5. False 7. 601 166 9. 87 295

Exercise 7

1. 17 523 184 3. 170 5. 498 651 7. 24 124 9. 9 852 332

Exercise 8

1. Tsh 1 021 200 3. Tsh 1 342 500 5. Tsh 246 637 500
7. 29 380 carrot seedlings 9. Tsh 1 637 500 11. Tsh 27 882 000
13. 65 824 iron sheets 15. Tsh 564 550

Exercise 9

1. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ 3. $7 \times 7 \times 7$
5. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$
7. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$
9. (a) 2 and 3 (b) 2 and 3 (c) 2 and 3 (d) 3 (e) 2 and 3 (f) 2, 3, and 7
11. 1, 2, 3, 5, 6, 10, 15, and 30
13. 1, 3, 5, 7, 9, 15, 21, 35, 45, 63, 105, and 315
15. 1, 2, 3, 4, 6, and 12

Exercise 10

1. 180 3. 360 5. 225 7. 288 9. 144
11. 2 250 13. 120 15. 252 17. 100 minutes

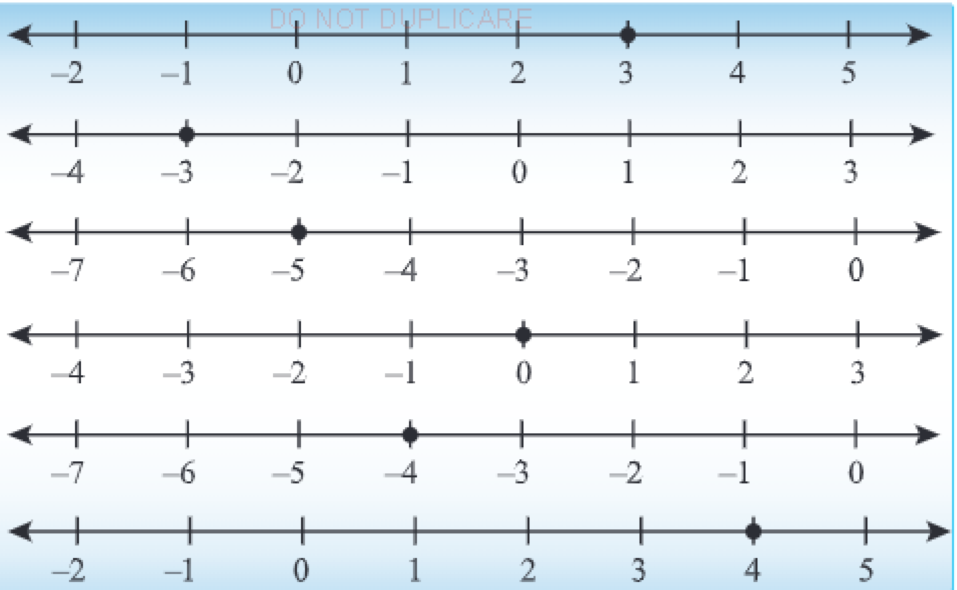
Exercise 11

1. 9 3. 14 5. 2 7. 8 9. 8 11. 6
13. (a) 21 (b) 36 (c) 120 15. 6 cm

Exercise 12

1. (a) (=) (b) (<)
   (c) (>) (d) (<)
   (e) (>) (f) (>)
   (g) (<) (h) (<)
   (i) (>) (j) (=)
3. John is taller than James.

5. (a) (b) (c) (d) (e) (f)



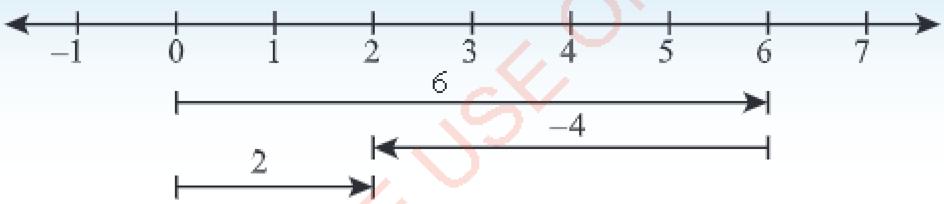
Exercise 13

1. 11 3. 158 5. –32 7. –248

9. 0 11. –3 13. 9 15. –28

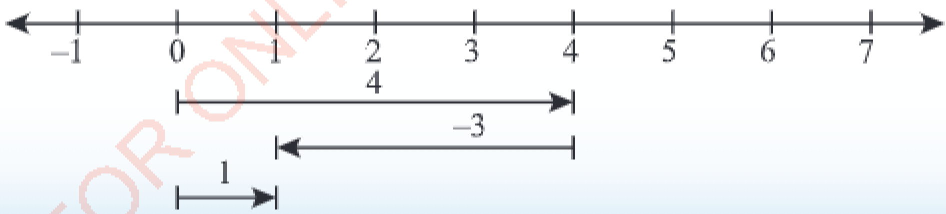
Exercise 14

1.

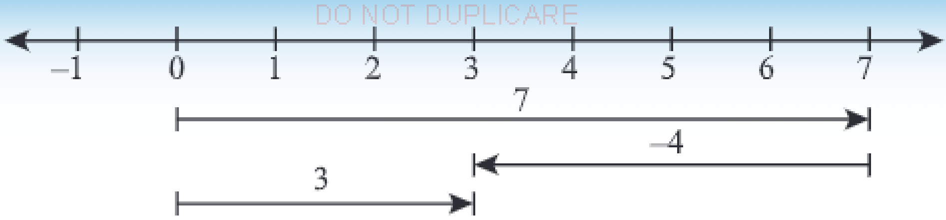


Therefore, 6 – 4 = 2.

3.



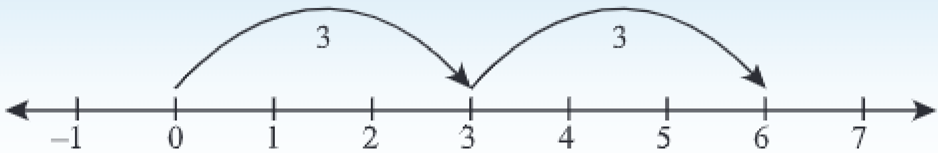
Therefore, 4 + (–3) = 1.

5. 

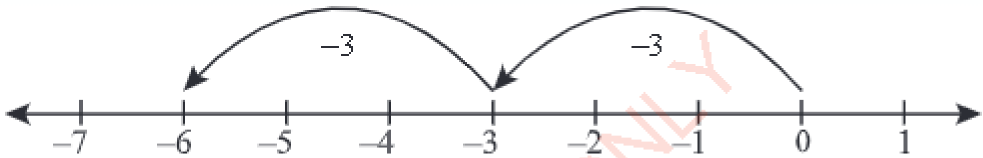
Therefore, 7 – (+4) = 3.

7. 234 9. 46 11. – 18 13. 11 15. –144

Exercise 15

1. 

Therefore, 3 × 2 = 6.

3. 

Therefore, –3 × 2 = –6.

5. 

Therefore, –2 × –3 = 6.

7. 12 9. 1 11. –2 13. 6 296 15. 1 705

Revision exercise

1. Tsh 1 229 000 3. 2

5. (a) 987 654 321 (b) 123 456 789

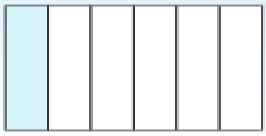
7. (a) 327 900 (b) 837 408 250
(c) 267 175 178 (d) 1 253

9.	726 packets	11. (a) 3	(b) 63	(c) 72 (d) 14
13.	(a) –240	(b) –100		(c) 24
15.	(a) 15	(b) 5		17. Positive (+)
19.	(a) 26	(b) – 152		(c) 369

Chapter Three

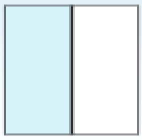
Exercise 1

1. (a)



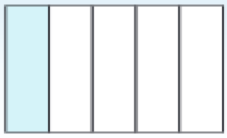
$\frac{1}{6}$

(b)



$\frac{1}{2}$

(c)

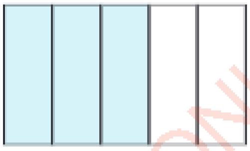


$\frac{1}{5}$

(d)

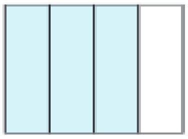
$\frac{1}{4}$

(e)



$\frac{3}{5}$

(f)



$\frac{3}{4}$

(g)

$\frac{3}{8}$

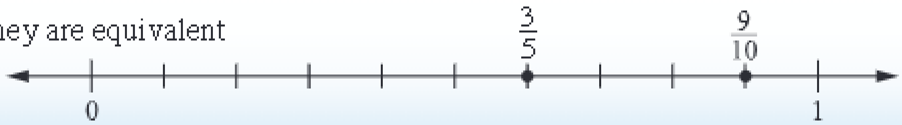
Exercise 2

1. (a) (i) and (ii) (b) (iii), (iv), and (v)

3. (a) $\frac{17}{5}$ (b) $\frac{47}{6}$ (c) $\frac{4279}{9}$ (d) $\frac{15395}{16}$

(e) $\frac{2263}{24}$ (f) $\frac{3208}{21}$ (g) $\frac{359}{19}$ (h) $\frac{9119}{25}$

Exercise 3

1. Proper fractions: (a), (b), (c), (f), (g), (h), (i), (j), (k), and (o)
   Improper fractions: (d), (e), (l), and (m)
   Mixed numbers: (n) and (p).
   The equivalent fractions are: (f), (g), (i), and (j)
3. In figure ABCD, a shaded part is one third and in figure EFGH, a shaded part is one quarter.
5. They are equivalent
7. 

9. (a) $1\frac{1}{2}$ (b) $8\frac{1}{6}$ (c) $51\frac{8}{17}$ (d) $1\frac{776}{1061}$

Exercise 4

1. (a) $\frac{1}{8}, \frac{3}{8}, \frac{3}{4}$ (b) $\frac{16}{41}, \frac{64}{91}, \frac{81}{93}$ (c) $\frac{18}{25}, \frac{84}{93}, \frac{71}{75}$ (d) $\frac{7}{40}, \frac{11}{55}, \frac{16}{30}$
3. (a) $\frac{81}{94}$ is greater than $\frac{3}{564}$ (b) $\frac{48}{155}$ is greater than $\frac{30}{465}$
5. (a) $1\frac{1}{4}$ is less than $3\frac{1}{2}$ (b) $\frac{1}{5}$ is less than $2\frac{1}{7}$

Exercise 5

1. $\frac{1}{2}$ 3. $5\frac{6}{7}$ 5. $\frac{10}{11}$ 7. $\frac{10}{7}$ or $1\frac{3}{7}$ 9. $12\frac{24}{35}$

11. $\frac{7}{6}$ or $1\frac{1}{6}$ 13. $8\frac{5}{6}$ 15. $\frac{17}{18}$ 17. $\frac{64}{21}$ or $3\frac{1}{21}$ 19. $\frac{248}{35}$ or $7\frac{3}{35}$

Exercise 6

1. $\frac{3}{4}$ 3. $\frac{19}{40}$ 5. $\frac{5}{24}$ 7. $2\frac{6}{7}$

9. $3\frac{1}{4}$ 11. $3\frac{7}{8}$ 13. $10\frac{13}{15}$ 15. $3\frac{5}{9}$

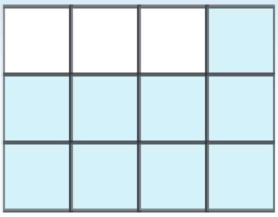
Exercise 7

1. $\frac{4}{15}$ 3. $\frac{9}{20}$ 5. $\frac{1}{12}$ 7. $\frac{2}{9}$

9. 6 11. 14 13. 9 15. $7\frac{1}{3}$

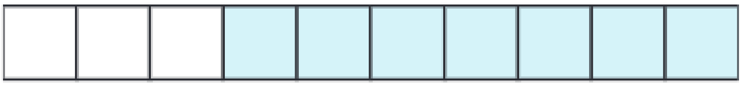
Exercise 8

1.



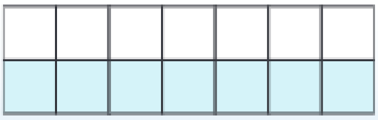
$\frac{1}{3} \times \frac{3}{4} = \frac{3}{12}$ or $\frac{1}{4}$

3.



$\frac{4}{5} \times \frac{3}{8} = \frac{12}{40}$ or $\frac{3}{10}$

5.



$\frac{1}{2} \times \frac{7}{7} = \frac{7}{14}$ or $\frac{1}{2}$

Exercise 9

1. 21   3. 16   5. 49   7. $\frac{9}{5}$ or $1\frac{4}{5}$ 9. $\frac{3}{8}$ 11. $\frac{9}{10}$

13. $\frac{16}{3}$ or $5\frac{1}{3}$ 15. 6   17. $4\frac{37}{48}$ 19. 4   21. $\frac{8}{3}$ or $2\frac{2}{3}$

Exercise 10

1. $\frac{7}{24}$ 3. $\frac{1}{16}$ 5. $\frac{26}{9}$ or $2\frac{8}{9}$

7. $\frac{27}{25}$ or $1\frac{2}{25}$ 9. $\frac{361}{105}$ or $3\frac{46}{105}$

Exercise 11

1. 1 044 cm   3. $13\frac{3}{4}$ kg   5. 184 packets of sweets

7. 126 gm   9. 40 cm

Revision exercise

1. $\frac{17}{10}$ or $1\frac{7}{10}$ 3. $1\frac{1}{2}$ 5. $\frac{29}{15}$ or $1\frac{14}{15}$ 7. $2\frac{31}{55}$

9. $1\frac{17}{18}$ 11. $2\frac{1}{2}$ 13. $2\frac{3}{4}$ 15. $83\frac{5}{12}$ kg

17. $22\frac{1}{2}$ hours 19. Chemistry $= \frac{5}{8}$, History $= \frac{1}{3}$, French $= \frac{5}{12}$ and Biology $= \frac{5}{19}$

Chapter Four

Exercise 1

1. (a) Four point one two eight.
   (b) Three hundred and sixty-seven point zero one.
   (c) Forty-three point four five six eight.

3. (a) $2 \times 10 + 3 \times 1 + \left(1 \times \frac{1}{10}\right) + \left(0 \times \frac{1}{100}\right) + \left(5 \times \frac{1}{1000}\right)$
   (b) $4 \times 1 + \left(5 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{100}\right) + \left(0 \times \frac{1}{1\ 000}\right) + \left(1 \times \frac{1}{10\ 000}\right)$
   (c) $0 \times 1 + \left(7 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{100}\right)$
   (d) $7 \times 1 + \left(4 \times \frac{1}{10}\right)$

Exercise 2

1. (a) 0.125 (b) 0.8333. . . (c) 0.235294. . . (d) 0.25
3. (a) 0.1111... (b) 3.2857. . . (c) 0.142857. . . (d) 5.3

Exercise 3

1. (a) $\frac{19}{40}$ (b) $1\frac{7}{1000}$ (c) $11\frac{1}{100}$ (d) $\frac{7}{20}$
3. (a) $\frac{8}{9}$ (b) $\frac{31}{90}$ (c) $\frac{241}{333}$ (d) $2\frac{4}{9}$

5. (a) $\frac{9}{20}$ (b) $\frac{17}{50}$ (c) $\frac{93}{100}$ (d) $\frac{43}{200}$ (e) $\frac{97}{100}$ (f) $\frac{517}{5000}$

7. (a) $0.\dot{3}$ (b) $0.8\dot{3}$ (c) $0.\dot{3}$ (d) $0.\dot{1}$ (e) $0.\dot{5}3846\dot{1}$

Exercise 4

1. 7.57 3. 47.028 5. 9.13 7. 4.01 9. 1.304

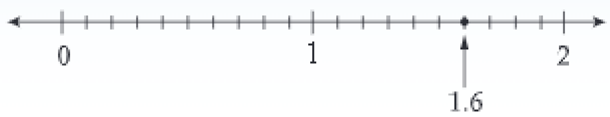
Exercise 5

1. 7 610 3. 0.5 5. 0.105 7. 2.1352 9. 54.5616 11. 22

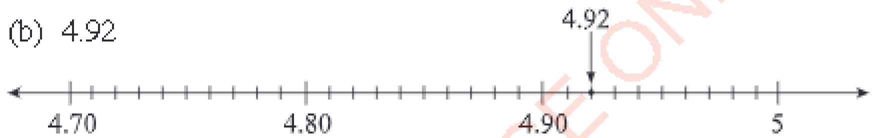
Exercise 6

1.

(a) 1.6

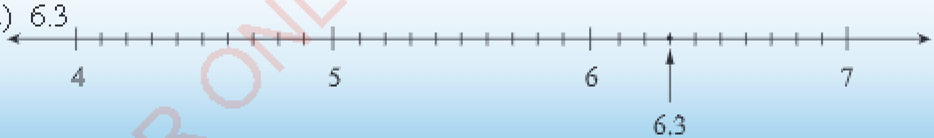


(b) 4.92



(c) 2.235

(d) 6.3



Exercise 7

1. 0.125

3. (a) 0.375 (b) 0.625 (c) 1

5. 300 people

Exercise 8

1. (a) 60% (b) 50% (c) 25% (d) 50% (e) 96%
   (f) 105% (g) 325% (h) 566.7% (i) 875% (j) 44.4%

3. (a) (i) $\frac{7}{20}$ (ii) 0.35 (b) (i) $\frac{3}{5}$ (ii) 0.6 (c) (i) $\frac{1}{4}$ (ii) 0.25
   (d) (i) $\frac{4}{25}$ (ii) 0.16 (e) (i) $\frac{3}{20}$ (ii) 0.15 (f) (i) $\frac{3}{100}$ (ii) 0.03
   (g) (i) $\frac{1}{50}$ (ii) 0.02 (h) (i) $\frac{1}{30}$ (ii) $0.0\dot{3}$ (i) (i) $\frac{49}{50}$ (ii) 0.98

5. 75% 7. (a) 7 820% (b) 340% (c) 400% (d) 75%

Exercise 9

Fractions	$\frac{1}{2}$	$\frac{1}{40}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{2}{3}$	$\frac{3}{4}$
Percentages	50%	$2\frac{1}{2}$%	10%	5%	$66\frac{2}{3}$%	75%
Decimals	0.5	0.025	0.1	0.05	$0.\dot{6}$	0.75

3. He had 40 000 Tanzanian shillings.

Revision exercise

1. (a) two thirds or two over three
   (b) three over five
   (c) one over ten

3. 0.048

5. (a) 0.25 (b) $1.8\dot{3}$ (c) $0.\dot{2}$

7. (a) $\frac{23}{9}$ (b) $\frac{49}{99}$ (b) $\frac{41}{333}$

9. 40%

11. 0.125

13. 93%

Chapter Five

Exercise 1

1. (a) 35 cm (b) 3.5 dm
3. 0.1 m
5. (a) 0.375 dm (b) 0.0375 m
7. (a) 0.9685 hm (b) 0.09685 km (c) 9685 cm
9. (a) 80 metres (b) 8 metres; 80 dm is shorter
11. (a) 687.5 m (b) 6 280 m (c) 28 600 m (d) 862 m
    (e) 68.75 m (f) 8.62 m (g) 0.021008 m (h) 105 m
13. 8 750 000 m, 8 750 m, 18.75 m, 0.000875 m.
15. 970 metres
17. 3 500 m

Exercise 2

1. 23 m 3 dm 6 cm
3. 21 km 23 dam 9 m
5. 26 m 6 dm 4 cm
7. 24 km 25 dam 4 m
9. 24 km 24 m 24 mm
11. (a) 1 487 m (b) 233 km (c) 89.02 m

Exercise 3

1. 10 km 6 hm 5 dam
3. 9 dm 9 cm
5. 8 m 9 dm
7. 1 km 918 m 9 dm
9. 9 km 9 hm 8 dam
11. 0.19 m

Exercise 4

1. 170 m 52 cm
3. 137 dam 5 m
5. 58 dm 9 cm

Exercise 5

1. (a) 10.1 m (b) 2.01 m (c) 2.5 m
   (d) 280.01 m (e) 3 700 m (f) 1375.5 m
3. 19 pieces, 0.0066 m
5. 300 trees
7. 250 cm

Exercise 6

1. (a) 30 000 g (b) 30 000 000 mg
3. 2 000 g
5. 0.006 kg
7. (a) 360 hg (b) 360 000 dg (c) 3 600 dag
9. 80 kg

Exercise 7

1. 23 g 3 dg 6 cg
3. 26 g 6 dg 4 cg
5. 23 hg 3 dag 1 g
7. 24 kg 26 dag 4 g
9. 24 kg 24 g 24 mg

Exercise 8

1. 1 g 8 dg
3. 9 hg 1 dag
5. 901 g 7 dg
7. 1 dag 90 dg 93 mg
9. 4 hg 9 dag

Exercise 9

1. 210 t
3. 137 dag 5 g
5. 58 dg 9 cg
7. 961 kg 800 g
9. 647 kg 240 g

Exercise 10

1. (a) 10 g 1 dg (b) 2 g 1 cg
3. 15 spoons of sugar
5. 120 g

Exercise 11

1. (a) 0630 hours (b) 1737 hours (c) 1520 hours
   (d) 1053 hours (e) 2250 hours (f) 2145 hours
3. (a) Twenty minutes to nine at night.
   (b) Eleven minutes to twelve midnight.
   (c) Fifteen minutes to six in the evening.
   (d) Twenty-four minutes to seven in the evening.
   (e) One o'clock in the night.

Exercise 12

1. 336 hours
3. 8 784 hours
5. 18 840 hours
7. 17 544 hours
9. 672 hours
11. (a) 432 000 s (b) 604 800 s (c) 43 200 s

Exercise 13

1. 9 hrs 16 min
3. 17 hrs 58 min 24 sec
5. 17 hrs 57 min

Exercise 14

1. Anna by 0.05 litres
3. 50 bottles
5. (a) 7 cows (b) 11 700 Tanzanian shillings

Exercise 15

1. (b) millilitre (d) litre (e) kilolitre
3. (a) 4.8 litres (b) 3 600 litres (c) 5.64 litres
5. There are 10 000 cl in 1 hl

Revision exercise

1. 9 kg 243 g
3. 1 tonne 993 kg
5. 6 m 15 cm
7. 999 dam
9. (a) 1 kg 50 g (b) 0.8 km 4 m (c) 4 min 20 s
11. 52 years
13. 92 days
15. 494.4 km
17. Yes, it can fit the length of a shape, the length of the space left is 8 mm.

Chapter Six

Exercise 1

1. (a) 8 000 (b) 12 000 (c) 14 000
   (d) 101 000 (e) 18 000 (f) 2 350 000
   (g) 61 000 (h) 10 000 (i) 1 235 000

3. (a) 30 (b) 20 (c) 30 (d) 10
   (e) 0 (f) 30 (g) 20 (h) 310

5. 237 000

7. (a) $157.8823529 \approx 157.9$ (b) $10.03255814 \approx 10.0$
   (c) 0.3 (d) $294.5 \approx 294.5$

9. (a) 688.9341

(b)

6	8	8	.	9	3	4	1
Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten thousandths

(c) (i) 688.93 (ii) 688.9 (iii) 700

Exercise 2

1. 1200 3. 4 800 5. 750 7. 800 9. $0.8\dot{3}$

11. Tsh1 200 000 13. (a) 0.1 (b) 60 (c) 2 000 000 (d) 3 (e) 500

Exercise 3

1. 0.285 3. 2.01 5. 10.7
7. (a) 1.923 (b) 0.714
9. (a) 712 000 (b) 24.7 (c) 133 (d) 102
11. (a) 7.333 (b) 7.8 (c) 0.4896

Exercise 4

1.	(a) 0.08	(b) 5.07	(c) 1.70
	(d) 0.72	(e) 12.05	(f) 3.35
	(g) 3.61	(h) 0.01	(i) 72.79
3.	(a) 3.1	(b) 0.7	(c) 250.7
	(d) 0.1	(e) 0.7	(f) 10.4
5.	18.03		
7.	(a) 0.0028	(b) 20.04	(c) 0.172
	(d) 6.010	(e) 2.1	
9.	(a) 4	(b) 7	(c) 6

Revision exercise

1. (a) 8.65 (b) 1.05 (c) 0.34
   (d) 31.78 (e) 19.67 (f) 0.45

3. (a) 5 000 000 (b) 5 268 000 (c) 5 267 900

5. 2 800 7. 100 9. 35.82 11. 240 13. 252

15. (a) 86.5 (b) 86.46 (c) 90 (d) 86

17. (a) (i) 17.8 (ii) 17.1 (iii) 2.0 (iv) 0.0
    (b) (i) 23.75 (ii) 23.08 (iii) 0.05 (iv) 0.08

19. (a) 0.69 (b) 0.95 (c) 0.09

21. (a) 35.00 (b) 35.0

Chapter Seven

Exercise 1

1. 3 line segments

3. (a) $\overrightarrow{BA}$ (b) $\overrightarrow{DA}$, $\overrightarrow{AC}$, $\overrightarrow{DC}$

5. There are 16 line segments.
   $\overline{AF}$, $\overline{AB}$, $\overline{BC}$, $\overline{AC}$, $\overline{AG}$, $\overline{GF}$, $\overline{AD}$, $\overline{GD}$, $\overline{DB}$, $\overline{GB}$, $\overline{GE}$, $\overline{EC}$, $\overline{GC}$, $\overline{FE}$, $\overline{EB}$, $\overline{FB}$

7. (a) 3 line segments (b) 6 line segments
(c) 10 line segments (d) 15 line segments

9. (a) A ray has one end point
(b) A line has no end points
(c) A line segment has two end points

11. Flat surfaces: (a) and (c)

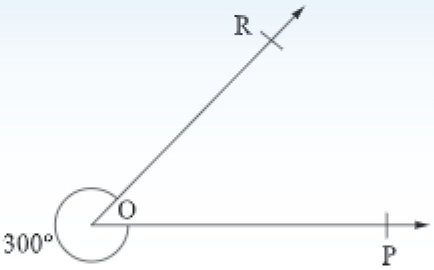
Exercise 2

1. Obtuse angle 3. Right angle (90°) 5. Acute angle

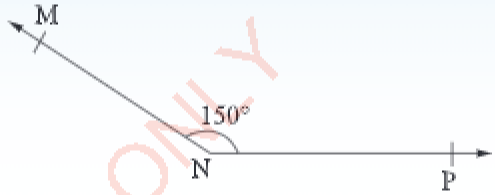
7. Acute angle 9. Obtuse angle

Exercise 3

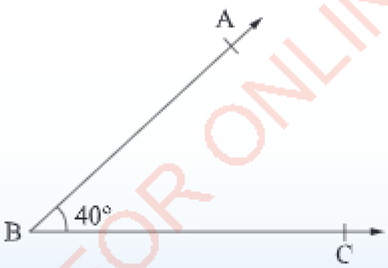
1. (a) Reflex angle



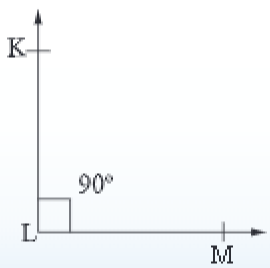
(b) Obtuse angle



(c) Acute angle



(d) Right angle

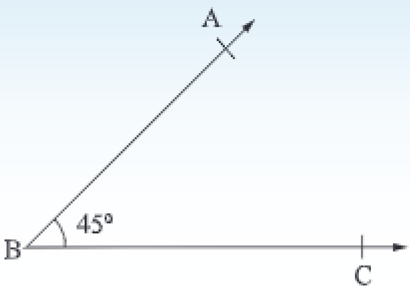


3. (a) 97° (b) 330° 5. $y = 130°$

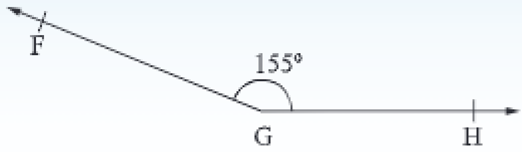
7. $y = 80°$ 9. $y = 30°$

Exercise 4

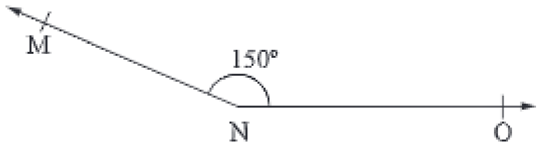
1.



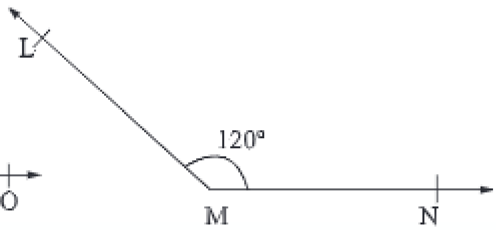
3.



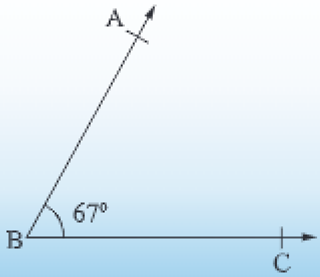
5.



7.



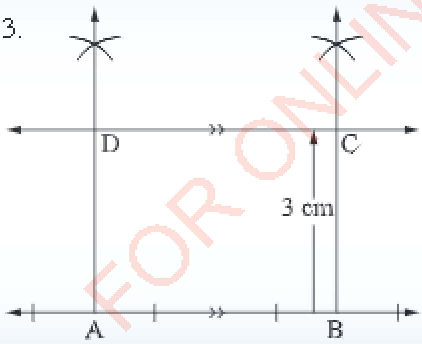
9.



Exercise 5

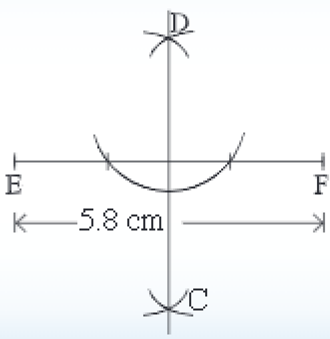
1. 8 lines

3.



Therefore, $\overline{AB}//\overline{CD}$ so that $\overline{BC}//\overline{AD}$

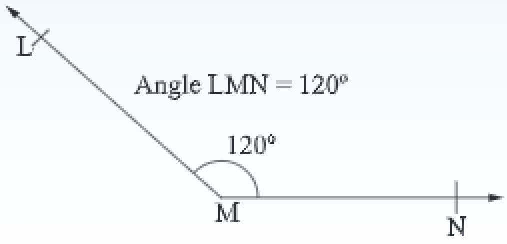
5.



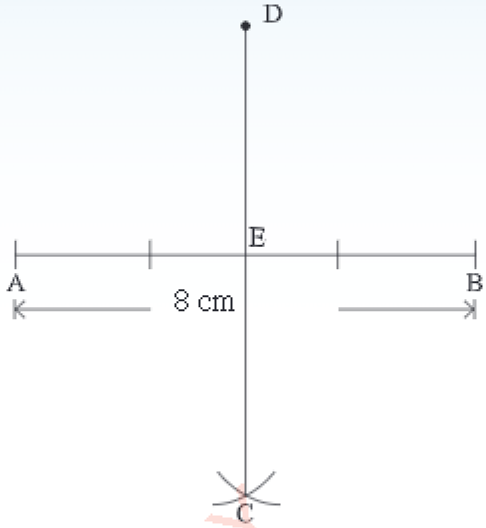
Exercise 6

1. (a) Vertically opposite angles: a and c, b and d, h and f, e and g
   (b) Corresponding angles: h and d, e and a, g and c, f and b
   (c) Alternate interior angles: e and c, f and d
   (d) Alternate exterior angles: h and b, a and g

3.

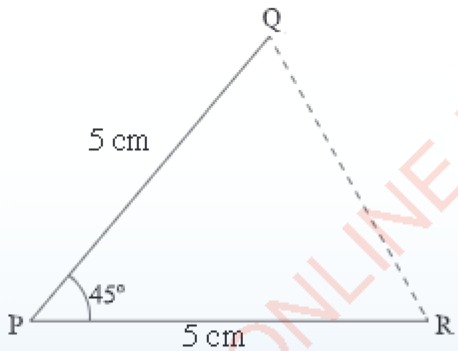


5.



$\overline{AB}$ = 8 cm, $\overline{AE}$ = $\overline{EB}$ = 4 cm

7.



   (a) $\overline{QR}$ = 3.8 cm (b) $P\hat{Q}R$ = 67.5°

9. $c = 70°$

11. (a) $x = 35°$, $h = 120°$, $z = 25°$
    (b) $x = 70°$, $y = 85°$, $k = 25°$

Exercise 7

1. Scalene Δ : (a), (b), (d), and (e)

   Isosceles Δ : (c) and (f)

   Equilateral Δ : None

   Acute angled Δ : (a), (b), (c), (d), and (f)

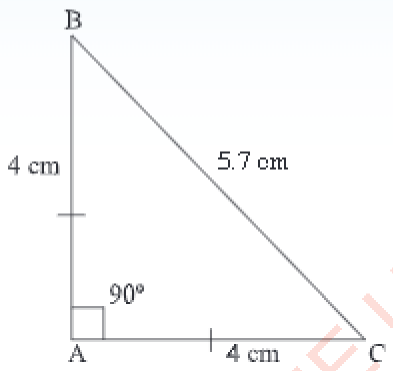
   Obtuse angled Δ : (d)

   Right-angled Δ: (e)

3. 10 vertices, Obtuse triangles: ΔSTR, ΔRXP, ΔRQY, ΔTQV, and ΔSYV

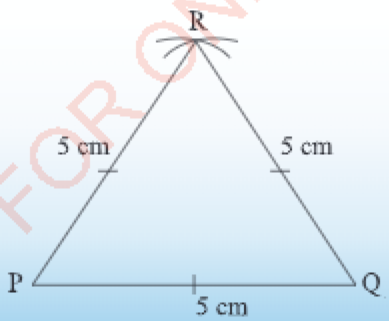
Exercise 8

1. $\overline{AB} = \overline{AC} = 4$ cm, the otherside $\overline{BC} = 5.7$ cm

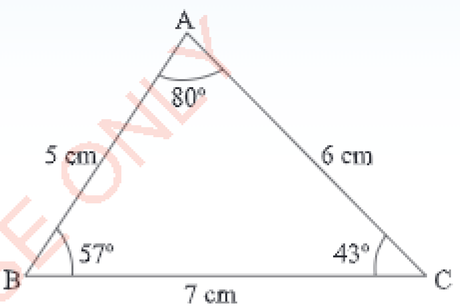


The base angles measure 45°.

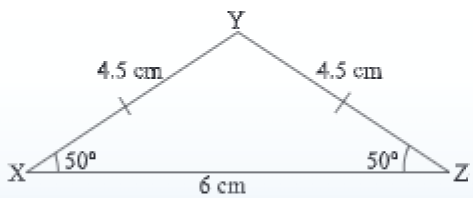
3. $P\hat{Q}R = P\hat{R}Q = R\hat{P}Q = 60°$.



5. The type of the triangle is acute triangle and a scalene triangle.



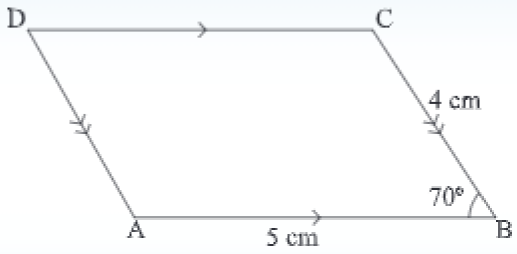
7. (a)



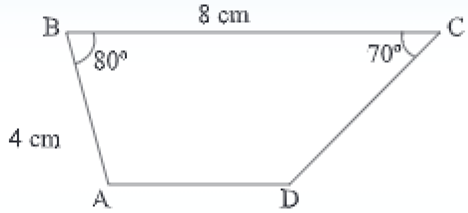
(b) $X\hat{Y}Z = 80°$, $\overline{YZ} = 4.5$ cm

Exercise 9

1.



3.

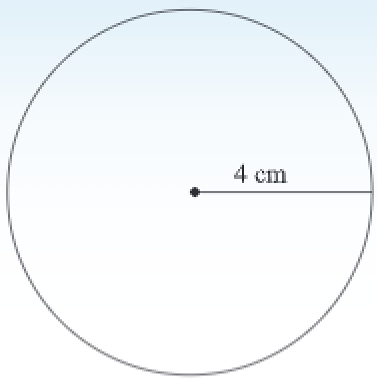


$\overline{AD} = 6$ cm

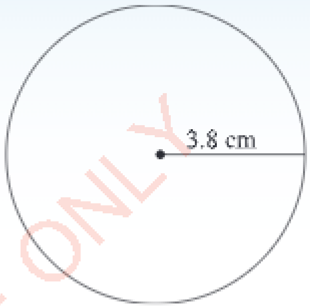
5. $x = 3.3$ cm, $y = 2$ cm, $S\hat{O}R = 90°$

Exercise 10

1. (a) Radius = 4 cm

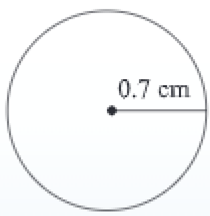


(b) Radius = 3.8 cm

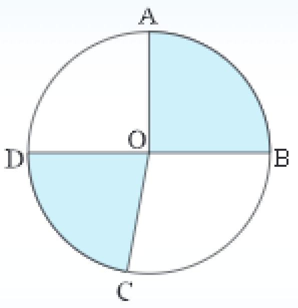


(c) Radius = 5.2 cm

(d) Radius = 0.7 cm

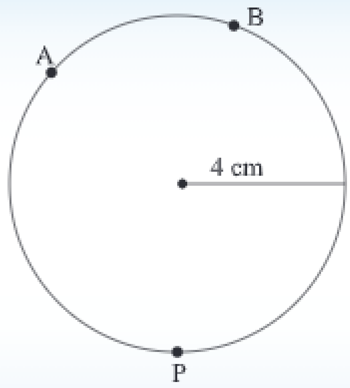


3.



The shaded regions represent:
Sector AOB and sector COD.

5.



Arc AB = $\overset{\frown}{AB}$ is called minor arc.
Arc APB = $\overset{\frown}{APB}$ is called major arc.

Revision exercise

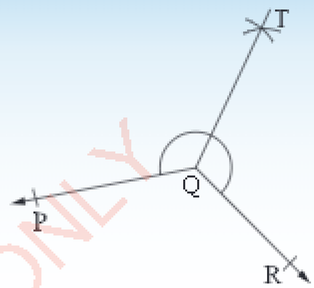
1. A point; a tip of a pin (d)
   A ray; a ray of light (a)
   A line segment; a ruler's edge (b) A surface plane; a top of a table (c)

3. (a) $a = A\hat{D}B$ and $A\hat{B}C = c + d$
   (b) $D\hat{B}C = d$
   (c) $A\hat{D}C = a + b$
   (d) $B\hat{D}C = b$
   (e) $A\hat{B}D = c$

5. (a) $\overline{BE}$ and $\overline{AC}$ or $\overline{AB}$ and $\overline{BC}$ or $\overline{AE}$ and $\overline{EB}$, $\overline{EC}$ and $\overline{CB}$
   (b) $a = 48°$
   (c) $A\hat{E}B = 90°$

7. $E\hat{D}G$ and $G\hat{D}H$, $E\hat{D}F$ and $F\hat{D}H$

9.



$P\hat{Q}T = T\hat{Q}R = 120°$

$P\hat{Q}R = 360° - 120°$
$= 240°$

11. $A\hat{O}D = 130°$, $C\hat{O}B = 130°$, $B\hat{O}D = 50°$

13. $b = 75°$, $c = 105°$

15. Semi-circular arc

17. Yes

19. All angles of square are equal (90° each) while angles of a rhombus are not equal. All diagonals in a square are equal while diagonals in a rhombus are not equal.

21. $a = 40°$, $b = 120°$, $c = 140°$

Chapter Eight

Exercise 1

1. (a) $5n + 3k + 2x$
   (i) There are three terms.
   (ii) The coefficient of n is 5, the coefficient of k is 3, and the coefficient of x is 2.

   (b) $6x - 3y - 5z$
   (i) There are three terms.
   (ii) The coefficient of x is 6, the coefficient of y is -3, and the coefficient of z is -5.

   (c) $10\frac{1}{4}x$
   (i) One term.
   (ii) The coefficient of x is $10\frac{1}{4}$.

3. $8y$	5. $-4w$	7. $6n$	9. $6k$	11. $y+1$
13. -12	15. 29	17. 118	19. -50	21. -3.6

Exercise 2

1. $25x + 30m + 20y$	3. $4p - 10q + 6$
5. $ar - 7br + 9cr$	7. $15ax + 9ay - 9az$
9. $12abx - 4aby - 14abz$	11. $10ax + 5bx + 15cx$
13. $-10np + 16nq + 6n$	15. $8amn - 16bmn + 40cmn$
17. $12auv - 28uv$	19. $-6ax + 2ab$
21. $ax + nm - 2cz$	23. $3(3a - 2b + c)$
25. $5(a + 4b - 2c)$	27. $2a(2t - 3r + 4m)$
29. $2a(6p - q + 4r - 3s)$	

Exercise 3

1. $12h - 20$	3. $4p - 3$	5. $2x - 3y$
7. $\frac{1}{2}b - 4c$	9. $\frac{a + 3b}{8}$	11. $\frac{34y + 28}{15}$

Exercise 4

1. $5x = 20$
3. $21 + x = 125$
5. $99 - x = 63$
7. $2x + 8 = 3x - 7$
9. $5x + 8 = 4(x + 8)$

Exercise 5

1. $x = 13$
3. $x = 5$
5. $x = 6\frac{1}{2}$
7. $x = 6$
9. $x = 10$
11. $x = 10$
13. $x = 3\frac{6}{7}$
15. $x = 10$

Exercise 6

1. 12 years
3. 18 and 19
5. (a) $x = 50°$ (b) $x = 28°$
7. $y = 10$
9. $m = 28$, or $m = 20$
11. 16
13. Daughter's age = 4 years, woman's age = 32 years.
15. 9

Exercise 7

1. $x = 2, y = 1$
3. $x = 4, y = 2$
5. $x = 3, y = 1$
7. $x = 3, y = 2$
9. $x = 31.2, y = 2.4$
11. $x = 2, y = 3$
13. $x = 2, y = 2$
15. $x = 6, y = 10$
17. $x = \frac{14}{9}, y = \frac{-35}{3}$
19. $x = 12, y = 3$
21. $x = \frac{212}{29}, y = -\frac{155}{29}$

Exercise 8

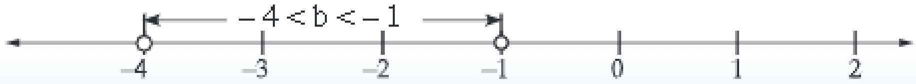
1. 69 and 40
3. 16 girls and 20 boys
5. A pen costs Tsh 800 and a pencil costs Tsh 500
7. 80 and 70
9. $x = \frac{-4}{5}$ and $y = \frac{16}{5}$

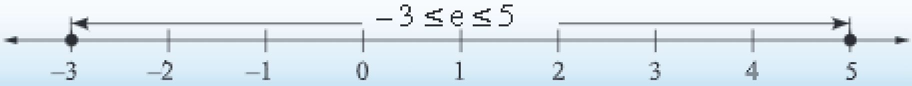
Exercise 9

1. (a), (b), (d), (e), (f), (g), (j), (k) and (n)

3. (a) $x = 1, 2, 3, 4, 5$ (b) $x = 0, 1, 2, 3, 4.$
(c) $x = 5, 7, 9, 11, 13, \ldots$ (d) $x = -2, -1, 0, 1, 2, 3, 4.$

5. (a) $15 > -24$ (b) $-26 < -14$ (c) $-40 < -10$
(d) $-24 < 12$ (e) $-9a < -9b$

7. 

9. 

Exercise 10

1. $w > 5$ cm 3. $x < \frac{2}{3}$ 5. $x \geq 3$ 7. $x < 2$

Revision exercise

1. (a) $16a$ (b) $7m$ (c) $-0.7x + 4.1y$ (d) $5x + \frac{3}{4}$

3. (a) $10m - n$ (b) $9x + y$ (c) $-6k - 5n - 5$ (d) $-5x + 5y$

5. (a) $120abmn$ (b) $-42nm$ (c) $-ax + 6bx$

7. (a) 80 (b) -4 (c) $\frac{15}{2}$ or 7.5

9. (a) $\frac{15}{13}$ (b) $x = -6$ (c) $x = 30$
(d) $x = 10$ (e) $x = \frac{7}{8}$ (f) $x = \frac{5}{2}$ or $2\frac{1}{2}$
(g) $x = \frac{25}{4}$ or $6\frac{1}{4}$ (h) $x = \frac{29}{19}$ or $1\frac{10}{19}$

11. Father 49.5 years; James 16.5 years.

13. (a) $x = 3, y = 6$ (b) $x = 2.2, y = 4.4$ or $x = \frac{11}{5}, y = \frac{22}{5}$
(c) $x = 2, y = 6$ (d) $x = 10, y = 7$

15. (a) $x = 10,\ y = 11$ (b) $x = 8,\ y = 5$

(c) $x = \frac{3}{2},\ y = \frac{5}{4}$ (d) $x = 2,\ y = 6$

17. $x = 15,\ y = 32$

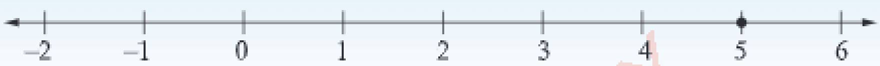
19. (a) $x > 2\frac{1}{2}$ (b) $x \le 1$ (c) $x < -1$ (d) $x \ge \frac{1}{4}$

(e) $x \ge \frac{9}{8}$ (f) $x < -2$

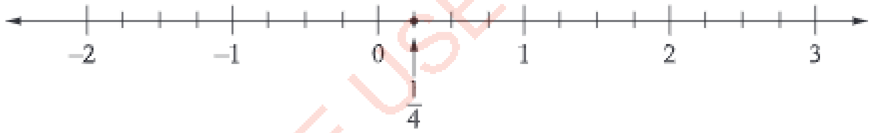
21. $x = 3,\ y = 2.5$ 23. $\frac{3}{6}$ 25. $x + y = 102$

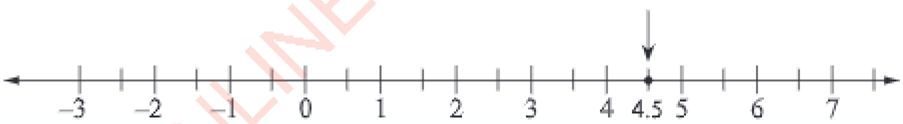
Chapter Nine

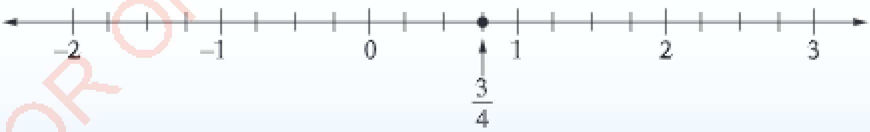
Exercise 1

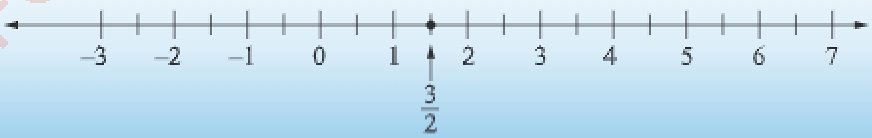
1. (a) 

(b) 

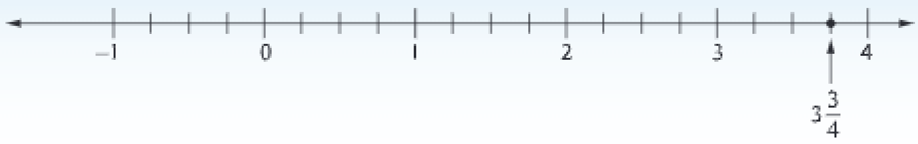
(c) 

(d) 

(e) 

(f) 

(g) 

(h) 

3. (a) 

(b) 

(c) 

5. (a) $\frac{1}{25}$ (b) $\frac{4}{1}$ (c) $-\frac{27}{1}$ (d) $\frac{28}{5}$

(e) $-\frac{3}{100}$ (f) $\frac{17}{4}$ (g) $-\frac{49}{8}$ (h) $-\frac{8}{3}$

Exercise 2

1. (a) $-\frac{1}{6}$ (b) $-1\frac{1}{4}$ (c) 0 (d) $-\frac{11}{12}$

3. (a) 2.8 (b) –1.4 (c) –5.6 (d) 2.2

(e) 1.4 (f) 25.194 (g) 0.875 (h) 19.73

Exercise 3

1. $\frac{56}{45} = 1\frac{11}{45}$ 3. $-\frac{8}{21}$ 5. $3\frac{3}{5}$ 7. $\frac{1}{20}$ 9. $-\frac{7}{180}$

11. –1.9 13. $\frac{4}{5}$ 15. $2\frac{7}{9}$ 17. $-\frac{1}{4}$ 19. 4.848336

Exercise 4

1. $-\frac{12}{17}$ 3. –12 5. $-\frac{14}{17}$ 7. $\frac{11}{12}$ 9. 111

11. $\frac{7}{6}$ or $1\frac{1}{6}$ 13. $-\frac{3}{2}$ 15. $\frac{31}{7}$ or $4\frac{3}{7}$ 17. $-\frac{41}{220}$

19. $-12\frac{3}{5}$ or $-\frac{63}{5}$

Exercise 5

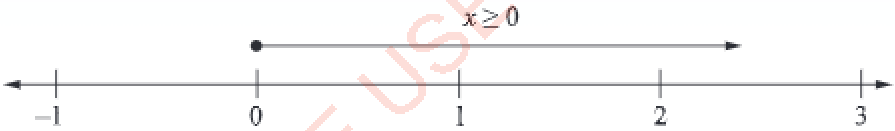
1. (a) $\frac{7}{8}$ (b) $\frac{59}{40}$ (c) $\frac{226}{495}$ (d) $\frac{191257}{9999}$ (e) $\frac{78}{99}$

3. (i) Whole numbers are: (d), (l), (m), and (q).
   (ii) Integers are: (d), (e), (g), (l), (m), and (q).
   (iii) Rational numbers are: (b), (c), (d), (e), (g), (h), (i), (j), (k), (l), (m), (n), (p), and (q).
   (iv) Irrational numbers are: (a), (f), (o), and (r).

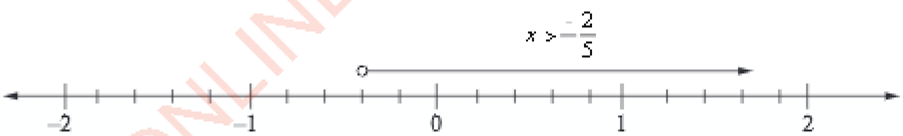
Exercise 6

1. (a) $0.432 < 0.437$ (b) $-0.127 < 0.001$
   (c) $3.724 > 3.716$ (d) $-0.129 < -0.128$
   (e) $\pi > 3.14$ (f) $-\sqrt{7} < -\sqrt{5}$

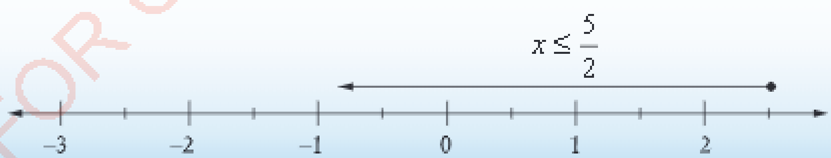
3. (a)



(b)



(c)



Exercise 7

1. (a) 62 (b) $\sqrt{2}$ (c) 8 (d) $\frac{4}{7}$ (e) 12.5 (f) pq

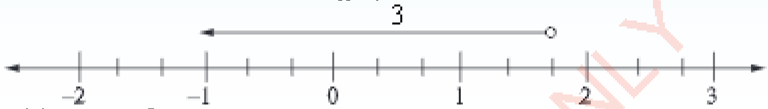
3. (a) $x = 3$ or $x = -3$ (b) $x = 7$ or $x = -3$ (c) $x = 3$ or $x = -3$
(d) $x = 9$ or $x = 11$ (e) $x = 3$ or $x = 9$
(f) $x = 2$ or $x = -\frac{4}{3}$

5. (a) $x > 2$ or $x < -2$ (b) $5 < x < 7$ (c) $-3 \leq x \leq 3$
(d) $-3 \leq x \leq -1$ (e) $-\frac{5}{2} < x < \frac{5}{2}$ (f) $x > 0$ or $x < -4$

Revision exercise

1. (a) $\frac{13}{2}$ (b) $-\frac{102}{11}$ (c) $\frac{81}{99}$ (d) $\frac{387}{2}$ (e) $\frac{23}{1}$
(f) $\frac{2207}{9}$ (g) $-\frac{43}{10}$ (h) $\frac{5174}{45}$ (i) $\frac{3}{250}$ (j) $\frac{123}{999}$

3. $x < \frac{5}{3}$

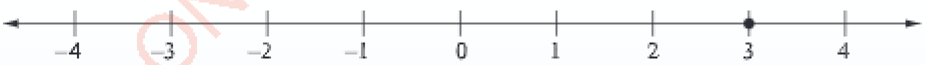


5. $-\frac{11}{4} < x \leq \frac{3}{4}$

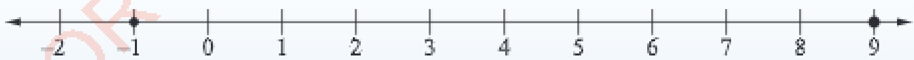
7. (a) $\frac{19}{8}$ or $2\frac{3}{8}$ (b) $-\frac{7}{12}$ (c) $-\frac{20}{33}$
(d) $8\frac{1}{2}$ or $\frac{17}{2}$ (e) $-\frac{282}{175}$ or $-1\frac{107}{175}$
(f) $7\frac{9}{22}$ (g) $3\frac{11}{13}$ (h) $3\frac{34}{67}$ (i) $-\frac{4}{15}$

9. (a) > (b) > (c) < (d) = (e) > (f) >

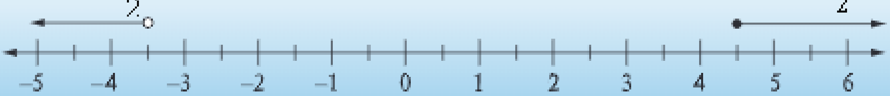
11. (a)

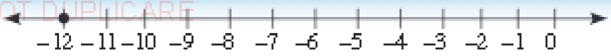


(b)



(c) $x < -3\frac{1}{2}$ $x > 4\frac{1}{2}$



13. Tsh 240 000
17. Tsh 850 500
15. 

Chapter Ten

Exercise 1

1.	2 : 3	3.	2 : 1	5.	4 : 3	7.	16 : 9
9.	28 : 1	11.	7 : 4	13.	17 : 100		
15.	1000 : 1	17.	34 : 1	19.	3 : 2		
21.	25 : 30 = 5 : 6	23.	24 : 8 = 21 : 7	25.	$\frac{4}{7} = \frac{20}{35}$		
27.	12 : 28 = 3 : 7	29.	3 : 10	31.	15 : 12 : 10		

33. First part is Tsh 12 000, second part is Tsh 5 000, third part is Tsh 20 000

Exercise 2

1. (a) 70, 30 (b) 45, 30 (c) 6, 6, 4 (d) $\frac{2}{5}, \frac{1}{10}$

3. Juma - Tsh 30 000, Ali - Tsh 18 000 and John - Tsh 12 000.

5. 2.1 kg 7. 40 kg of A, 32 kg of B 9. 50 kg

11. 9 100 litres 13. 312, 1 170, 3 900 15. $\frac{9n^2}{2}$

Exercise 3

1. (a) Profit Tsh 600 (b) Loss Tsh 200 (c) Profit Tsh 800
 (d) Profit Tsh 250 (e) Loss Tsh 500

3. (a) Tsh 31 000 (b) Tsh 23 400 (c) Tsh 165 000
 (d) Tsh 639 200 (e) Tsh 74 000

5. 35% 7. Tsh 252 000 9. Tsh 300 000

Exercise 4

1. (a) Tsh 5 200 (b) Tsh 96 600 (c) Tsh 4 800 (d) Tsh 24 000
 (e) Tsh 304 000 (f) Tsh 298 667 (g) Tsh 1 920 000 (h) Tsh 56 250
 (i) Tsh 106 250 (j) Tsh 4 200 000

3. (a) 7 % p.a. (b) 8% p.a. (c) 12% p.a. 5. 14% 7. 4 yrs 5 months

9. Tsh 200 000 11. (a) Tsh 1 080 000 (b) Tsh 7 000 000

13. Tsh 106 250 15. 4 years 3 months

Revision exercise

1. (a) 52.25 years (b) $1\frac{1}{2}$ hours (c) Tsh 1 200 000

3. Tsh 1 824 000 5. $2\frac{1}{2}$ years or 2 years six months

7. 24° : 48°: 120° : 168°

9. P = Tsh 125 000

Chapter Eleven

Exercise 1

1. (a) H(1, 2), I(2, 0), J(0, –3), K(3, –3), L(–1, –2), M(–2, –3), N(–2, 1), Q(–1, 0), P(–1, 3)
   (b) H belongs to the first quadrant, L belongs to the third quadrant
   K belongs to the fourth quadrant N belongs to the second quadrant

3. (a) Quadrant III (b) Quadrant II
   (c) Quadrant IV (d) Quadrant I

Exercise 2

1. (a) 1, positive (b) 7, positive (c) $\frac{2}{3}$, positive
   (d) –1, negative (e) $-\frac{7}{2}$, negative (f) $\frac{7}{5}$, positive
   (g) 0, zero (h) 2, positive (i) undefined
   (j) undefined (k) 0, zero

3. (a) P(3, –2), Q(0, 4), and R(–4.5, –2)
   (b) Gradient of line PQ is –2, gradient of line PR is 0, and the gradient of line QR is $\frac{4}{3}$.
   (c) Isosceles triangle

Exercise 3

1. (a) $5y - 4x = 17$ (b) $5y - 2x = 5$ (c) $3y - 2x = 11$
   (d) $y + x = 7$ (e) $y = 0$ (f) $5y + 2x = -14$
   (g) $4y + 3x = 0$ (h) $3y - 2x = 20$ (i) $7y + 3x = -12$
   (j) $y + x = 0$

3. (a) $y = 2x - 3$ (b) $y = -2x + 5$ (c) $y = 3x - 6$
   (d) $y = \frac{3}{2}x - 1$ (e) $y = -3x$

5. (a) $y = -\frac{7}{4}x + \frac{11}{4}$, $m = -\frac{7}{4}$ and $c = \frac{11}{4}$

(b) $y = -\frac{14}{3}x + 4$, $m = -\frac{14}{3}$ and $c = 4$

(c) $y = 2x - 5$, $m = 2$ and $c = -5$

(d) $y = -\frac{4}{5}x + 8$, $m = -\frac{4}{5}$ and $c = 8$

(e) $y = 8x$, $m = 8$ and $c = 0$

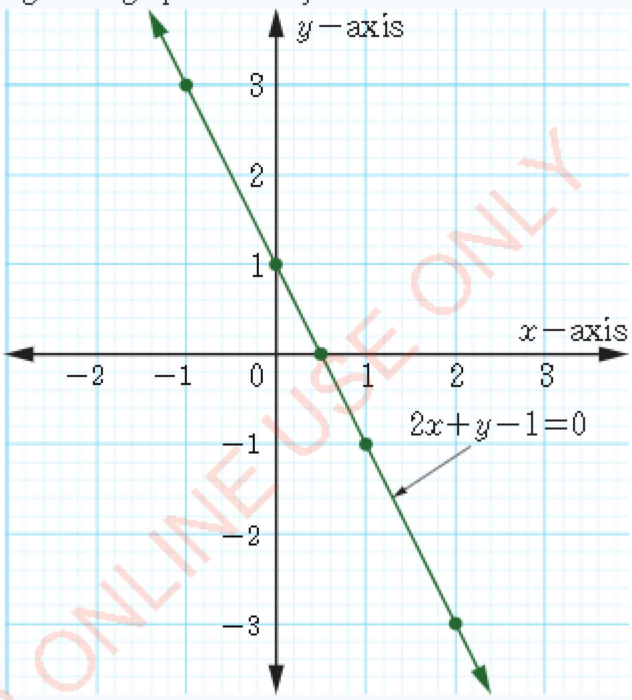
(f) $y = 3x + \frac{70}{13}$, $m = 3$ and $c = \frac{70}{13}$

Exercise 4

1. Table of values for $2x + y - 1 = 0$

x	–2	–1	0	1	2
y	5	3	1	–1	–3

The following is the graph of $2x + y - 1 = 0$.



The graph meet the coordinate axes at (0, 1) and (0.5, 0).

3. (a) Gradient of line CD is 0

Gradient of line BC is undefined

(b) $y = 3$ and $x = 4$

(c) $y = \frac{-3}{4}x + 3$, slope $= \frac{-3}{4}$, y–intercept = 3, x–intercept = 4,

Exercise 5

1. Solution is (1, 2),
   that is $x = 1$, $y = 2$

   Verification:

   (i) $2x + y = 4$

   $2(1) + 2 = 4$

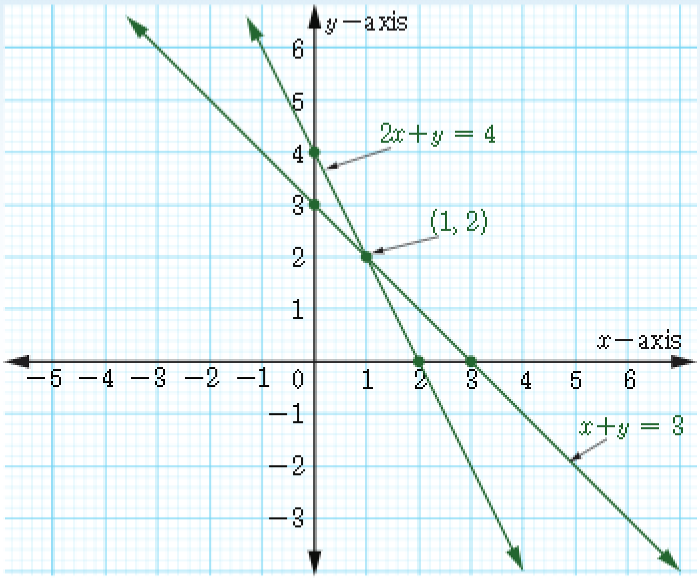
   $4 = 4$

   (ii) $x + y = 3$

   $1 + 2 = 3$

   $3 = 3$

   Thus, the point (1,2) satisfies both equations



3. Solution is (–2, 3)
   that is $x = -2$, $y = 3$

   Verification

   (i) $2x + 3y = 5$

   $2(-2) + 3(3) = 5$

   $-4 + 9 = 5$

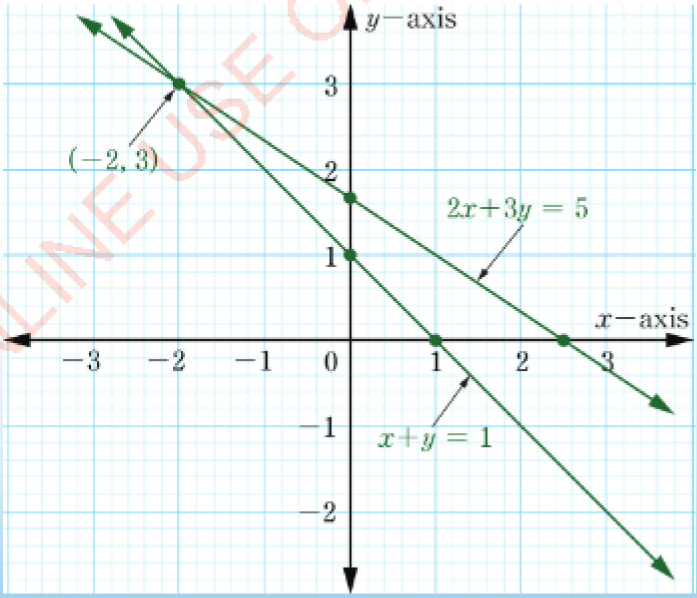
   $5 = 5$

   (ii) $x + y = 1$

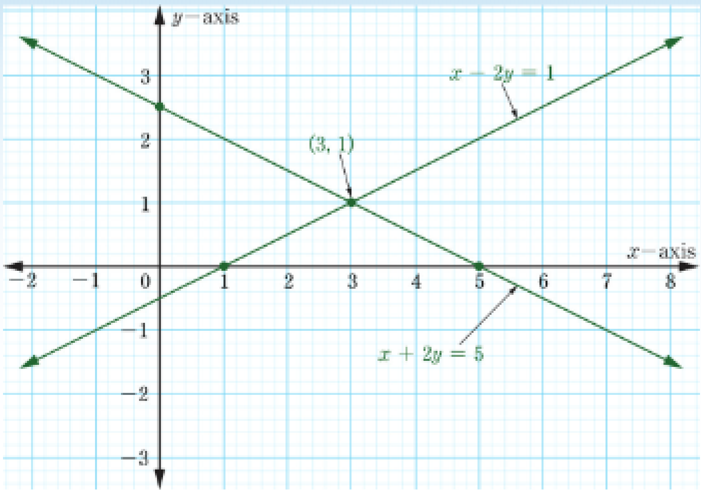
   $-2 + 3 = 1$

   $1 = 1$

   Thus, the point (–2,3) satisfies both equations



5. Solution is (3, 1)
that is $x = 3,\ y = 1$



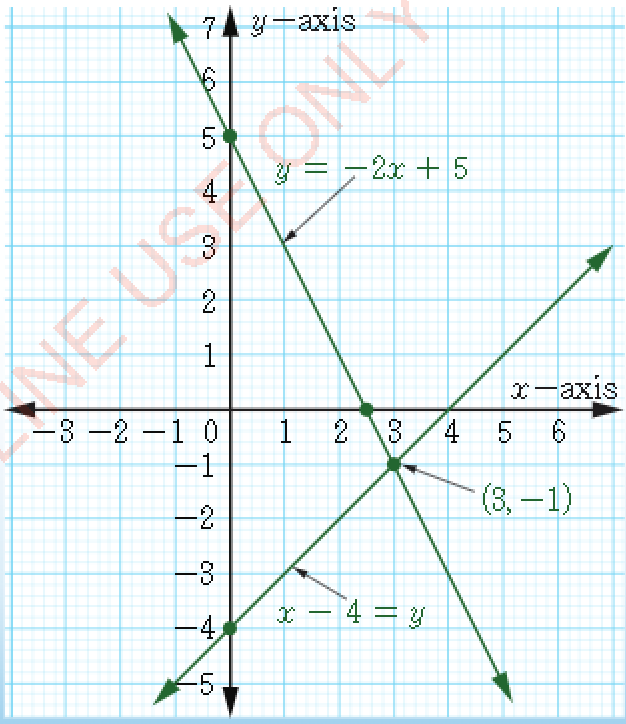
Verification

(i) $x + 2y = 5$
$3 + 2(1) = 5$
$3 + 2 = 5$
$5 = 5$

(ii) $x - 2y = 1$
$3 - 2(1) = 1$
$3 - 2 = 1$
$1 = 1$

Thus, the point (3,1) satisfies both equations

7. Solution = (3, −1),
that is, $x = 3,\ y = -1$



Verification

(i) $x - 4 = y$
$3 - 4 = -1$
$-1 = -1$

(ii) $y = -2x + 5$
$-1 = -2(3) + 5$
$-1 = -6 + 5$
$-1 = -1$

Thus, the point (3,−1) satisfies both equations

9. Solution is (2, 5),
that is, $x = 2$, $y = 5$

Verification

(i) $y + 1 = 3x$

$5 + 1 = 3(2)$

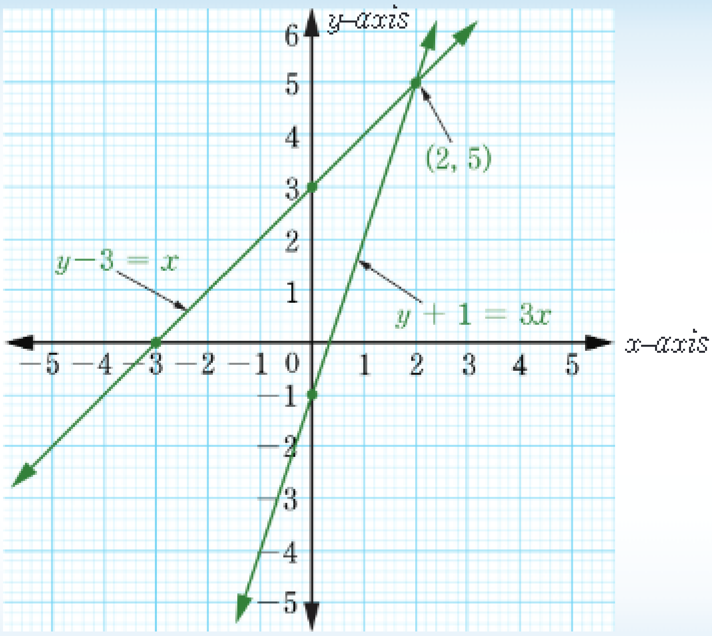
$6 = 6$

(ii) $y - 3 = x$

$5 - 3 = 2$

$2 = 2$

Thus, the point (2,5) satisfies both equations



Revision exercise

1. (a) $m = 3$, positive (b) $m = 1$, positive (c) $m = -1$, negative

(d) $m = \frac{2}{5}$, positive (e) $m = -\frac{1}{4}$, negative (f) $m = -1$, negative

3. (a) $y = -4x + 8$, $m = -4$, $c = 8$

(b) $y = \frac{1}{6}x + \frac{1}{3}$, $m = \frac{1}{6}$, $c = \frac{1}{3}$

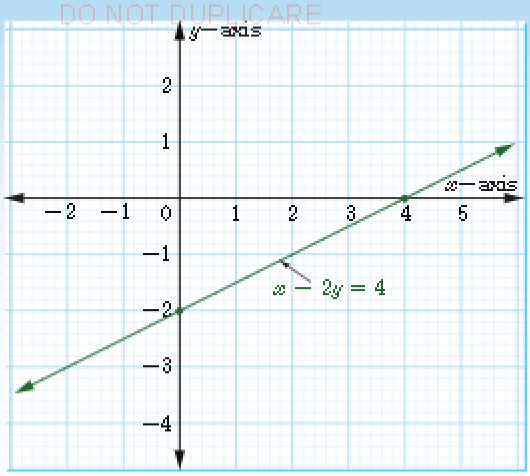
(c) $y = 2x - 2$, $m = 2$, $c = -2$

(d) $y = -\frac{3}{4}x + \frac{23}{4}$, $m = -\frac{3}{4}$, $c = \frac{23}{4}$

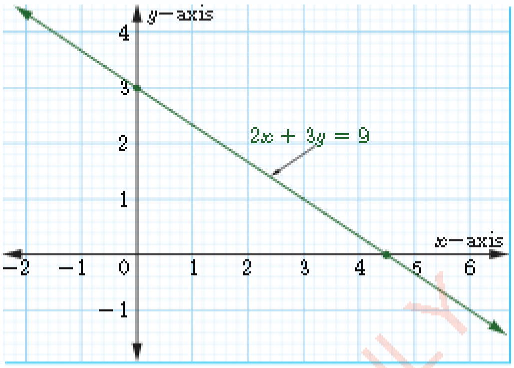
(e) $y = -\frac{3}{4}x - 1$, $m = -\frac{3}{4}$, $c = -1$

(f) $y = -\frac{4}{3}x - 3$, $m = -\frac{4}{3}$, $c = -3$

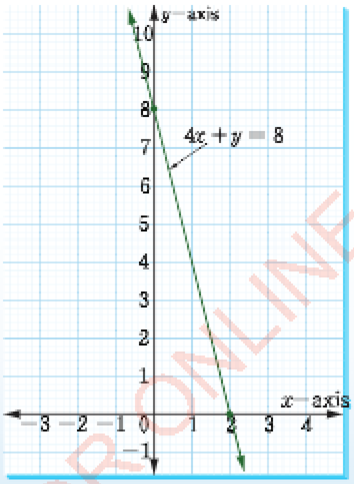
5. (a)



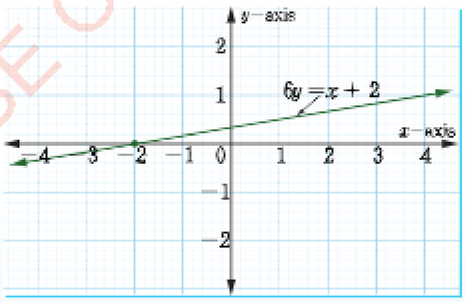
(b)



(c)



(d)



7. (a) $y = -\frac{4}{3}x - 2$ (b) $y = -6x + 6$

(c) $y = -\frac{3}{2}x - 3$ (d) $y = -\frac{3}{2}x$

9. (a) (1, 0), (0, 0), Horizontal
   (b) (1, 2), (2, 1), Neither vertical nor horizontal
   (c) (3, –2), (3, 5), Vertical
   (d) (– 4, 2), (– 4, –1), Vertical
   (e) (2, 8), (–2, 8), Horizontal
   (f) $\left(\frac{3}{4}, 1\right)$, $\left(\frac{7}{3}, 4\right)$, Neither vertical nor horizontal
   (g) (0.5, –1), (0.5, –2), Vertical
   (h) (0,2), (0, –3), Vertical

11. (a) $y = \frac{2}{7}x + \frac{20}{7}$, y–intercept $= \frac{20}{7}$
    (b) $y = \frac{5}{9}x + \frac{53}{9}$, y–intercept $= \frac{53}{9}$
    (c) $y = -\frac{11}{36}x + \frac{427}{186}$, y–intercept $= \frac{487}{186}$
    (d) $y = \frac{1}{3}x - \frac{1}{3}$, y–intercept $= -\frac{1}{3}$
    (e) $y = 6$, y–intercept $= 6$
    (f) $y = x - 386$, y–intercept $= -386$

Chapter Twelve

Exercise 1

1.	19 cm	3.	16.55 mm	5.	18.3 mm
7.	21.7 dm	9.	36 m	11.	70 dm
13.	60 m	15.	16 m	17.	4 cm

19. Length = 35 m and width = 12 m

Exercise 2

1. 65.94 mm 3. 109.9 mm 5. 95.8328 mm
7. 94.2 cm 9. 150.72 cm 11. 4 m
13. 60 cm 15. 3.85 dm 17. 21 dm 19. 40.82 cm

Exercise 3

1. 16 square units 3. 6 square units 5. 13 square units

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Exercise 4

1.	154 cm^2	3.	14 dm^2	5.	17.64 m^2
7.	120 cm^2	9.	35.2 cm^2	11.	7 125 m^2
13.	144 cm^2	15.	20 cm	17.	7.5 m
19.	1 260 mm^2	21.	6 cm^2		

Exercise 5

1.	52 m^2	3.	1 530 mm^2	5.	8 cm^2
7.	76 cm^2	9.	24 cm^2	11.	7 cm
13.	200 cm^2	15.	160 mm	17.	200 m

Exercise 6

1.	15 cm^2	3.	$\left(\frac{1}{8}x + \frac{1}{16}\right)cm^2$	5.	lh square units
7.	10 cm^2				

Exercise 7

1.	24.1152 cm^2	3.	14.13 cm^2	5.	226.08 cm^2
7.	115.395 cm^2	9.	14 cm	11.	346. 5 dm^2
13.	769.3 cm^2	15.	61 600 mm^2	17.	38.465 m^2
19.	384 650 m^2	21.	1962.5 cm^2		

Revision exercise

1.	28 cm	3.	96.712 cm	5.	28 cm
7.	120 dm^2	9.	(a) 42.5 cm (b) 165 cm	11.	96 cm^2
13.	121 cm^2	15.	125.7 dm, 1257.14 dm^2		
17.	94 286 mm^2	19.	63.3 cm		

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Glossary

Absolute value of a number: The value of a number without regard to its sign.

Algebra: A branch of mathematics that substitutes letters for numbers. It deals with representing numbers through variables.

Angle: The amount of turning.

Arc: A part of a circumference of a circle.

Area: The number of unit squares that cover the surface of a closed figure.

Chord: A line segment connecting two points on the circumference.

Circle: A closed path which is at equal distance from fixed point (centre).

Circular region: A surface bounded by a circle.

Circumference: The perimeter of a circle.

Coefficient: A number that is written along with a variable or it is multiplied by the variable.

Constant: A value or number that never changes in expression, it is constantly the same.

Decimal: A fraction of which its denominator is a multiple of 10, 100, 1 000, and so on.

Denominator: The bottom part of a fraction.

Diameter: A chord of a circle which passes through the centre of the circle.

Difference: An answer obtained after subtracting two numbers.

Digits: The numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Equation: A mathematical sentence with an equal sign (=).

Even number: A number which is exactly divisible by 2.

Integer: A number with no fractional part (no decimals).

Irrational number: Cannot be written in the form of $\frac{a}{b}$, where a and b are integers, but $b \neq 0$.

Line: A set of points which extend in both directions without an end.

Loss: Made by selling an item at a lower price than the buying price.

Natural numbers: Counting numbers starting with 1, 2, 3, 4, 5, . . .

Number line:	A straight line marked with equal intervals, and labelled with numbers.
Numeral:	A symbol which represents a number.
Numerator:	The top part of a fraction.
Odd number:	A number which is not exactly divisible by 2.
Parallelogram:	A quadrilateral which both of its pairs of opposite sides are parallel.
Perimeter:	A length of boundary that surrounds a shape.
Place value:	A position of a digit in a numeral.
Polygon:	A closed plane figure bounded by a finite number of line segments placed end to end successively.
Prime number:	A positive integer, except one, which is divisible by itself and one only.
Profit:	A gain amount from any business activity. It is made by selling an item at a higher price than the buying price.
Product:	An answer obtained by multiplication of two or more numbers or variables.
Proportion:	A statement that two ratios are equal. It is a mathematical comparison between two numbers.
Quotient:	An answer obtained by dividing two numbers.
Ratio:	A comparison by division between two or more quantities which are in the same unit and can be simplified like a fraction.
Rational number:	Any number written in the form of $\frac{a}{b}$, where a and b are both integers except that, $b \neq 0$.
Ray:	A line segment extended in one direction.
Real number:	Any number which is either rational or irrational number.
Reciprocal of a number:	This is 1 divided by that number. Product of a number x and its reciprocal yields 1.
Rectangle:	A parallelogram which all its angles measure 90°.
Rhombus:	A parallelogram whose all sides are equal.
Sector:	A part of a circular region bounded by two radii of a circle and an arc.
Segment:	A plane bounded by a chord of a circle and the intercepted arcs.

Square:	A rectangle with four equal sides and each angle measures 90°.
Sum:	An answer obtained by adding together numbers.
Trapezium:	A quadrilateral with one pair of parallel opposite sides.
Triangle:	A polygon with three edges and three vertices.
Variable:	A symbol used to represent a numerical value that can change. It represents an unknown number or unknown value or unknown quantity.
Whole numbers:	Natural numbers including digit zero.

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ISBN 978-9987-09-290-
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